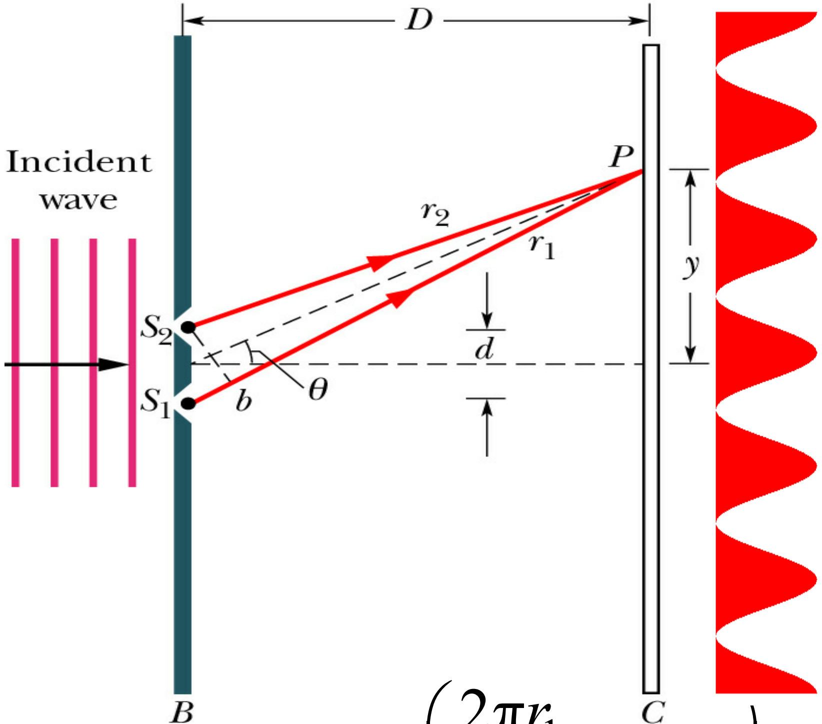
Announcements

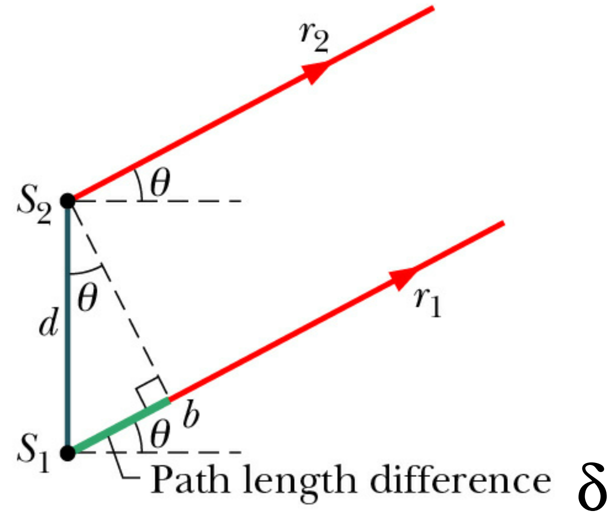
1. Physics colloquium this week – Professor Jonathan Lees, UNC-Chapel Hill "Molten Rock, Singing Fire: Physics of Exploding Volcanoes" . (Next week we will also have a very interesting visitor – Professor Luis Orozco, Distinguished Traveling Lecturer sponsored by the Division of Laser Sciences of APS.)
2. Today's topics

   Interference phenomena in light waves

   Diffraction

Diffraction pattern from a plane wave incident on a double slit:





