Announcements

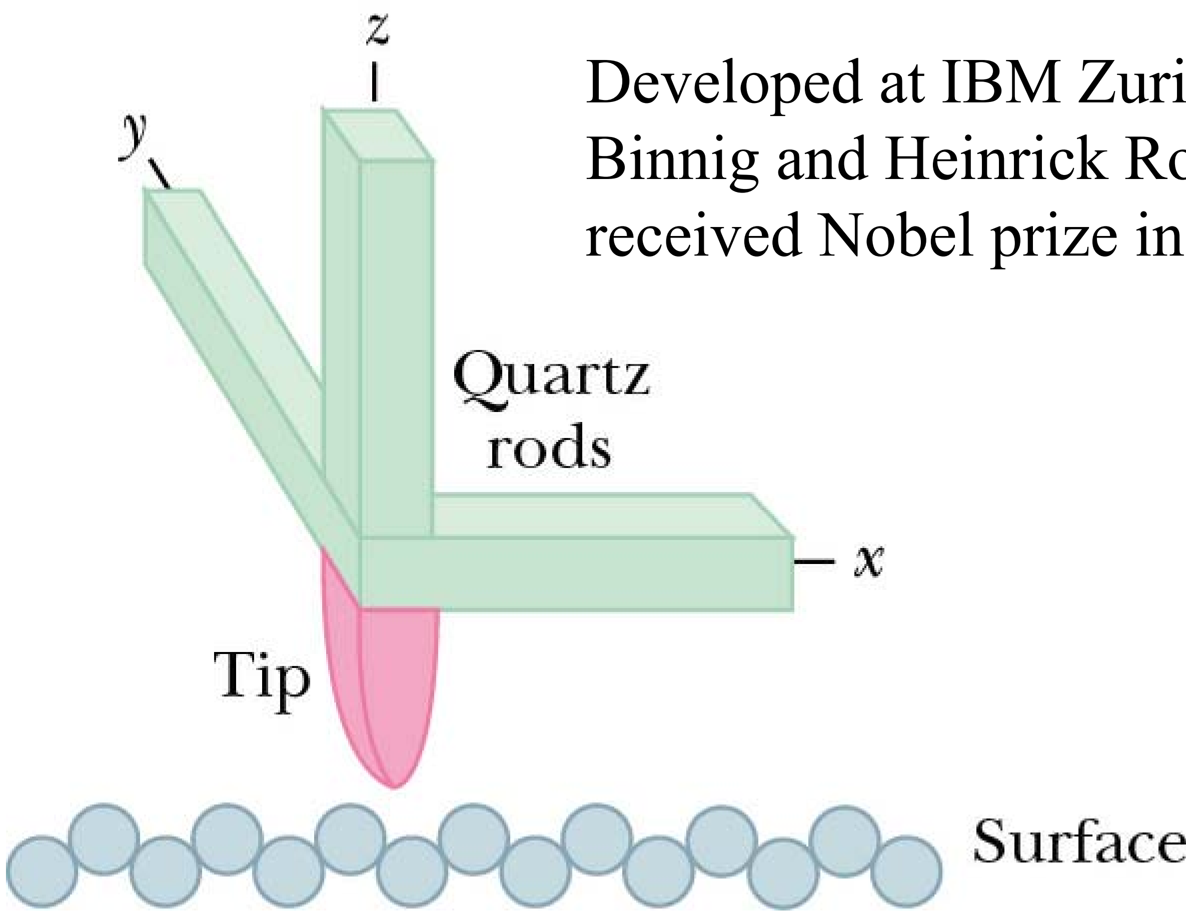
1. Hour exam scheduled for 11-11:50 AM on Wednesday, Apr. $16^{th}$. May bring 1 equation sheet (8.5" x 11"), to be turned in with exam papers, **AND YOUR TEXT BOOK.**

