Announcements

1. Schedule for the next 2 weeks –

   "Modern" Physics Topics

   4/7 – Special theory of relativity (Chap. 39)

   4/9 -- Quantum Physics (Chap. 40)

   4/11 -- Quantum Physics (Chap. 41)

   4/14 -- Review Chap. 34-41

   4/16 – Third exam -- format??

2. Last 2 weeks of classes – some topics in materials physics & nuclear physics Chap. (42-46)

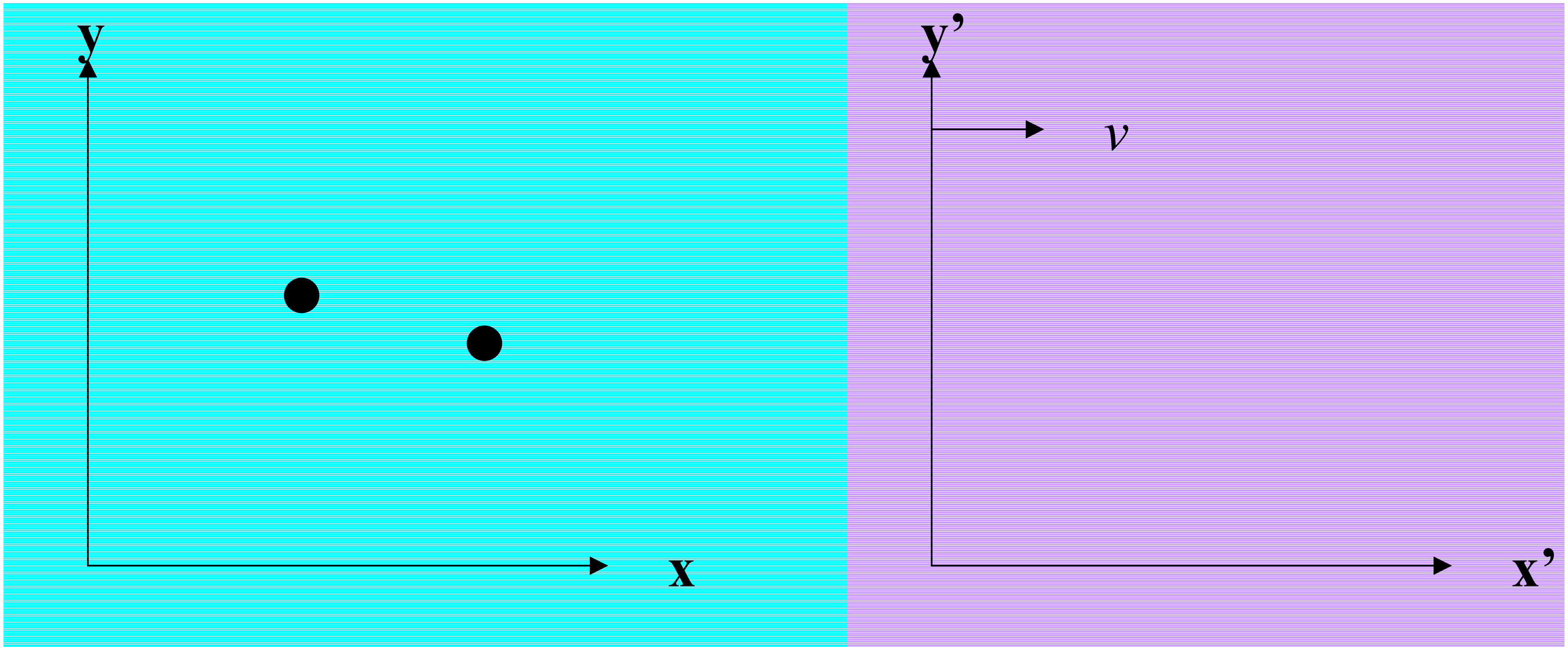
Some results from the special theory of relativity

Postulates:

- The laws of physics are the same in all inertial reference frames. (inertial reference frame ➔ traveling at constant velocity)
- The speed of light in vacuum c = 299792458 m/s is measured to be the same in all inertial reference frames.

The effects:

- c is the limiting speed in vacuum.
- The Lorentz transformation describes position and time relationships between frames of reference
- New formulations of momentum, and energy.