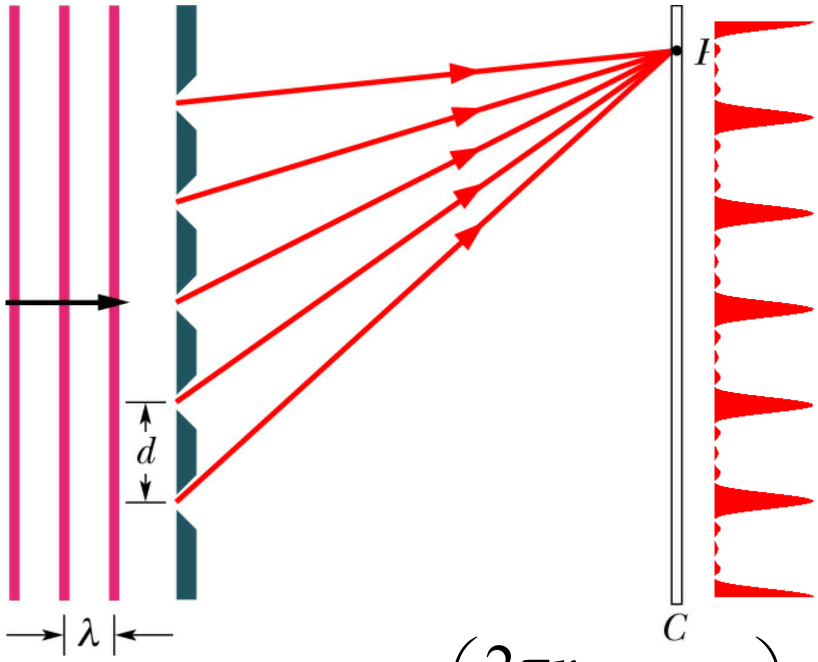
$$r_1 - r_2 = \delta = d \sin\theta$$

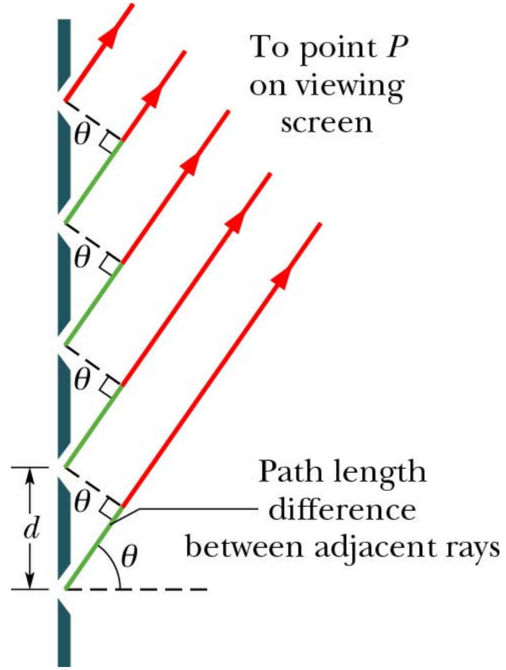
$$E(P,t) = E_{\max} \sin\left(\frac{2\pi r_1}{\lambda} - 2\pi f t\right) + E_{\max} \sin\left(\frac{2\pi r_2}{\lambda} - 2\pi f t\right)$$

$$= 2E_{\max} \sin\left(\frac{\pi(r_1 + r_2)}{\lambda} - 2\pi f t\right) \cos\left(\frac{\pi(r_1 - r_2)}{\lambda}\right)$$

➔ intensity maxima occur for $\frac{\pi(r_1 - r_2)}{\lambda} = m\pi \quad \Rightarrow d \sin\theta = m\lambda$

