Announcements

1. Thursday – 4/10: Special physics lectures – Distinguished Traveling Lecturer sponsored by the the Division of Laser Physics of APS – Professor Luis Orozco

   Physics Colloquium at 4 PM "Quantum Feedback in Cavity QED or the Capture and Release of a Quantum Butterfly"

   Public Lecture at 8 PM "Waves and Particles in Light"

2. Vote Friday for form of 3rd exam – April 16, 2003

3. Topics for today –

   Brief review of relativity results (Chap. 39)

   Quantum Physics (Chap. 40)

   Particle nature of electromagnetic waves & wave nature of particles

Important results from the Special Theory of Relativity

Notice new physics at velocities v comparable to c (speed of electromagnetic waves in a vacuum). Note: Maxwell's equations are already consistent with notions of relativity.

New energy – momentum relationships within a single reference frame:

New zero of energy: If a particle has mass m and has zero velocity, its "rest mass energy" is mc^2. We can define a new "total" energy (not including potential energy) as

$$E = K + mc^2 = \gamma mc^2, \qquad \text{where } \gamma \equiv 1 \Big/ \sqrt{1 - \frac{v^2}{c^2}}$$

$$\text{In this same scheme, momentum becomes: } \mathbf{p} = \gamma m \mathbf{u}$$

$$\Rightarrow E^2 = p^2c^2 + m^2c^4; \qquad p^2c^2 = K^2 + 2Kmc^2$$

New relationships for transforming measurements between different inertial frames of reference moving at a relative velocity *v*. Lorentz transformation:

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

$$k'_x = \gamma\left(k_x - v\omega / c^2\right)$$

$$k'_y = k_y$$

$$\omega' = \gamma(\omega - vk_x)$$

Doppler effect for electromagnetic waves:

$$\omega' = \gamma\omega\left(1 - \frac{v}{c}\right) \quad (\text{since} \quad k_x = \frac{\omega}{\mathrm{c}})$$

$$\omega' = \omega\frac{1 - \frac{v}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} = \omega\sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}}$$

or $\boxed{f' = f\sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}}}$

Peer Instruction Question

When should we use the relativistic formulation of physical laws instead of the non-relativistic Newtonian formulation?

Doppler effect: (A) Relativistic (B)Non-relativistic

Manned space flight: (A) Relativistic (B) Non-relativistic

Electrons hitting your TV screen: (A) Relativistic (B) Non-relativistic

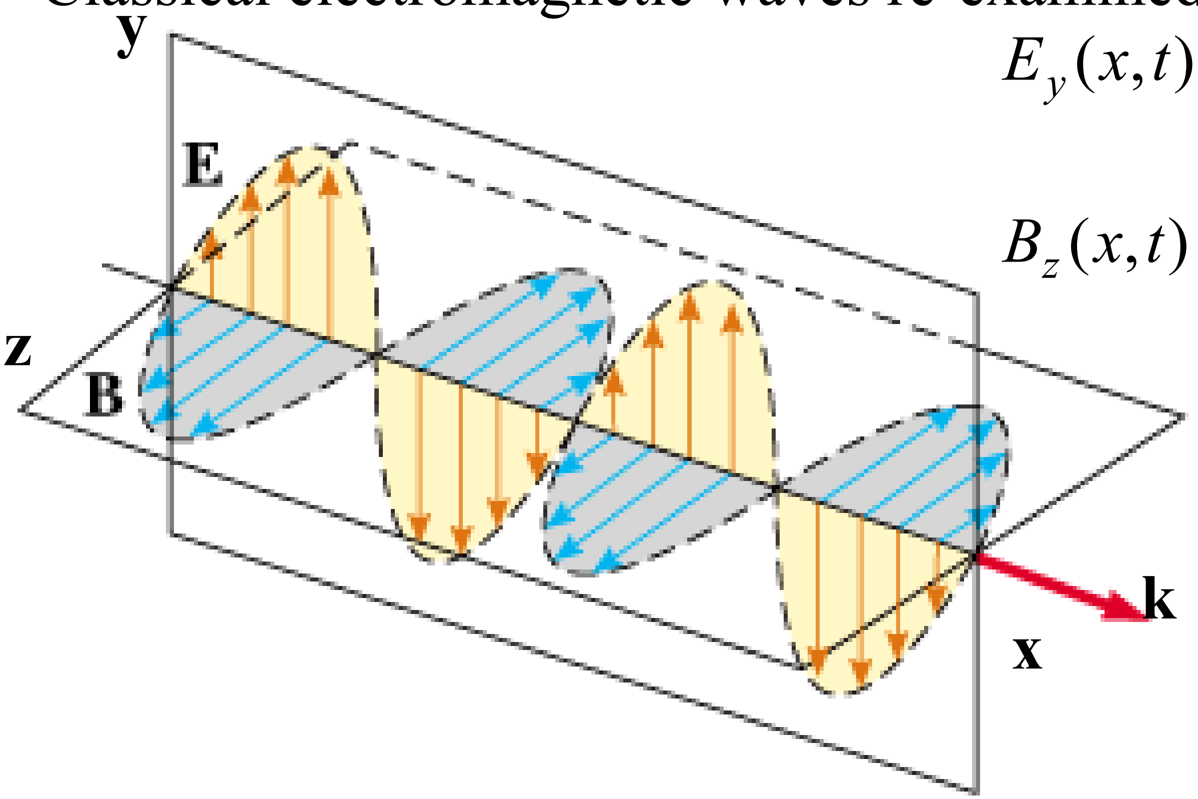
An electron from cosmic sources with kinetic energy of 1 MeV
(A) Relativistic (B) Non-relativistic