2. Advice for studying – focus on material covered in lecture and/or in homework problems.

3. Review session Tuesday 6 PM in Olin 107
   other?

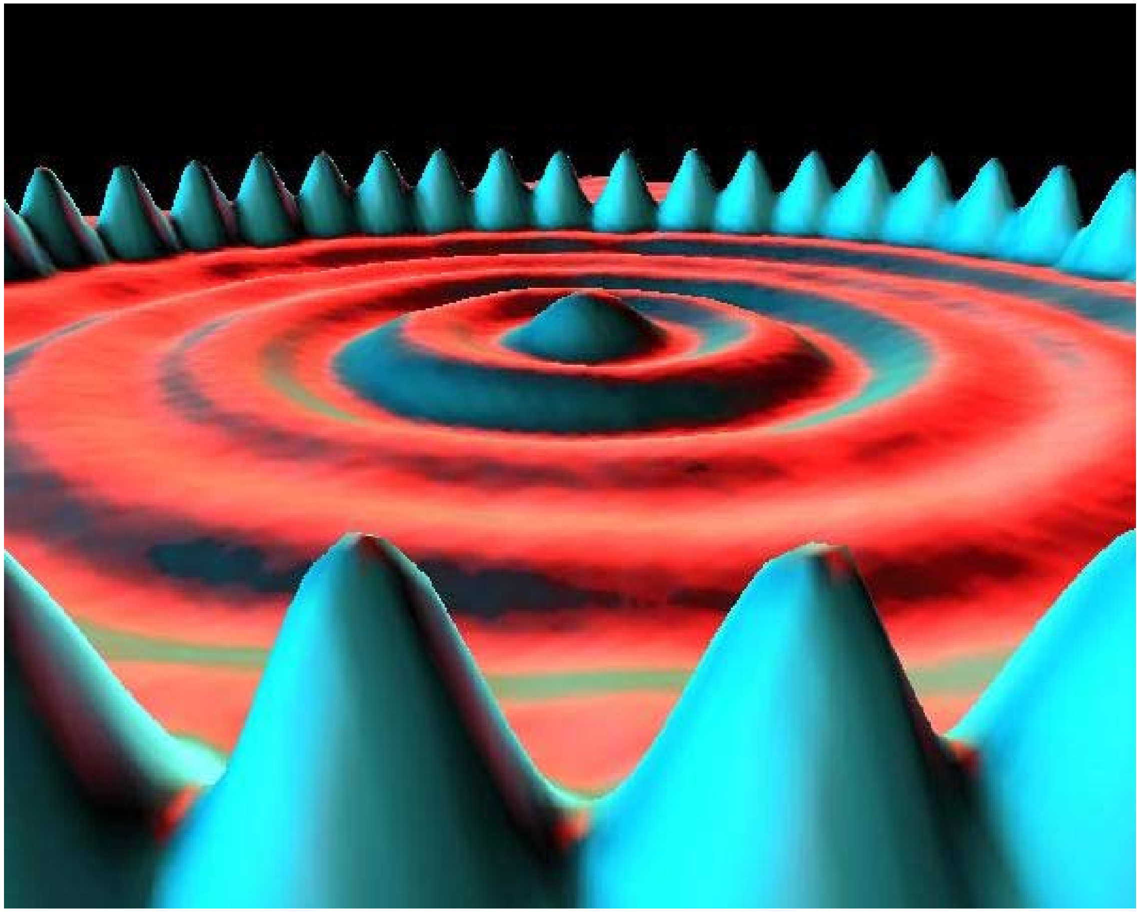
4. Material for today's lecture
   - Comments on STM and AFM
   - Systematic review

How a scanning tunneling microscope works:




Developed at IBM Zurich by Gerd Binnig and Heinrick Rohrer who received Nobel prize in 1986.

Scanning tunneling microscope (STM) images of $|\psi(x)|^2$ :

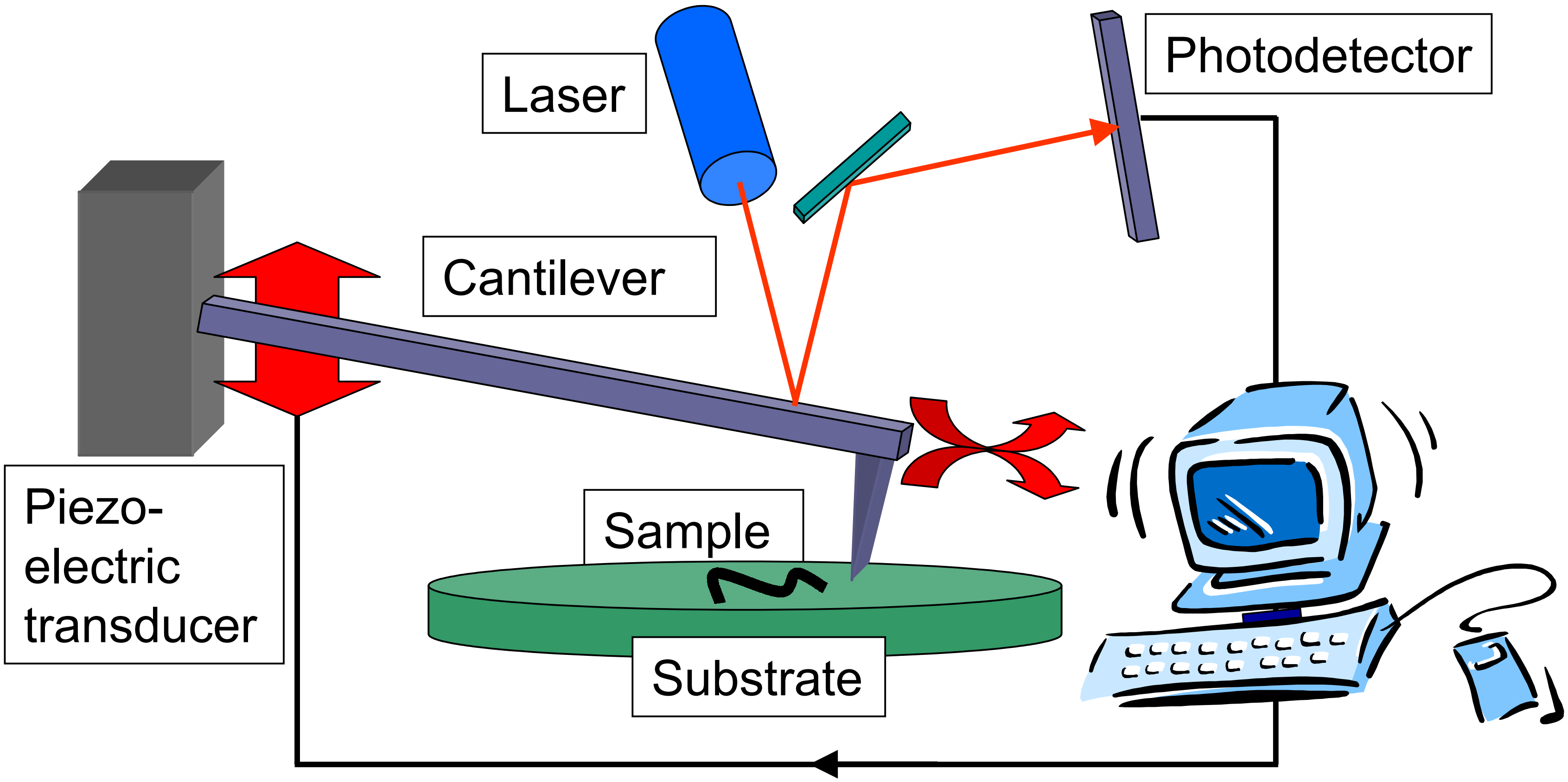


STM image of 48 iron atoms on a Cu surface.

