Announcements

1. Vote on form of 3rd exam, scheduled for Wed. 4/16/03, covering material in Chaps. 34-41

   In class ☐ Take home ☐

   Review sessions -- Monday's class
   Tuesday 6 PM
   Other??

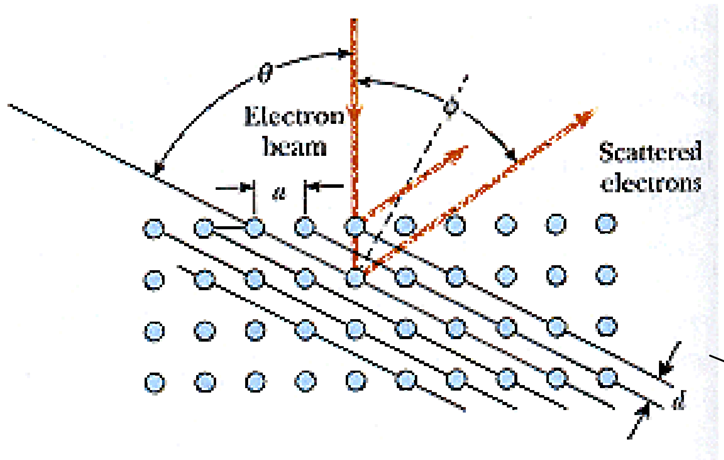
2. Today's topics: (Chaps. 40-41)

   Wave-particle descriptions of electromagnetic and mechanical phenomena – Heisenberg uncertainty principle

   Particle wave equation

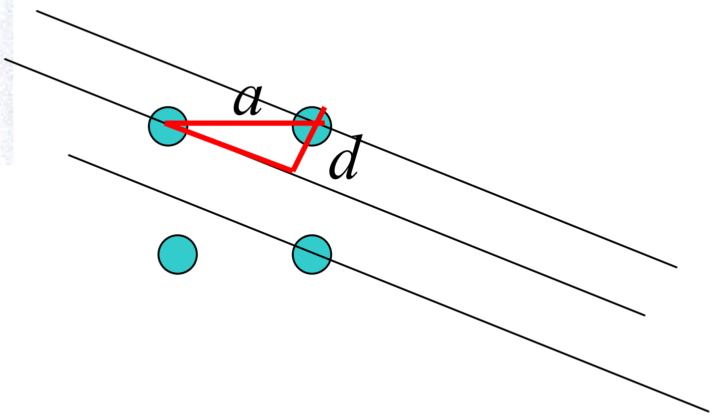
Homework problem:

(Davisson-Germer electron diffraction experiment)



Bragg condition:

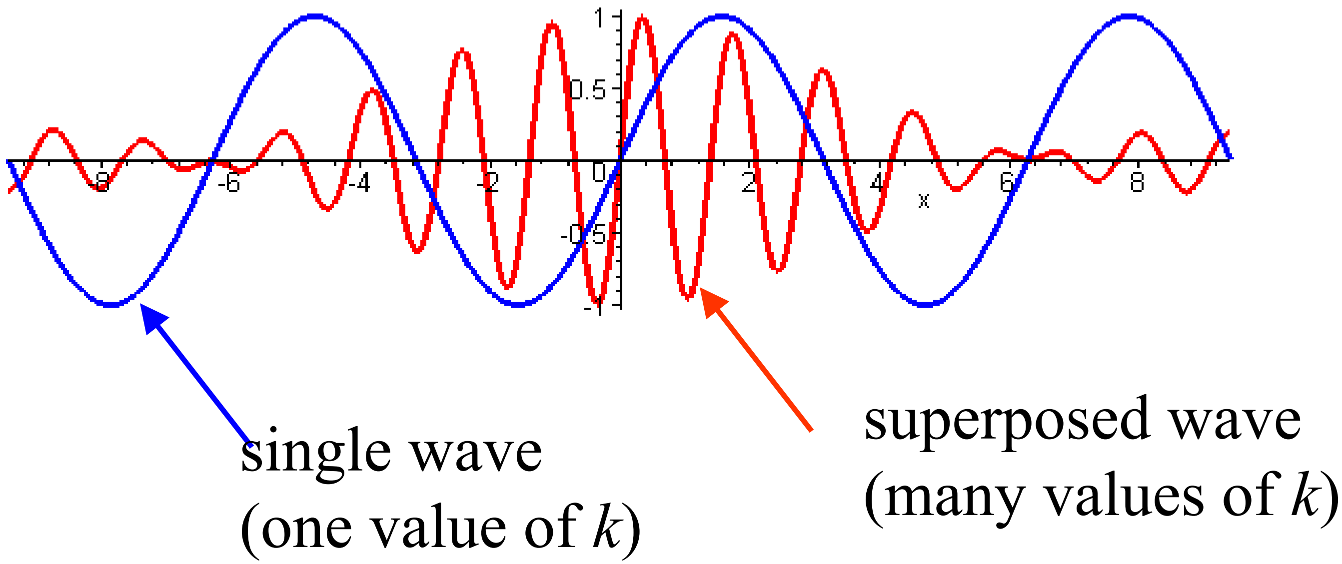
$2d \sin\theta = m\lambda$



Mathematical representation of particle and wave behaviors.

Consider a superposition of periodic waves at $t=0$:

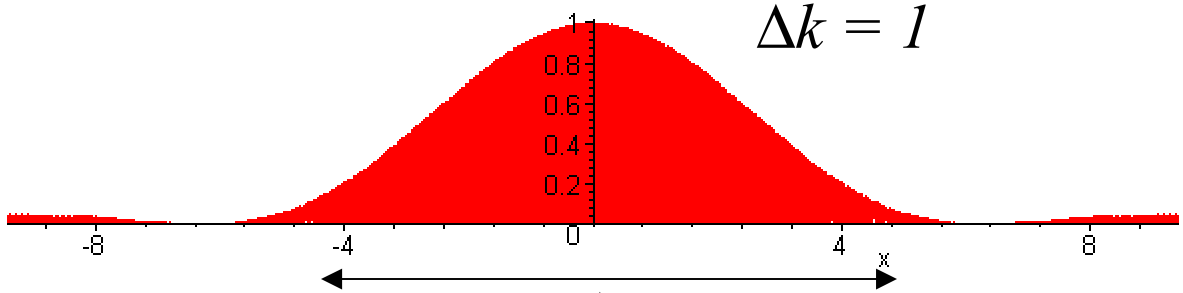
$$E(x,t) = \sum_i E_{\max} \sin(k_i x)$$



$$[E(x,0)]^2 = \left(\sum_i E_{\max} \sin(k_i x) \right)^2$$

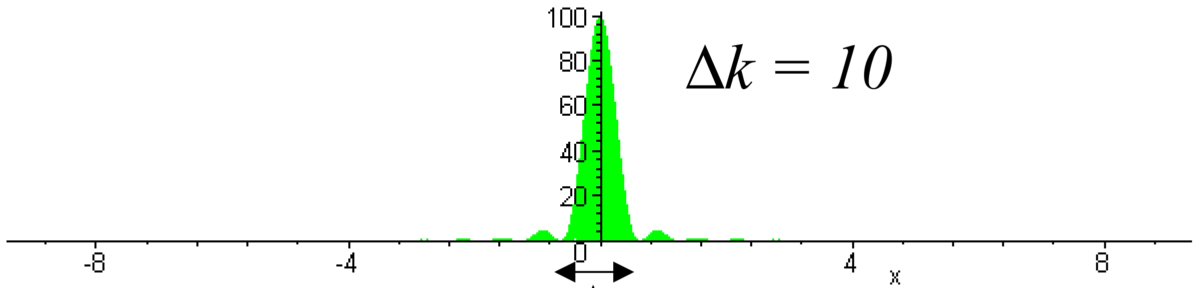
$$\Delta x \, \Delta k \approx 2\pi$$

$\Delta k = 1$



Δx

$\Delta k = 10$



Δx

Δx smaller ➔ more particle like

Δk smaller ➔ more wave like

$\Delta x\ \Delta k \approx 2\pi$ ➔ Heisenberg's uncertainty principle

De Broglie's particle moment – wavelength relation:

$$p = \frac{h}{\lambda} = \frac{h/2\pi}{\lambda/2\pi} = \hbar k$$

Heisenberg's hypotheses: $\Delta x \Delta p \geq \frac{\hbar}{2}$

$$\Delta t \Delta E \geq \frac{\hbar}{2}$$

Wave equations

Electromagnetic waves:

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} = c^2 \frac{\partial^2 \mathbf{E}}{\partial x^2}$$

Matter waves: (Schrödinger equation)

$$-i\hbar \frac{\partial}{\partial t} \Psi(x,t) = \left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) \right] \Psi(x,t)$$

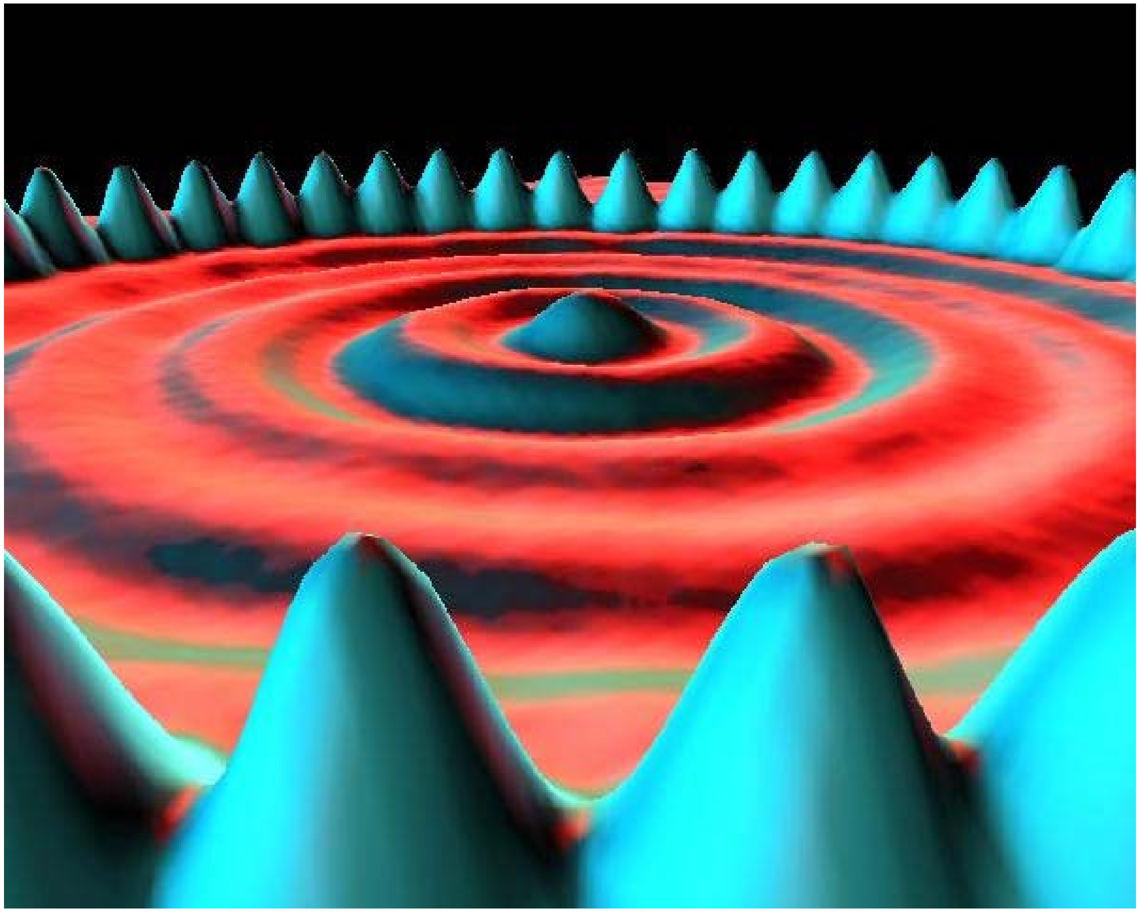
Electromagnetic waves	Matter waves
Vector – **E** or **B** fields	Scalar – probability amplitude
Second order *t* dependence	First order *t* dependence
Examples:	Examples:
$E_y(x,t) = E_{\max}\sin(kx - \omega t)$	$\Psi(x,t) = \Psi_0 \sin(kx) e^{-iEt/\hbar}$
$B_z(x,t) = \frac{E_{\max}}{c}\sin(kx - \omega t)$	"bound" states