A proton from cosmic sources with kinetic energy of 1 MeV
(A) Relativistic (B) Non-relativistic

Quantum Physics

- Changes in physical laws for very small distances, short times
- Electromagnetic waves have some particle-like properties
  - discrete energies – photoelectron effect
  - transfer momentum in collisions – Compton effect
- Particles have some wave-like properties:
  - de Broglie wavelength
  - interference effects – Davisson-Germer experiment
  - Bohr model of the atom

Classical electromagnetic waves re-examined:



$$E_y(x,t) = E_{\max} \sin\left(\frac{2\pi x}{\lambda} - 2\pi f t\right)$$

$$B_z(x,t) = \frac{E_{\max}}{c} \sin\left(\frac{2\pi x}{\lambda} - 2\pi f t\right)$$

Energy density associated with electromagnetic wave:

$$u = \tfrac{1}{2}\varepsilon_0 E^2 + \tfrac{1}{2} B^2/\mu_0 = \tfrac{1}{2}\left(\varepsilon_0 E_{\max}^2 + B_{\max}^2/\mu_0\right)\sin^2\left(\frac{2\pi x}{\lambda} - 2\pi f t\right)$$

$$\langle u \rangle_{avg} = \tfrac{1}{2}\varepsilon_0 E_{\max}^2$$

Poynting vector: $\langle \mathbf{S} \rangle_{avg} = \dfrac{\hat{\mathbf{x}}}{2\mu_0 c} E_{\max}^2$

Quantum theory of electromagnetic waves -- apparent at low intensities

$$\langle u \rangle_{avg} = hf\left(n + \tfrac{1}{2}\right) \qquad n = 0,1,2,3\cdots$$

$$h = 6.6261 \text{ x } 10^{-34} \text{ J s}$$

$$= 4.1323 \text{ x } 10^{-15} \text{ eV s}$$

$$\langle \mathbf{S} \rangle_{avg} = \hat{\mathbf{x}} h \frac{f}{c}\left(n + \tfrac{1}{2}\right) = \hat{\mathbf{x}} \frac{h}{\lambda}\left(n + \tfrac{1}{2}\right)$$

➔one "photon" has a quantum of energy *hf*

momentum h/λ

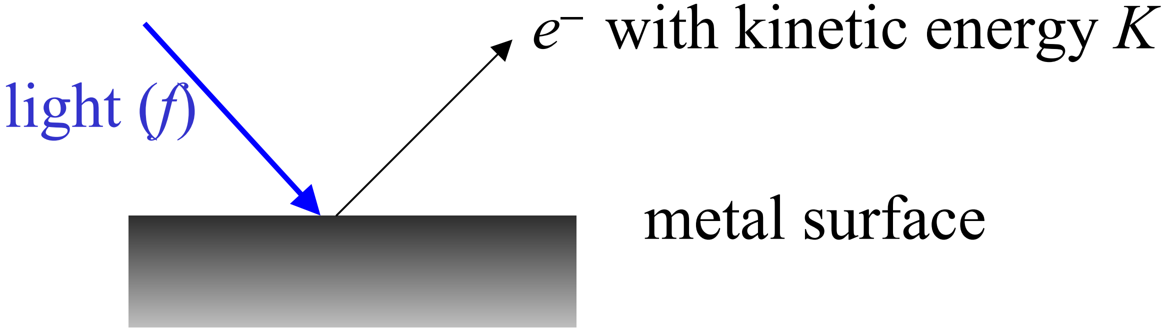
➔ one "photon" has a quantum of energy hf