Diffraction pattern for *N* slits – diffraction grating



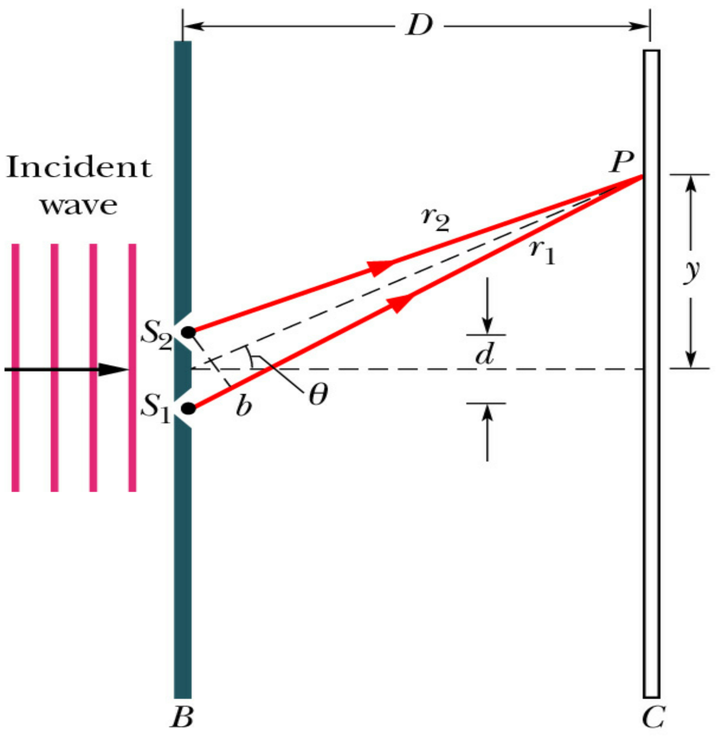


$$E(P,t) = \sum_i E_{\max} \sin\left(\frac{2\pi r_i}{\lambda} - 2\pi f t\right)$$

$$= E_{\max} \sin\left(\frac{2\pi r_{av}}{\lambda} - 2\pi f t\right) \frac{\sin\left(\frac{N\pi d \sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi d \sin\theta}{\lambda}\right)}$$

Intensity maxima at $d \sin\theta = m\lambda$

Summary of results:



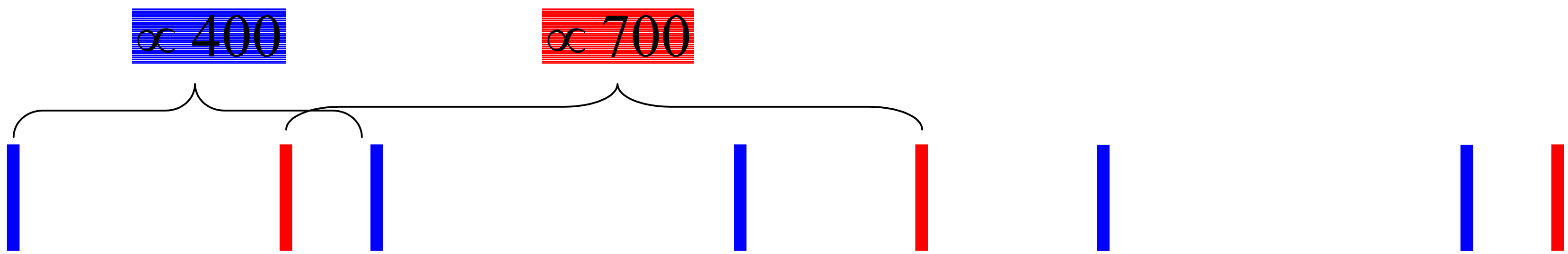
Constructive interference:

$$d \sin\theta = m\lambda$$

Image on screen:

$$y \approx m\lambda \frac{D}{d}$$

Question from last time --



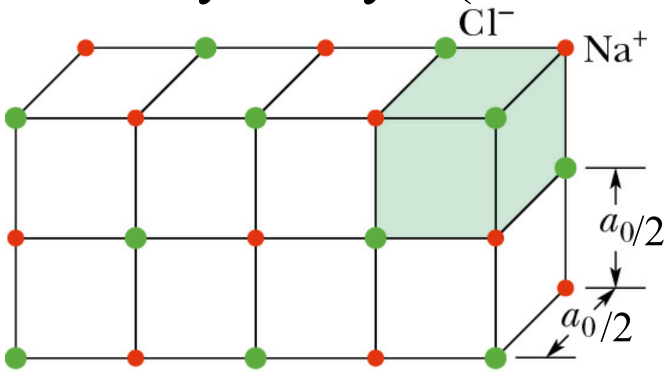
The above lines show the diffraction pattern of blue light from a given grating. Where would you expect red light to be diffracted?