From: M.F. Crommie, C.P. Lutz, D.M. Eigler. "Confinement of electrons to quantum corrals on a metal surface". Science **262**, 218-220 (1993).

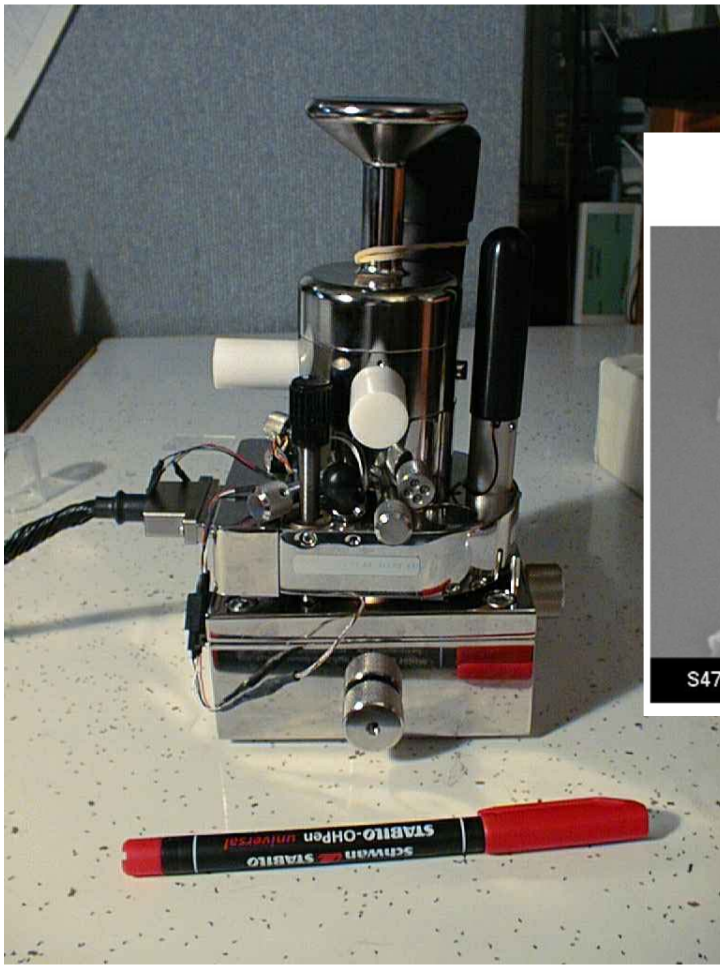
Related technology – atomic force microscopy (slides from Profressor Martin Guthold)