What is the meaning of the matter wave function $\Psi(x,t)$?

- $\Psi(x,t)$ is not directly measurable
- $|\Psi(x,t)|^2$ is measurable – represents the density of particles at position x at time t.
- For a single particle system, $\int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = 1$
- For many systems of interest, the wave function can be written in the form $\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$

$$|\Psi(x,t)|^2 = |\psi(x)|^2$$

Visualization of $|\psi(x)|^2$

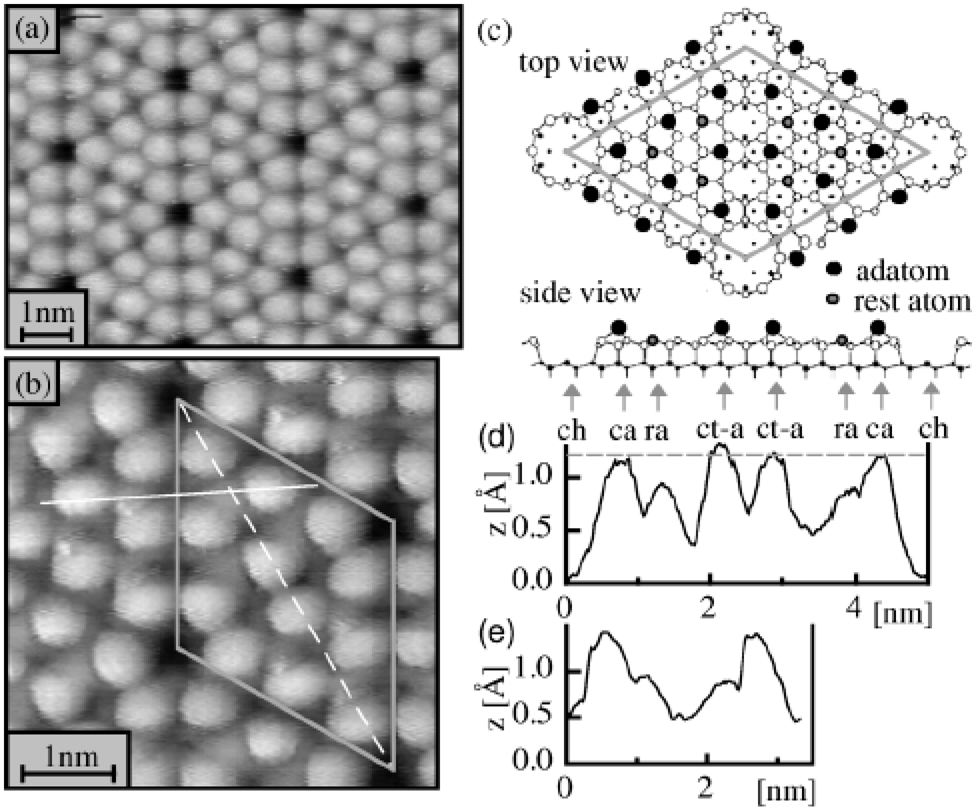


STM image of 48 iron atoms on a Cu surface.