(A) Approximately midway between each blue line

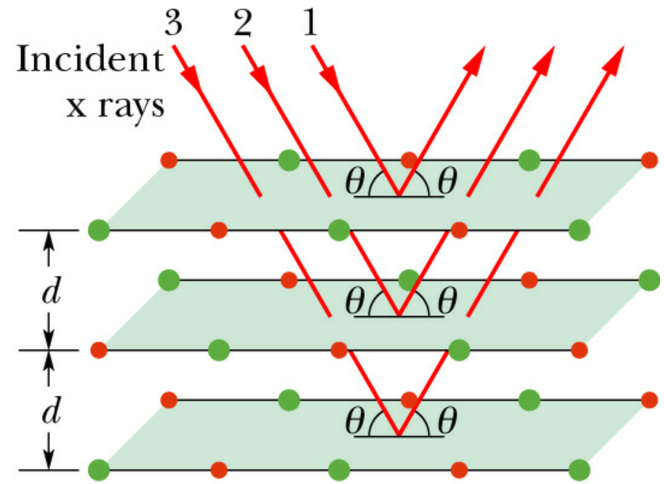
(B) Separation between red lines is greater than that of blue lines

(C) Separation between red lines is smaller than that of blue lines

Diffraction by X-rays ($\lambda \approx 0.1$ nm)

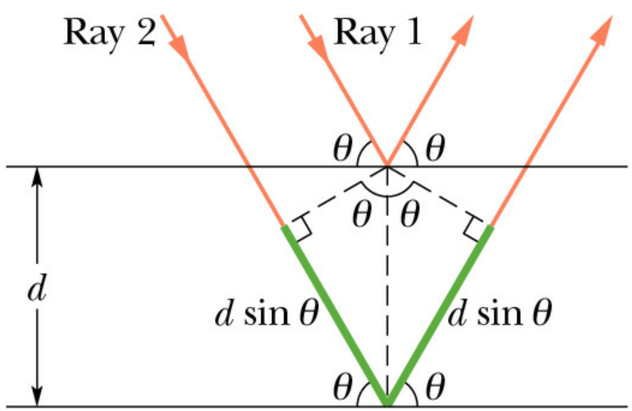


(a)

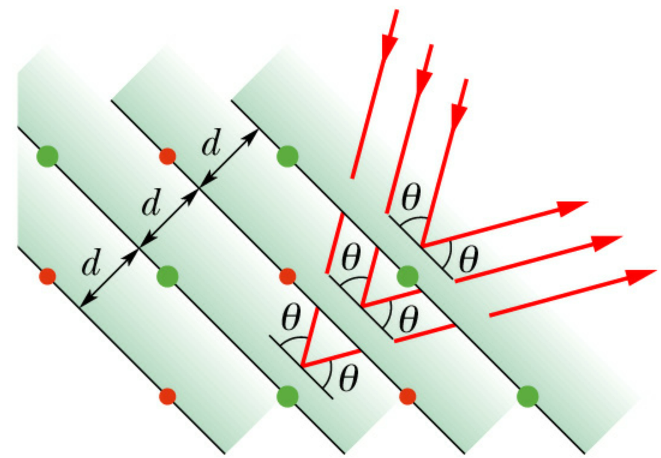


(b)

NaCl $a_0 \approx 0.56$ nm



(c)



(d)

Bragg condition:

$$2d \sin \theta = m\lambda$$

Bragg condition:

$$2d \sin\theta = m\lambda$$

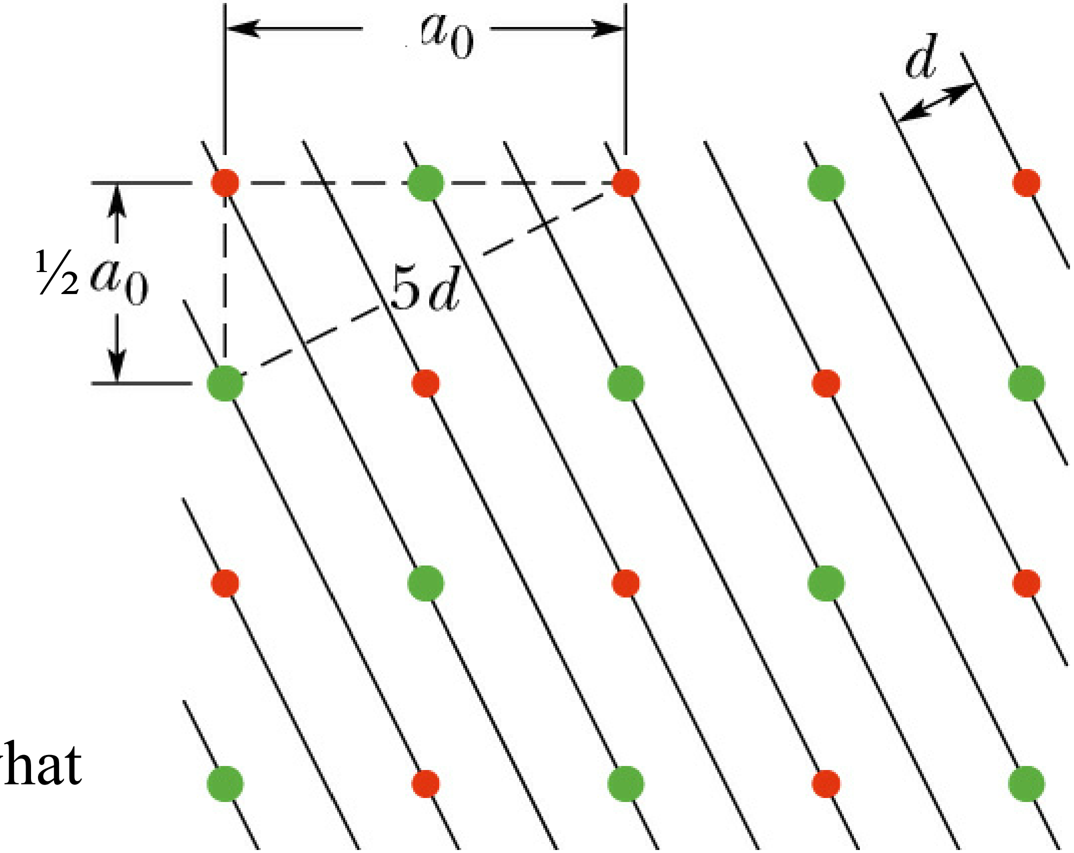
Example $\lambda = 0.154$ A^o

$$d = \frac{a_0}{2\sqrt{5}}$$

If $\theta = 37.95^o$ for $m=1$, what is a_0?

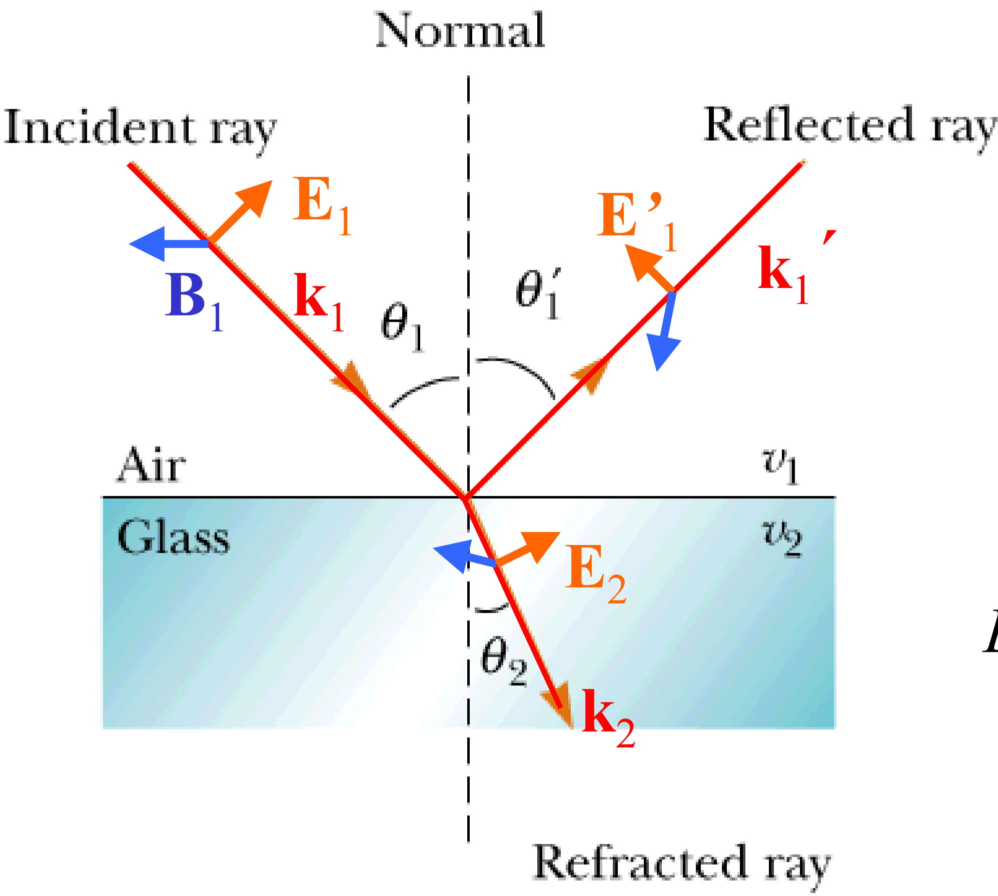
$$2d\sin\theta = 2\frac{a_0}{2\sqrt{5}}\sin\theta = \lambda$$