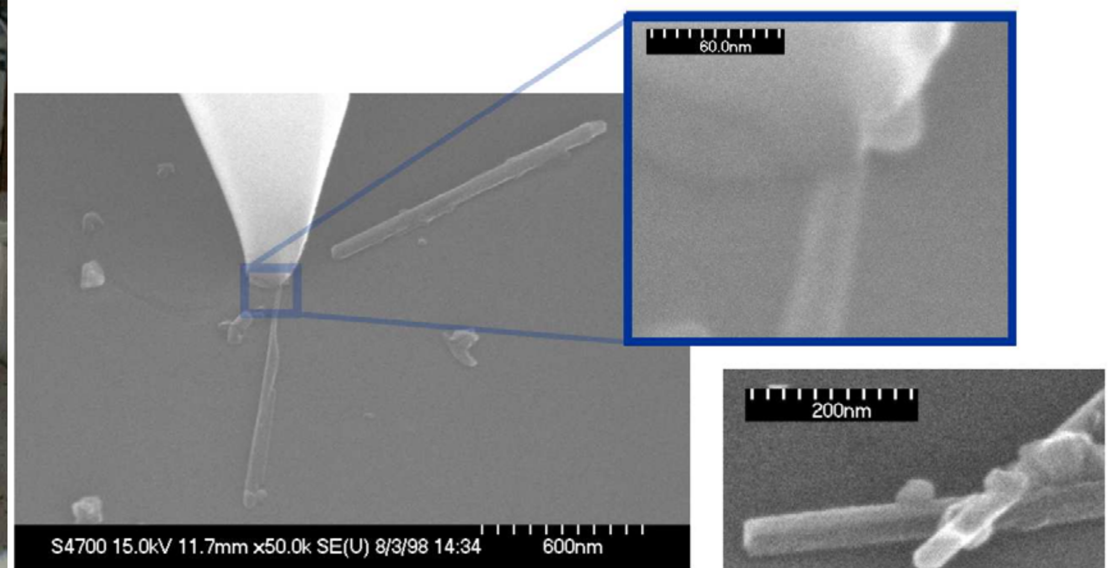
# Schematic of an AFM



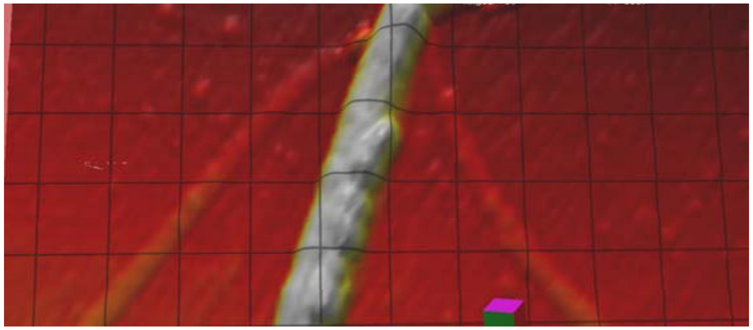

Force controlled by feedback

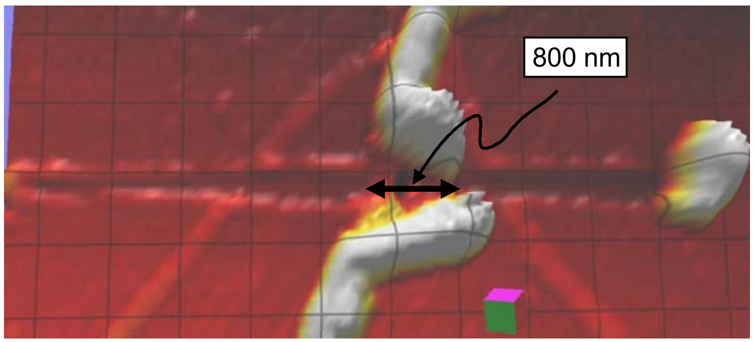
# Atomic Force Microscopy




(Slide from Martin Guthold)



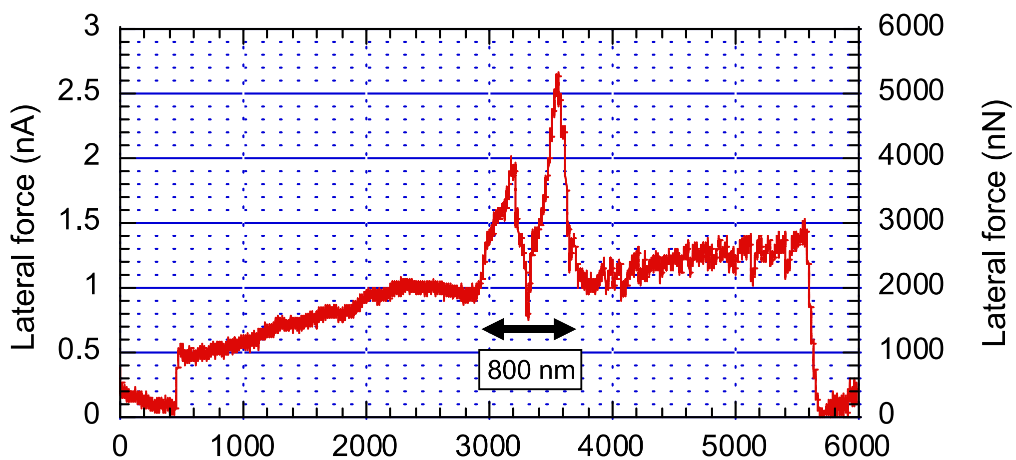



# Manipulation of fibrin

Fiber is

- •broken off the surface,
- •stretched,
- •ruptured

•Diameter: ~ 150 nm




(Slide from Martin Guthold)

Review of Chaps. 34-41:

Maxwell's equations:

Coulomb-Gauss law: $$\oint_{\text{area}} \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\varepsilon_0}$$

Gauss's for magnetic field: $$\oint_{\text{area}} \mathbf{B} \cdot d\mathbf{A} = 0$$

Biot-Savart-Ampere-Maxwell law: $$\oint_{\text{line}} \mathbf{B} \cdot d\mathbf{s} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\left(\int \mathbf{E} \cdot d\mathbf{A}\right)}{dt}$$

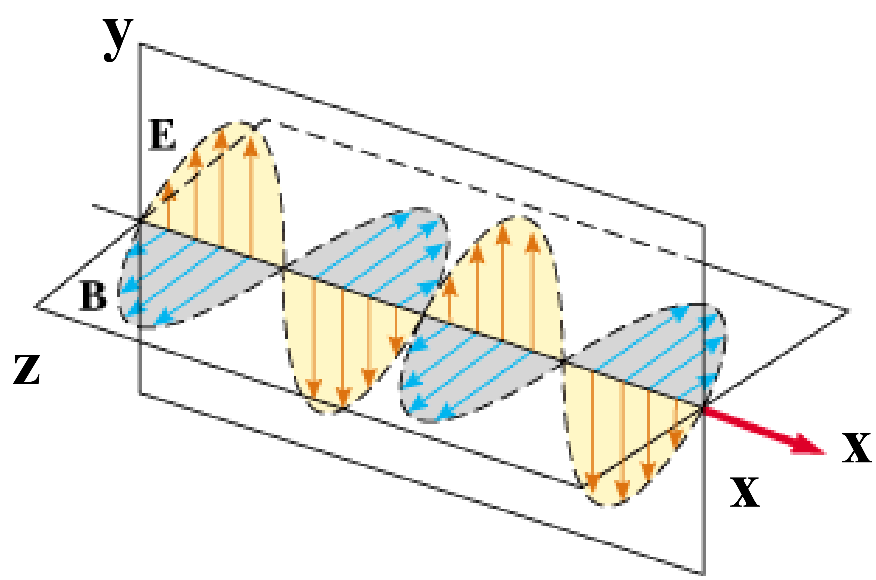
Faraday's law: $$\oint_{\text{line}} \mathbf{E} \cdot d\mathbf{s} = -\frac{d\left(\int \mathbf{B} \cdot d\mathbf{A}\right)}{dt}$$