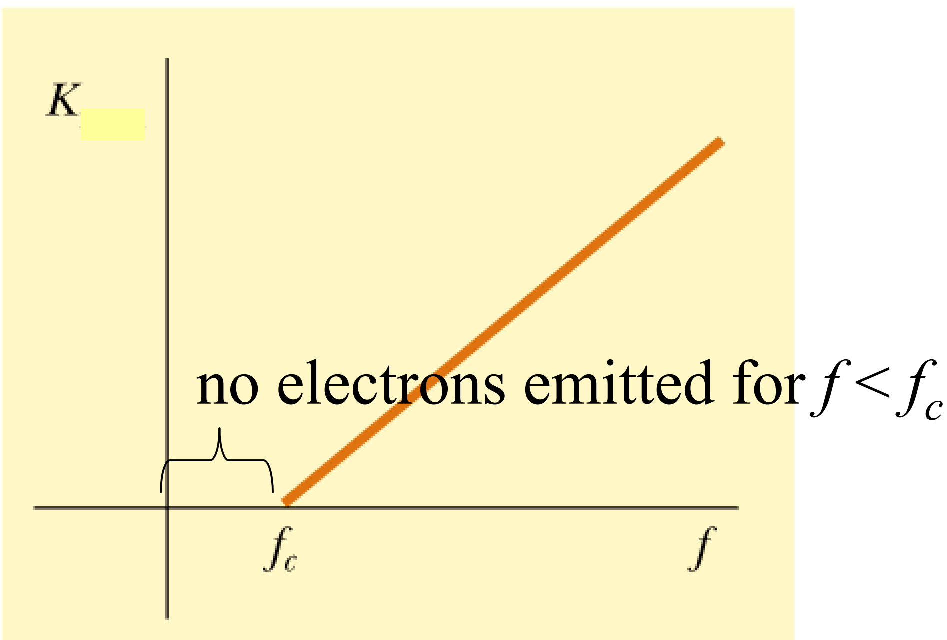
momentum h/λ

Experimental evidence:

1. Analysis by Max Planck of radiation distribution from thermal source (such as a glowing tungsten wire). (Requires assumptions from statistical formulation of thermal physics.)
2. Albert Einstein's analysis of the photoelectric effect
3. Arthur H. Compton's analysis of the scattering of light by an electron.

Photoelectron effect:





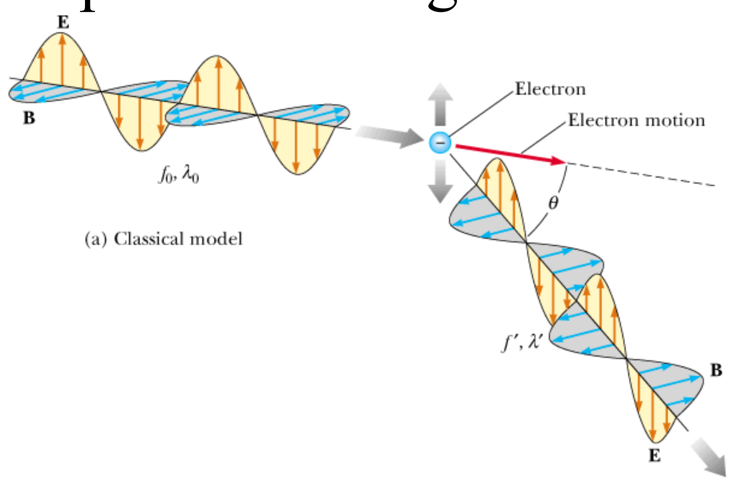
Einstein's idea based on the fact that each surface has a "work function" ϕ