$$a_0 = \frac{\lambda\sqrt{5}}{\sin\theta} = 0.56\,\text{nm}$$



Interference between reflected waves:

Recall -- the behavior of a plane-polarized electromagnetic wave near the surface of two materials:



Periodic waves:

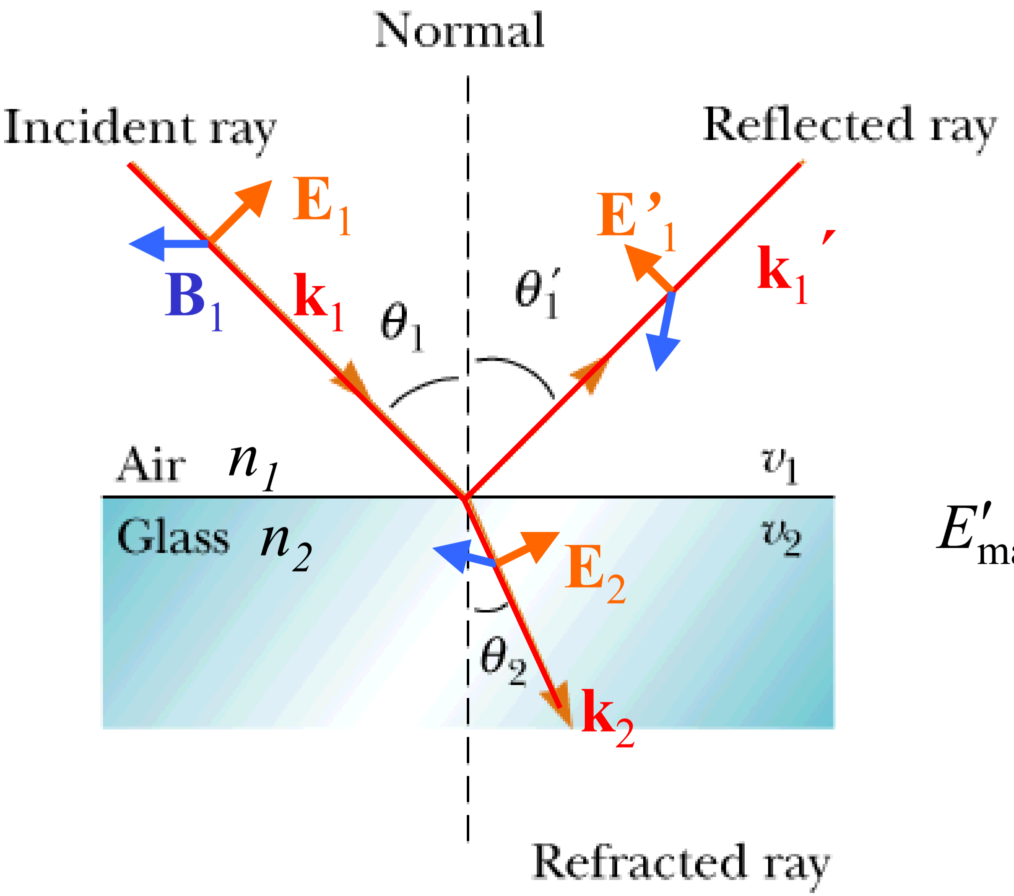
$$E_1 = E_{\max_1} \sin(\mathbf{k}_1 \cdot \mathbf{r} - \omega_1 t)$$