Lorentz transformation: $x' = \gamma(x - vt)$ $\quad x = \gamma(x' + vt')$

$$\gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \qquad y' = y \qquad y = y'$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \qquad t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$

Properties of γ factor: $\gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ $1 \le \gamma \le \infty$

For example:

If $v = 0.5c, \gamma = 1.1547$

$v = 0.994c, \gamma = 9.14243$

$v = 0.99994c, \gamma = 91.2885$

$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$

$$y' = y \qquad y = y'$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \qquad t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$

Note that:

$\Delta x = \gamma \Delta x'$ if $\Delta t' = 0$

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$\Delta t' = \gamma \Delta t$ if $\Delta x = 0$

Example:

Consider some measurable process such as a decay of a cosmic ray particle $\mu \rightarrow e + \nu + \bar{\nu}$ which is known to the follow the relationship, $N_\mu(t) = N_0 e^{-t/\tau}$ with t=2.2μs.

In a classic experiment, Rossi and Hall (*Phys. Rev.* **59**,223 (1941)), measured μ particles traveling with v=0.994c on the top and bottom of a mountain with Δx = 2000 m .

Δt=2000m/0.994c=6.7μs ➔ expect $\dfrac{N_\mu(6.7\mu s)}{N_0} = e^{-6.7/2.2} = 0.048$

➔ found $\dfrac{N_\mu}{N_0} \approx 0.72 = e^{-0.7/2.2}$

$$\text{Infer}: \quad \underbrace{\Delta t_\mu}_{0.7\ \mu\text{s}} = \underbrace{\Delta t_{Earth}}_{6.7\ \mu\text{s}} / \gamma \qquad \gamma = \frac{1}{\sqrt{1-(0.994)^2}} = 9.1$$

Peer instruction question

How can you explain this “time dilation” from the point of view of the meson?

(A) Who cares what the meson thinks.

(B) Meson doesn’t know and therefore is not effected by the fact that a crazy scientist measures a time interval of 9.1 times longer than 0.7 μs.

(C) Meson has a good reason to think that it traveled from the top to the bottom of the mountain in 0.7 μs.

What the meson thinks:

$$\Delta t_\mu = \frac{\Delta x_\mu}{v}$$

$$\Delta x_\mu = \frac{\Delta x_{Earth}}{\gamma}$$

$$\Delta t_\mu = \frac{\Delta x_{Earth}}{\gamma v} = \frac{\Delta t_{Earth}}{\gamma}$$ ➔ agrees with scientist

Moving clocks run slowly: $\Delta t' = \frac{\Delta t}{\gamma}$

Moving rulers are shortened: $\Delta x' = \frac{\Delta x}{\gamma}$

Lorentz transformation of velocities

$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$

$$y' = y \qquad y = y'$$

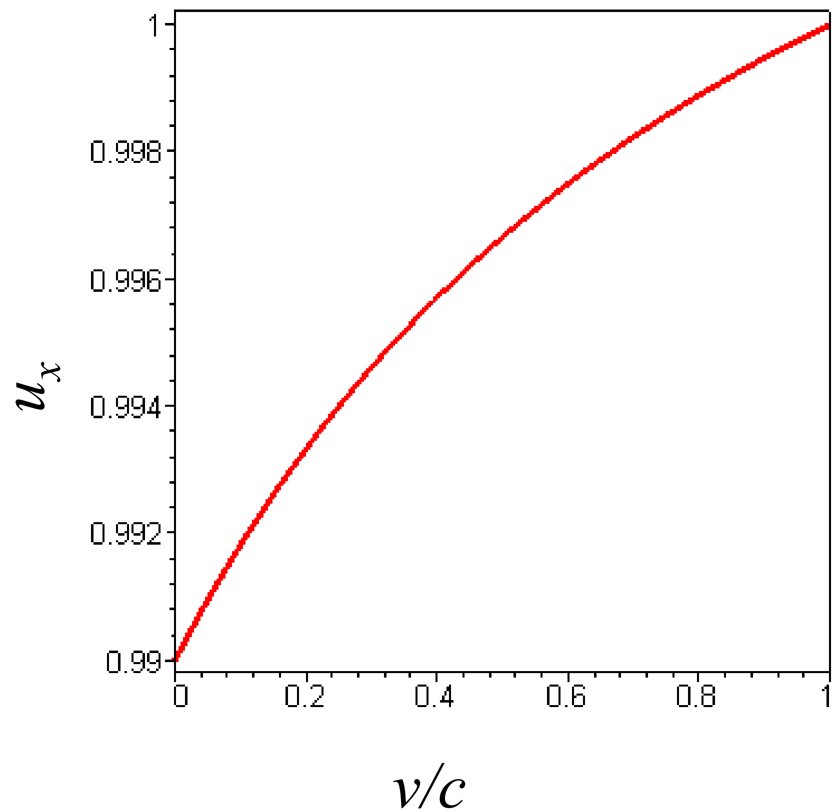
$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \qquad t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$