From: M.F. Crommie, C.P. Lutz, D.M. Eigler. “Confinement of electrons to quantum corrals on a metal surface”. Science **262**, 218-220 (1993).

Visualization of $|\psi(x)|^2$

A surface if a nearly perfect Si crystal



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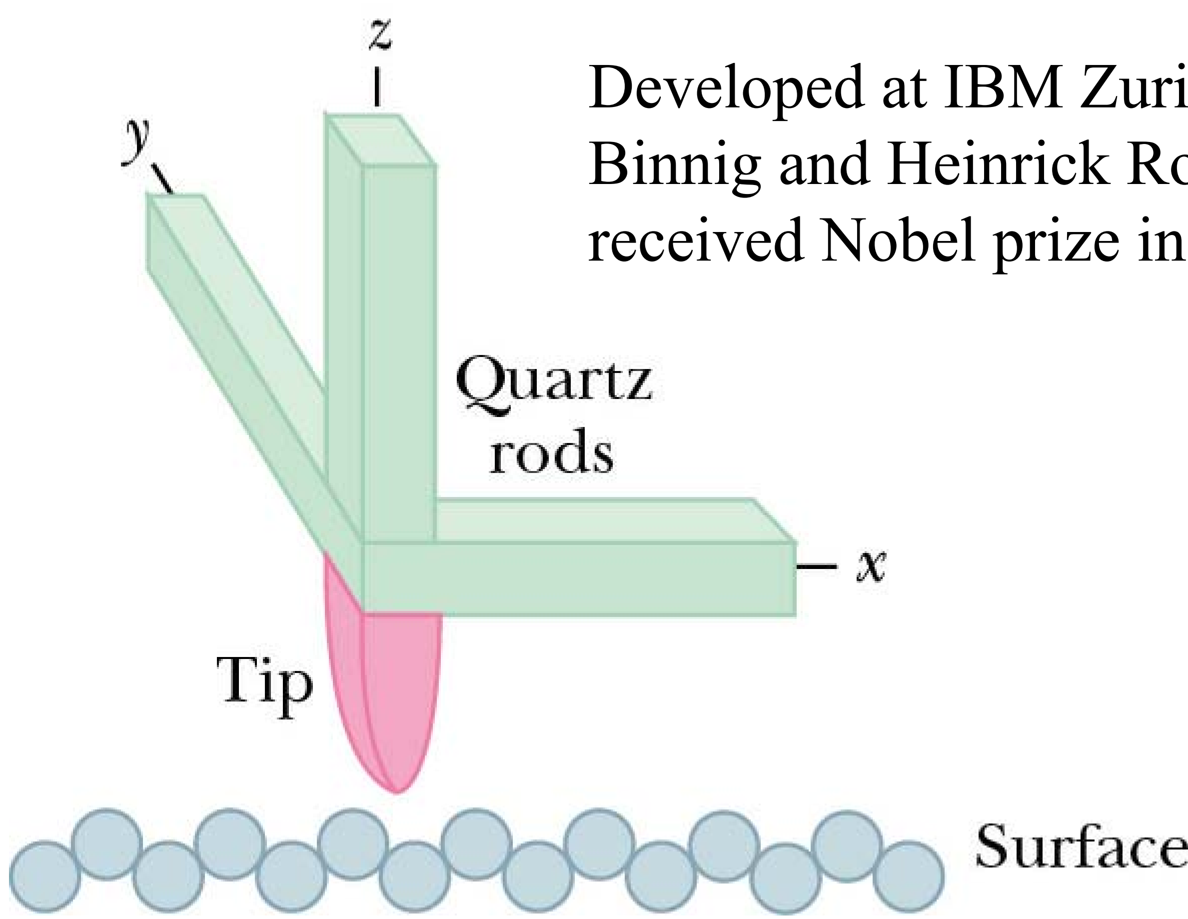
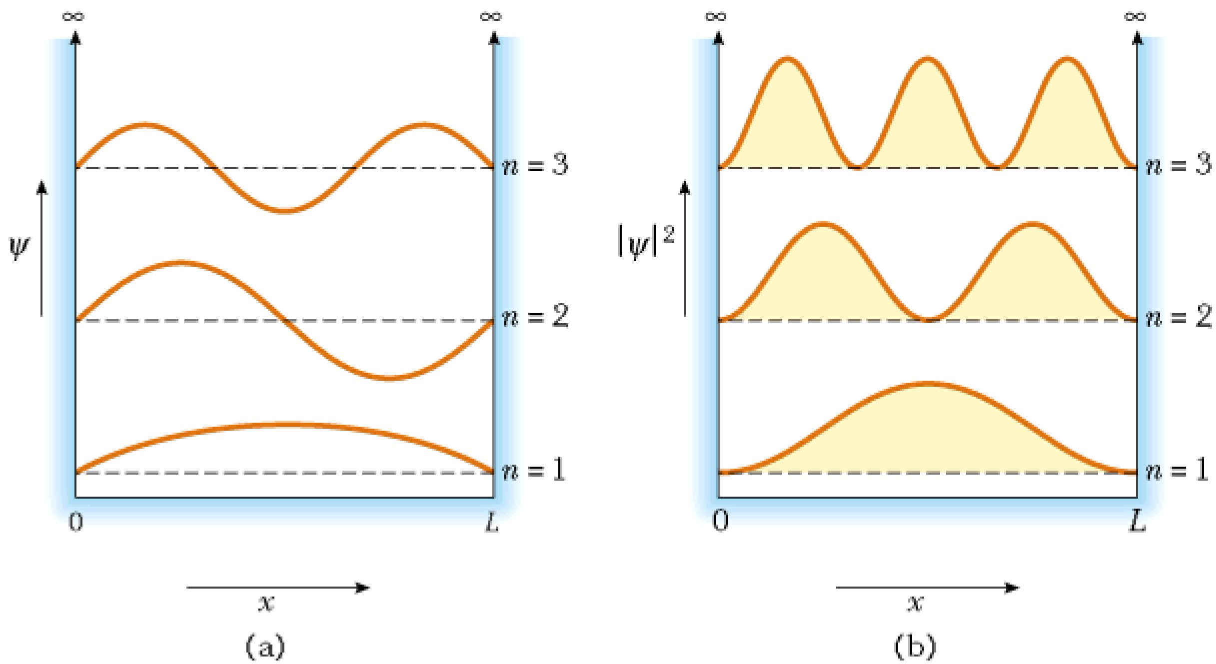
How a scanning tunneling microscope works:



Developed at IBM Zurich by Gerd Binnig and Heinrick Rohrer who received Nobel prize in 1986.

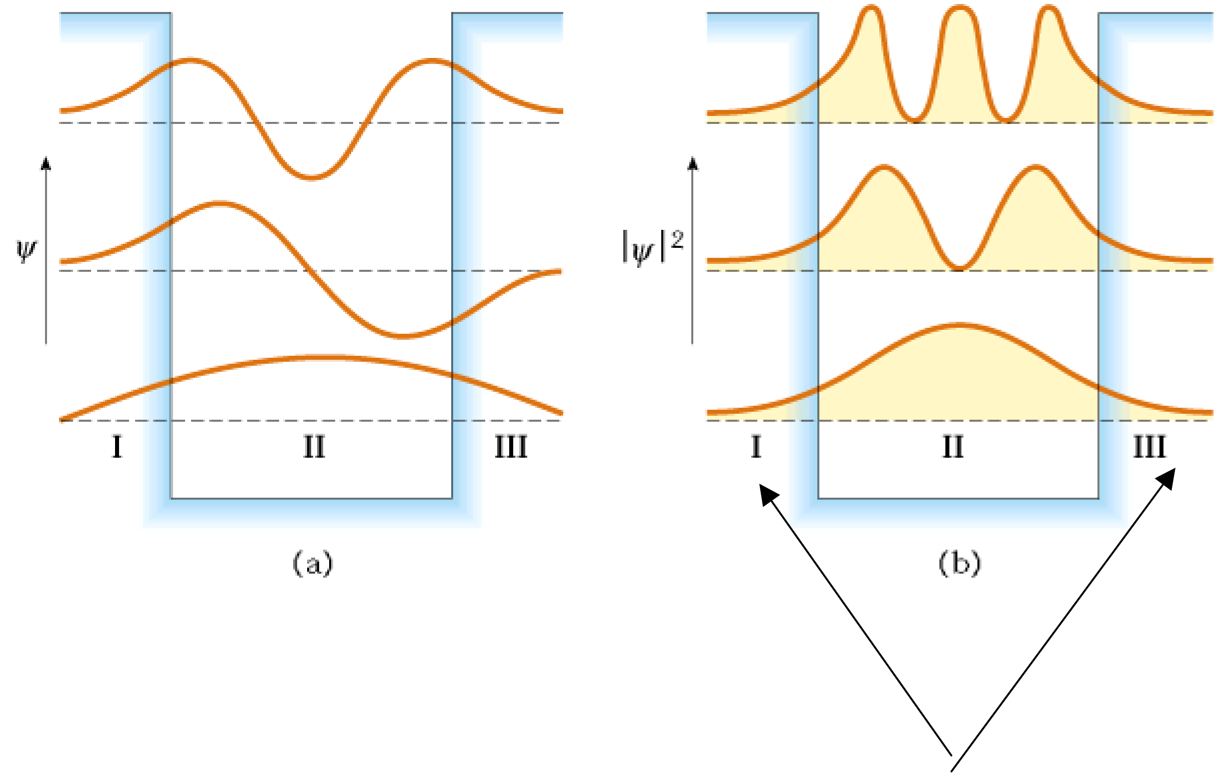
Electrons in an infinite box:



$$E\,\psi(x) = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\right]\psi(x) \quad \text{for } 0 \le x \le L$$

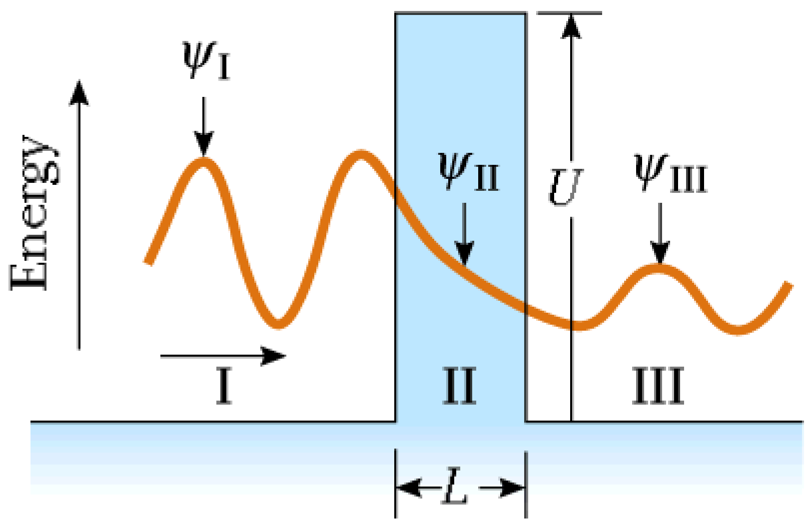
$$\psi(x) = \psi_0 \sin\left(\frac{n\pi x}{L}\right) \qquad n = 1,2,3\cdots$$

Electrons in a finite box:



finite probability of electron existing outside of classical region

Tunneling through a barrier



surface region tip

vacuum