$$E_1' = E_{\max_1}' \sin(\mathbf{k}_1' \cdot \mathbf{r} - \omega_1' t)$$

$$\frac{\omega_1}{k_1} = \frac{\omega_1'}{k_1'} = v_1$$

$$E_2 = E_{\max_2} \sin(\mathbf{k}_2 \cdot \mathbf{r} - \omega_2 t)$$

$$\frac{\omega_2}{k_2} = v_2$$



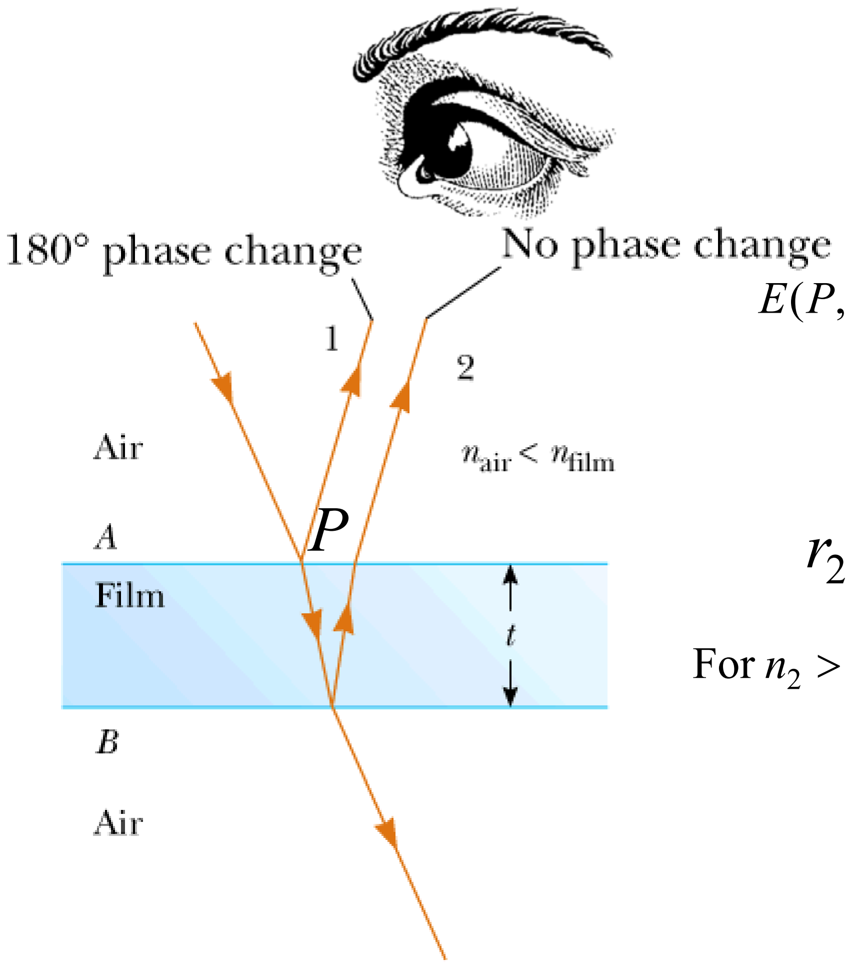
Matching electric and magnetic fields at boundary:

$$E'_{\max_1} = E_{\max_1} \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2}$$

or

$$E'_{\max_1} = E_{\max_1} \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