$$u'_x = \frac{dx'}{dt'} = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} \qquad u_x = \frac{dx}{dt} = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}}$$

$$u'_y = \frac{dy'}{dt'} = \frac{u_y}{\gamma\left(1 - \frac{u_x v}{c^2}\right)} \qquad u_y = \frac{dy}{dt} = \frac{u'_y}{\gamma\left(1 + \frac{u'_x v}{c^2}\right)}$$

Behavior of transformation for $u_x' = 0.99c$

$$u_x = \frac{dx}{dt} = \frac{u_x' + v}{1 + \frac{u_x' v}{c^2}}$$



Relativistic energies and momenta

$$\mathbf{p} = \gamma m \mathbf{u}$$

$$E = \gamma mc^2$$

Note that: $E^2 - p^2c^2 = (\gamma mc^2)^2\left(1 - \frac{u^2}{c^2}\right) = (mc^2)^2$

$$\boxed{E^2 = p^2c^2 + m^2c^4}$$

Kinetic energy: $K = E - mc^2$

$$K = \sqrt{p^2c^2 + m^2c^4} - mc^2 = \gamma mc^2 - mc^2$$

$$= mc^2\left(1 + \frac{p^2}{m^2c^2}\right)^{\frac{1}{2}} - mc^2$$

$$\approx \frac{p^2}{2m} - \frac{p^4}{8m^3c^2} + \cdots$$

Example:

What is the energy of an electron at rest?

$$E = mc^2 = 9.1 \text{ x } 10^{-31} \text{ kg } (3 \text{ x } 10^8 \text{m/s})^2 = 8.19 \text{ x } 10^{-14} \text{ J}$$

$$= 5.1 \text{ x } 10^5 \text{ eV}$$

$$= 0.51 \text{ MeV}$$

What is the speed of a 20 GeV electron?

$$\gamma = \frac{E}{mc^2} = \frac{20\times10^9}{0.51\times10^6} = 3.9216\times10^4$$

$$v = c - \Delta v$$

$$\gamma = \frac{1}{\sqrt{1-\left(1-\frac{\Delta v}{c}\right)^2}} \approx \frac{1}{\sqrt{2\frac{\Delta v}{c}}} \Rightarrow \Delta v \approx \frac{c}{2\gamma^2} = 3\times10^{-10}c$$

Example:

What is the energy of a proton at rest?

$E = mc^2 = 1.67 \times 10^{-27}\,\text{kg}\,(3 \times 10^8\,\text{m/s})^2 = 1.51 \times 10^{-10}\,\text{J}$

$= 9.4 \times 10^8\ \text{eV}$

$= 940\ \text{MeV}$

What is the speed of a 20 GeV proton?

$$\gamma = \frac{E}{mc^2} = \frac{20\times10^9}{940\times10^6} = 21.28$$

$$v = 0.9989c \qquad \Delta v = 1\times10^{-3}\,c$$

Peer instruction question

Suppose an electron has a speed $v=0.5c$. What is the ratio of $K_{Newtonian}/K_{relativistic}$

(A) 20%   (B) 80%   (C) 100%   (D) 200%

Extensions of Lorentz transformations

$$x' = \gamma(x - vt) \qquad p'_x = \gamma\left(p_x - vE/c^2\right) \qquad k'_x = \gamma\left(k_x - v\omega/c^2\right)$$

$$y' = y \qquad p'_y = p_y \qquad k'_y = k_y$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \qquad E'/c = \gamma\left(E/c - \frac{v}{c}p_x\right) \qquad \omega'/c = \gamma\left(\omega/c - \frac{v}{c}k_x\right)$$

$$k'_x = \frac{\omega'}{c} \Rightarrow \frac{\omega'}{c} = \gamma\frac{\omega}{c}\left(1 - \frac{v}{c}\right)$$

$$\omega' = \omega\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}} \text{ or } f' = f\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$