Solution to Maxwell's equations for linearly polarized electromagnetic plane waves:

$$E_y(x,t) = E_{max}\sin\left(\frac{2\pi}{\lambda}(x-ct)\right) = E_{max}\sin\left(\frac{2\pi x}{\lambda} - 2\pi f t\right) = E_{max}\sin(kx-\omega t)$$

$$B_z(x,t) = \frac{E_{max}}{c}\sin\left(\frac{2\pi}{\lambda}(x-ct)\right) = \frac{E_{max}}{c}\sin\left(\frac{2\pi x}{\lambda} - 2\pi f t\right) = \frac{E_{max}}{c}\sin(kx-\omega t)$$

$$\lambda f = \frac{\omega}{k} = c = \sqrt{\frac{1}{\mu_0\varepsilon_0}} = 2.99792458\times 10^8 \, m/s$$



Energy density (joules/m$^3$) of electromagnetic wave

$$u \equiv u_E + u_B = \tfrac{1}{2}\varepsilon_0 E^2 + \tfrac{1}{2} B^2/\mu_0$$

Time averaged value: $\langle u \rangle_{avg} = \tfrac{1}{2}\varepsilon_0 E_{\max}^2 = \tfrac{1}{2} B_{\max}^2/\mu_0$

Poynting vector [(Joules/s)/m$^2$=Watts/m$^2$]:

$$\mathbf{S} \equiv \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B}$$

Time averaged value:

$$\langle \mathbf{S} \rangle_{avg} = \frac{\hat{\mathbf{x}}}{2\mu_0 c} E_{\max}^2 = \frac{\hat{\mathbf{x}} c}{2\mu_0} B_{\max}^2$$

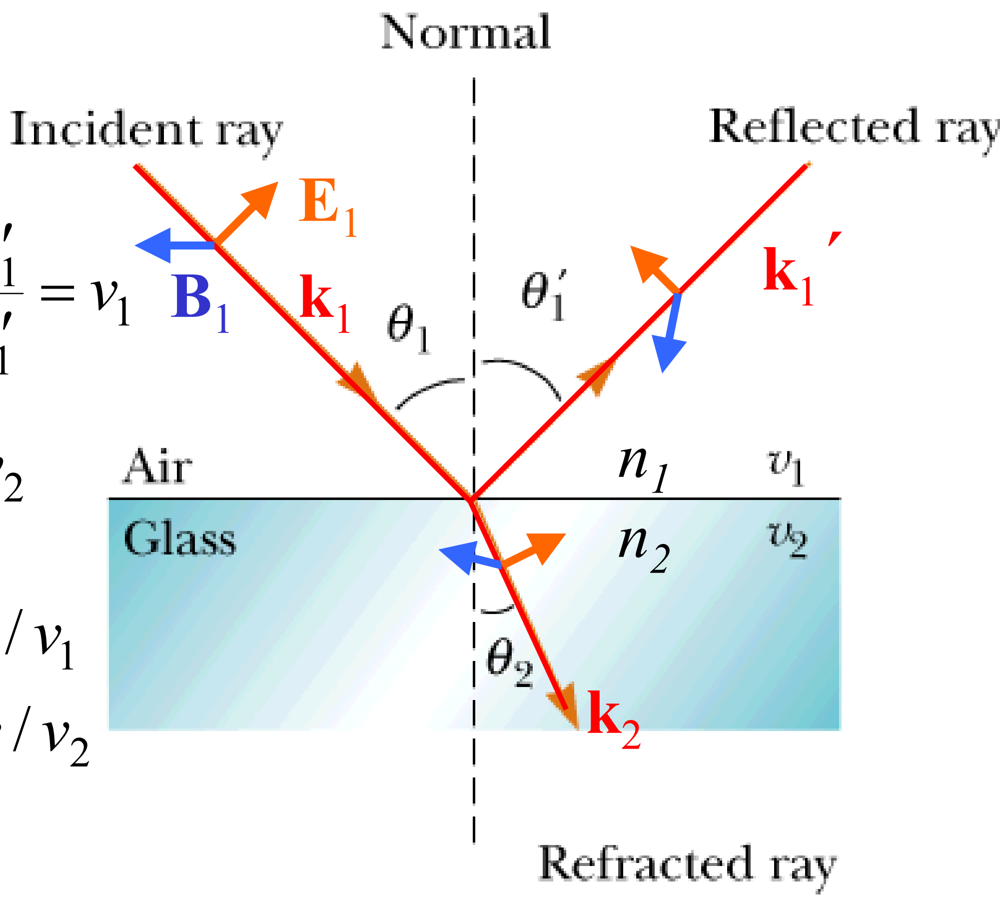
Consider the behavior of a plane-polarized electromagnetic wave near the surface of two materials

$$\frac{\omega_1}{k_1} = \frac{\omega_1'}{k_1'} = v_1$$

$$\frac{\omega_2}{k_2} = v_2$$

$$n_1 \equiv c / v_1$$

$$n_2 \equiv c / v_2$$




Relationships:

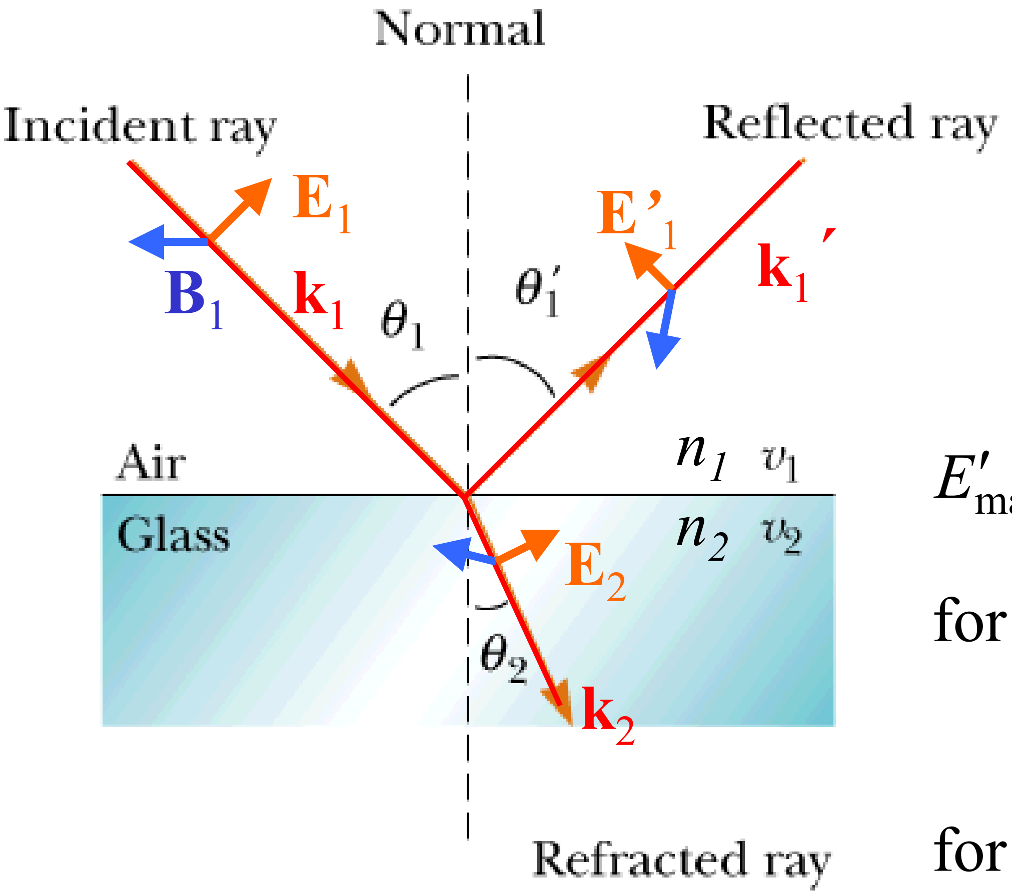
$$\theta_1 = \theta_1'$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

(Snell's law)

Note: in general $n = n(\omega)$

Plane waves reflected and refracted at surface:




Matching electric and magnetic fields at boundary:

For reflected waves:

$$E'_{\max_1} = E_{\max_1} \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2}$$