Multiple refractions and reflections in a thin film



$$E(P,t) = \pm E_{\max} \sin\left(\frac{2\pi r_1}{\lambda_1} - 2\pi f t\right) \pm E'_{\max} \sin\left(\frac{2\pi r_2}{\lambda_2} - 2\pi f t\right)$$

$$\approx 2E_{\max} \begin{Bmatrix} \sin \\ \cos \end{Bmatrix} \left(\frac{\pi(r_1 + r_2)}{\lambda_{av}} - 2\pi f t\right) \begin{Bmatrix} \cos \\ \sin \end{Bmatrix} \left(\frac{\pi(r_2 - r_1)}{\lambda_2}\right)$$

$$r_2 - r_1 \approx 2t$$

For $n_2 > n_1$ $\quad E_{\max_1} = -; E'_{\max_1} = +$

$$\Rightarrow \sin\left(\frac{\pi(2t)}{\lambda_2}\right) \quad \Rightarrow \text{max at} \quad 2nt = (m + \tfrac{1}{2})\lambda$$

$$\Rightarrow \text{min at} \quad 2nt = m\lambda$$

Example:

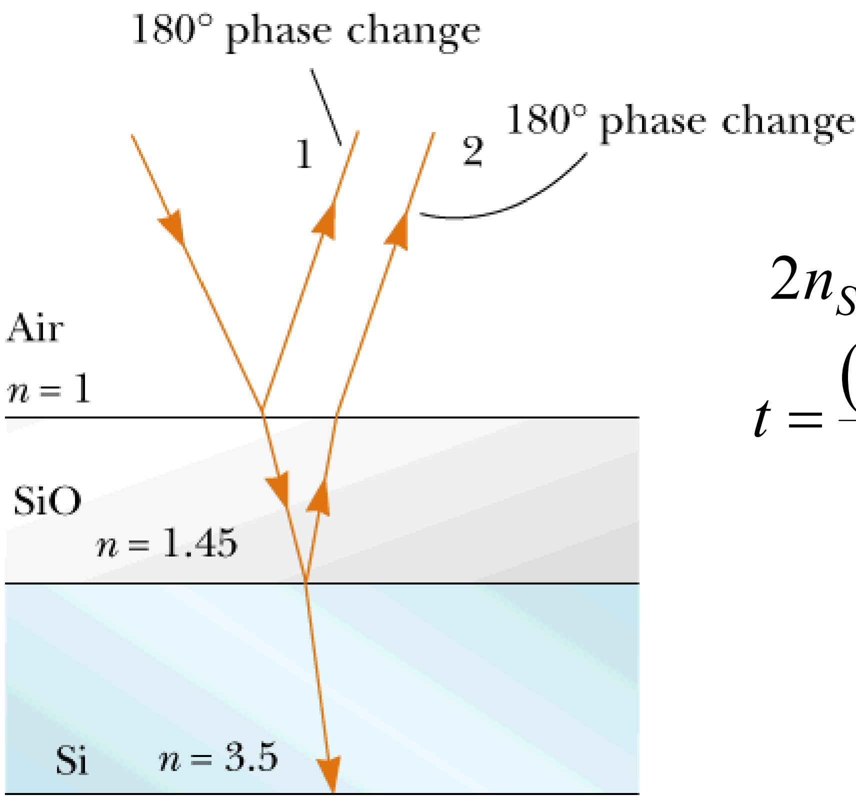
Suppose you see a rainbow pattern for oil on a pavement. What is the the approximate thickness of the oil in the red (λ=700nm) region. Assume that n_{oil}=1.4.

$$2n_{oil}t = \left(m + \tfrac{1}{2}\right)\lambda$$

$$t = \frac{\left(m + \tfrac{1}{2}\right)\lambda}{2n_{oil}} = \frac{\left(m + \tfrac{1}{2}\right)700\ \text{nm}}{2 \cdot 1.4} = 125\ \text{nm},\ 375\ \text{nm, etc.}$$

Example:

Suppose you want to prepare a surface so that it has minimal reflection such as in a solar cell where it is desirable to optimize refraction and minimize reflection at λ=550nm.



$$2n_{SiO}t = (m + \tfrac{1}{2})\lambda$$

$$t = \frac{(m + \tfrac{1}{2})\lambda}{2n_{SiO}} = \frac{(m + \tfrac{1}{2})550\ \text{nm}}{2 \cdot 1.45}$$

$$= 94.8\ \text{nm} \quad (\text{for } m = 0)$$