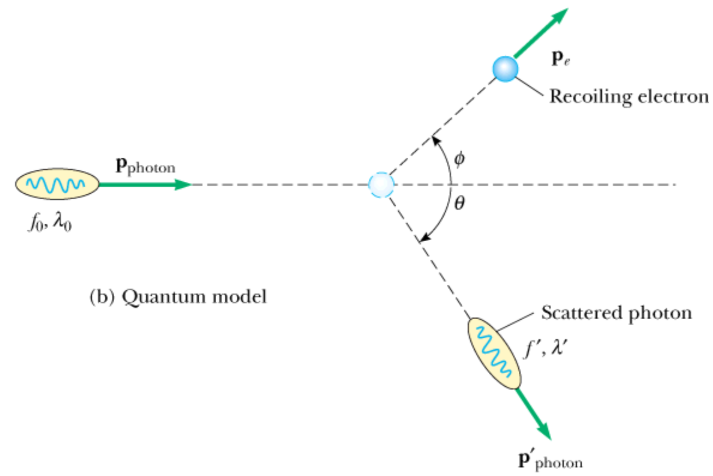
$$\boxed{hf_c = \phi}$$

$$K = hf - \phi$$

for $f > f_c$

Compton scattering





By analyzing the process with the assumption that the "photon" has an energy $E=hf$ and a momentum of magnitude $p=hf/c$, Compton was able to explain his data.

Note: Since the "photon" is traveling at the speed of light, we must treat this problem relativistically.

$$\text{Recall}: \quad E^2 = p^2c^2 + m^2c^4$$

$$\text{For photon}: \ \Rightarrow E = pc = hf = h\frac{c}{\lambda}$$

Analysis of Compton effect continued:

Conservation of energy: $$h\frac{c}{\lambda_0} = h\frac{c}{\lambda'} + \left(\gamma mc^2 - mc^2\right)$$

Conservation of momentum: $$\frac{h}{\lambda_0} = \frac{h}{\lambda'}\cos\theta + \gamma mv\cos\varphi$$

$$0 = \frac{h}{\lambda'}\sin\theta - \gamma mv\sin\varphi$$

Result: $$\boxed{\lambda' - \lambda_0 = \frac{h}{mc}\left(1-\cos\theta\right)}$$

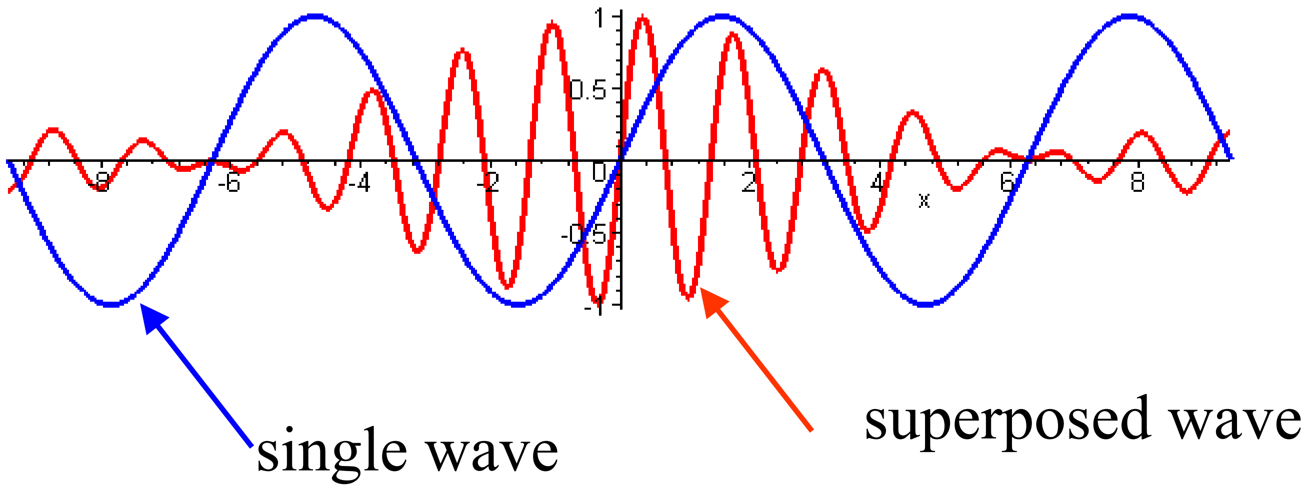
Compelling evidence of discrete "quanta" of light:

$E_{photon} = hf$ $\quad\quad$ $p_{photon} = h/\lambda$

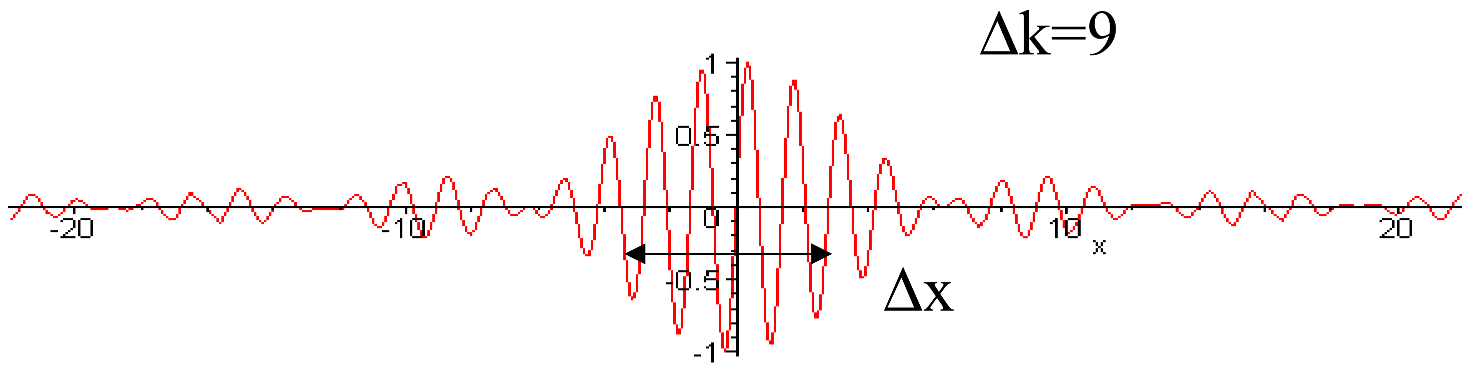
How can we mathematically reconcile particle and wave behaviors?

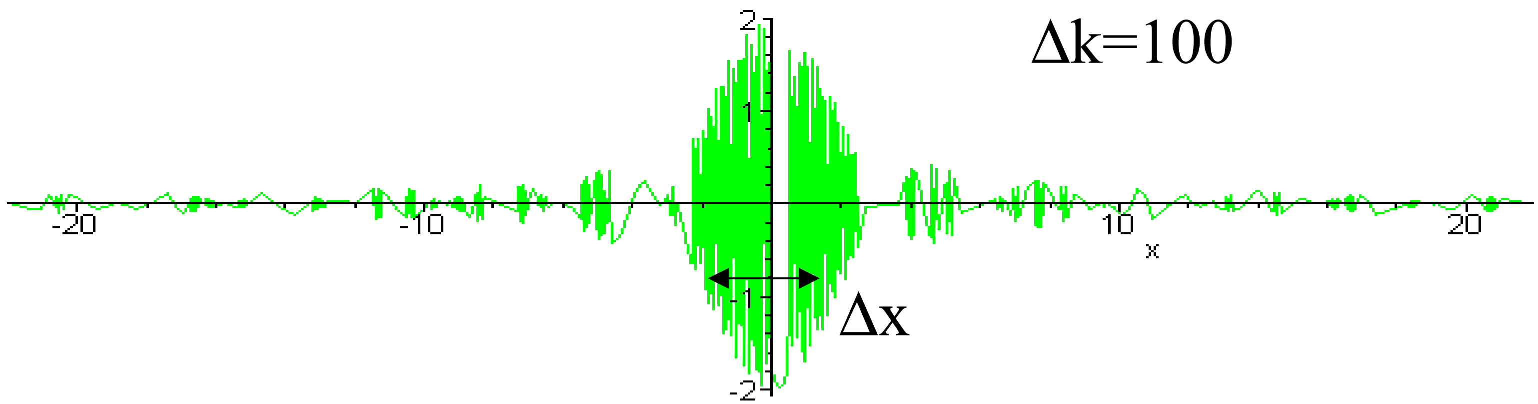
Consider a superposition of periodic waves at *t=0:*

$$E(x,t) = \sum_i E_{\max} \sin(k_i x)$$



More details about superposition:





For intensities, we find $\Delta x\,\Delta k \approx$ (constant)

Δx smaller ➔ more particle like

Δk smaller ➔ more wave like

Wave-like properties of particles

Louis de Broglie suggested that a wavelength could be associated with a particle's momentum

$$p = \frac{h}{\lambda}$$

"Wave" equation for particles – Schrödinger equation

$$\left[-\frac{h^2}{(2\pi)^2 m}\frac{\partial^2}{\partial x^2} + V(x)\right]\Psi(x,t) = -i\frac{h}{2\pi}\frac{\partial}{\partial t}\Psi(x,t)$$