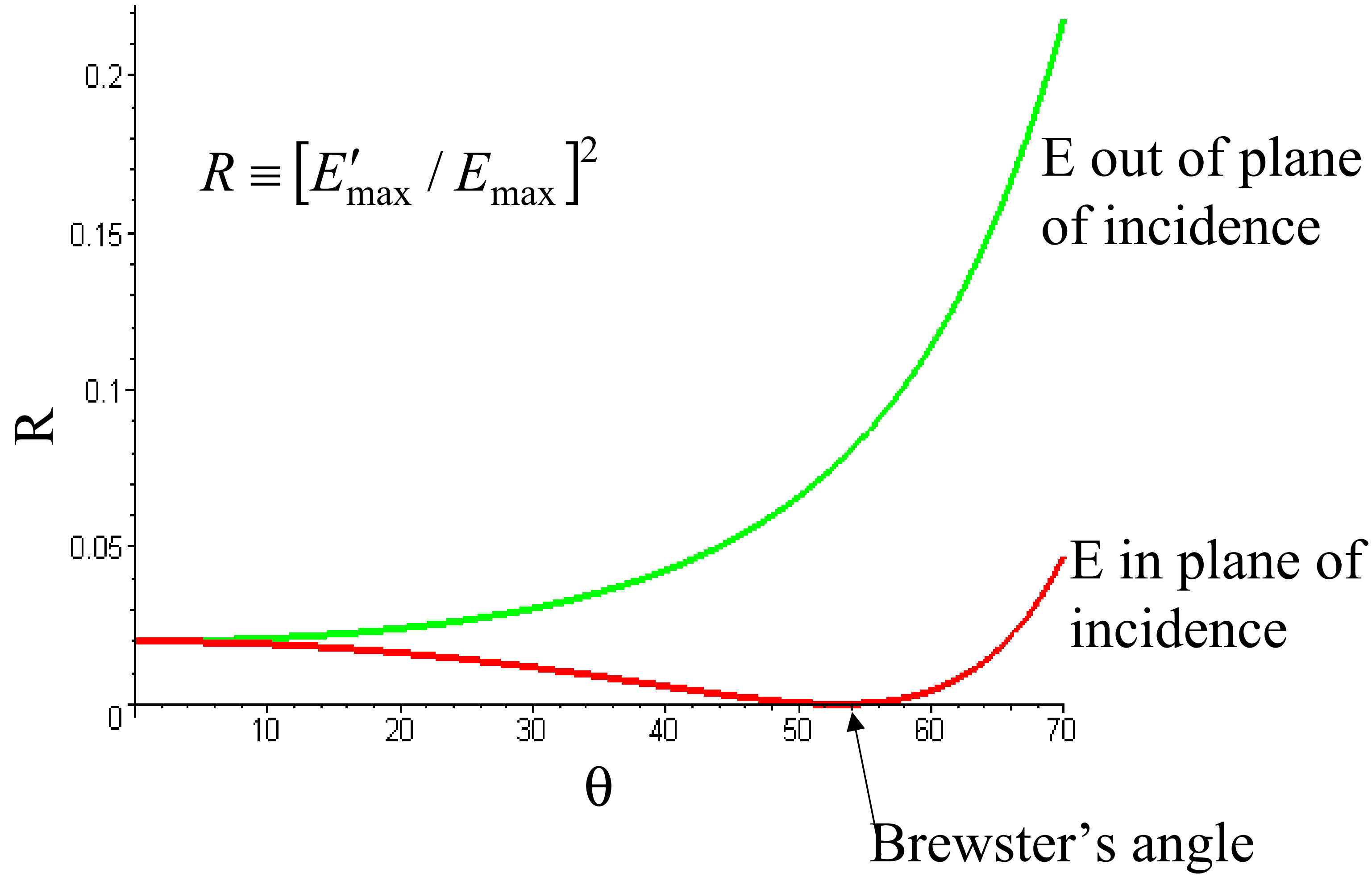
for E in plane of incidence

or

for E out of plane of incidence:

$$E'_{\max_1} = E_{\max_1} \frac{n_2 \cos\theta_2 - n_1 \cos\theta_1}{n_2 \cos\theta_2 + n_1 \cos\theta_1}$$

Example: $n_1 = 1, n_2 = 1.33$

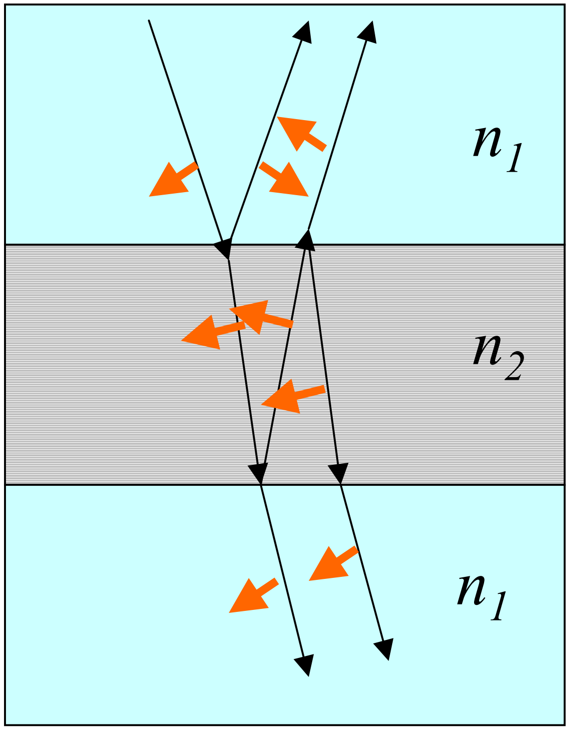



$\tan^{-1}(n_2/n_1)=53^o$

Interference of refracted and reflected waves in thin films

Example of interference with "–"

$n_1 > n_2$




$$\Rightarrow I = I_{\max}\left\{\sin\left(\tfrac{1}{2}(A-B)\right)\right\}^2$$

peaks at

$$\tfrac{1}{2}(A-B) \equiv \frac{\pi(r_2-r_1)}{\lambda} = \left(m+\tfrac{1}{2}\right)\pi$$

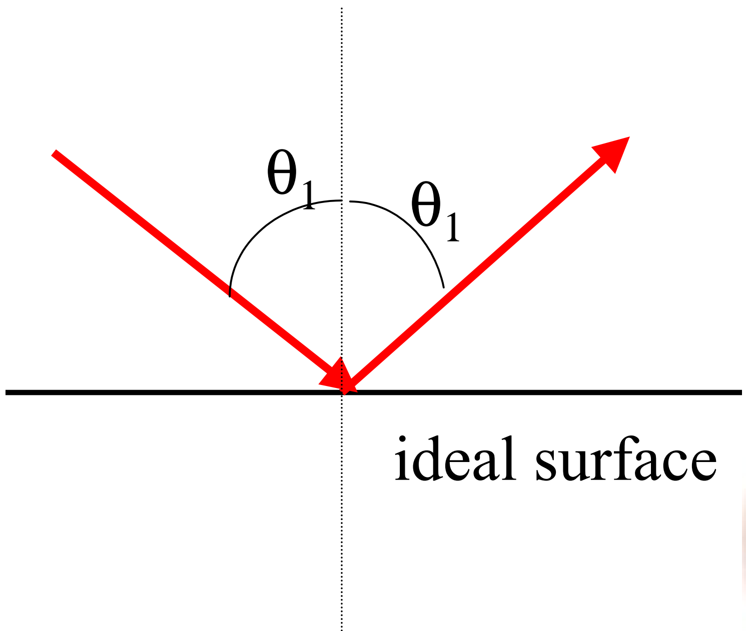
Example of interference with "+"

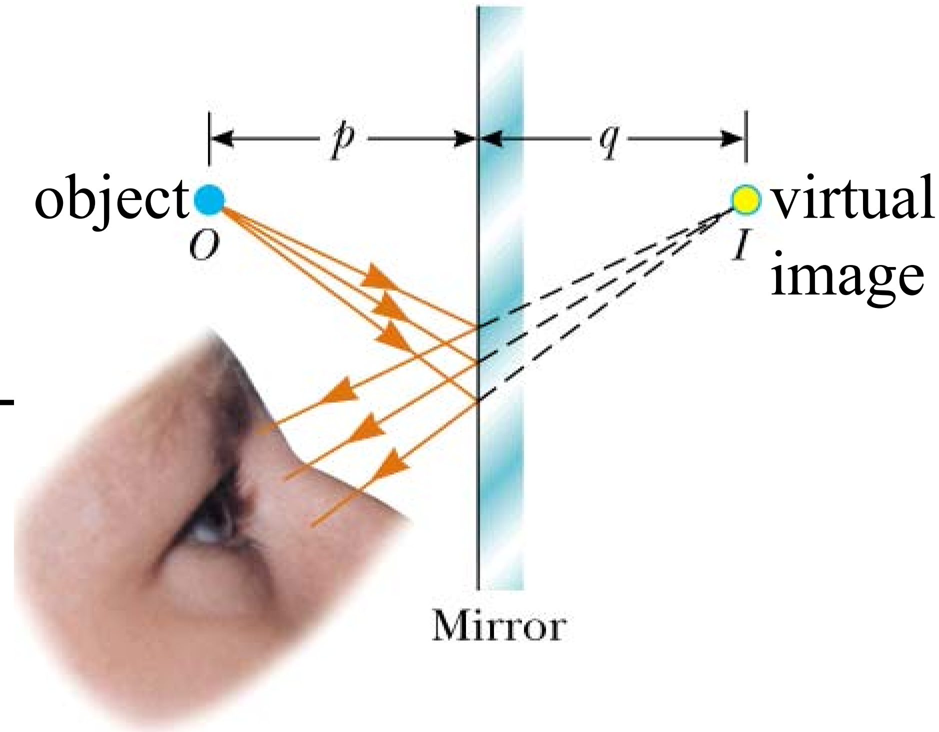
$$\Rightarrow I = I_{\max}\left\{\cos\left(\tfrac{1}{2}(A-B)\right)\right\}^2$$

peaks at

$$\tfrac{1}{2}(A-B) \equiv \frac{\pi(r_2-r_1)}{\lambda} = m\pi$$

# Images formed from reflected and refracted light



Mirror and lens equation: $$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

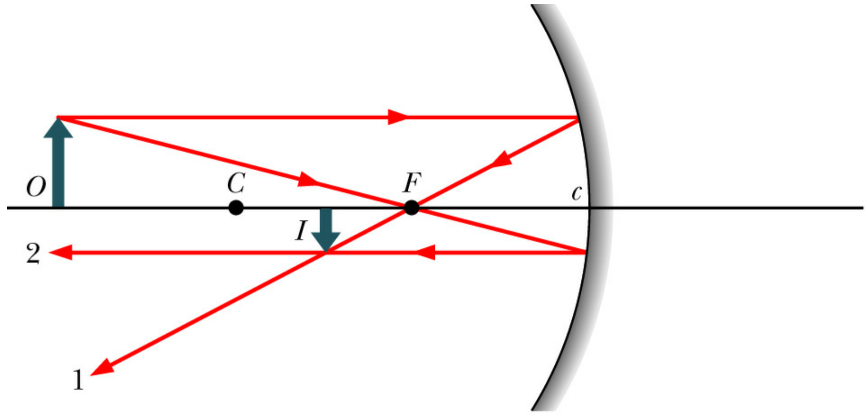
object distance

focal length (+ for converging, – for diverging )

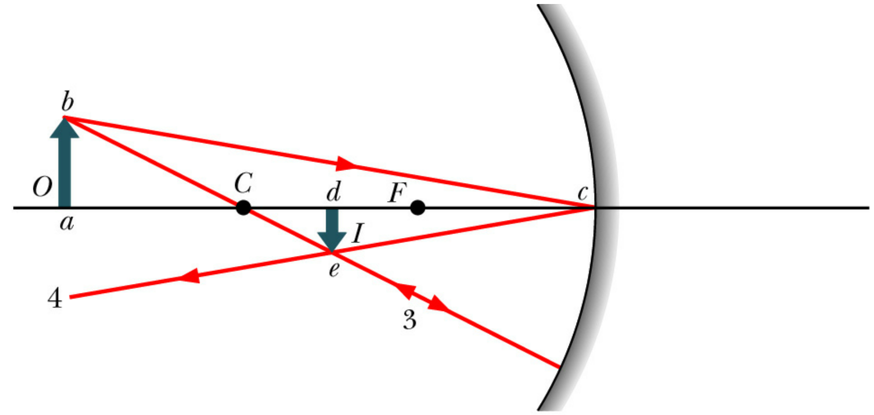
image distance (+ for real, – for virtual)

Magnification of image: $$M = \frac{h'}{h} = \frac{-q}{p}$$

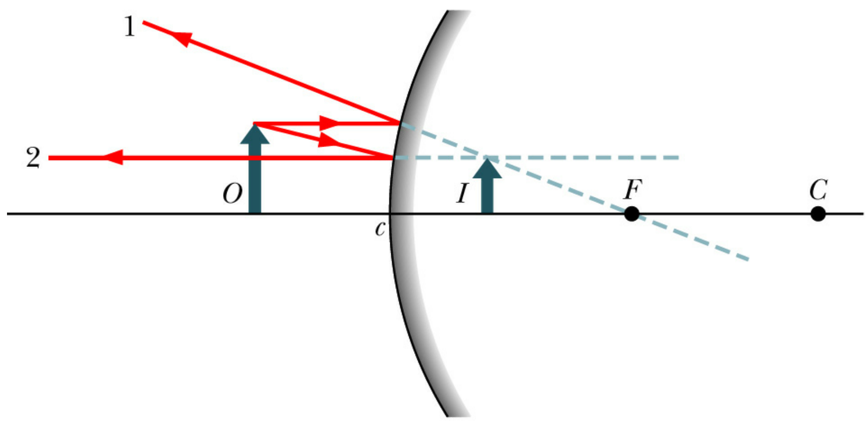
# Example of  ray diagrams for spherical mirrors:



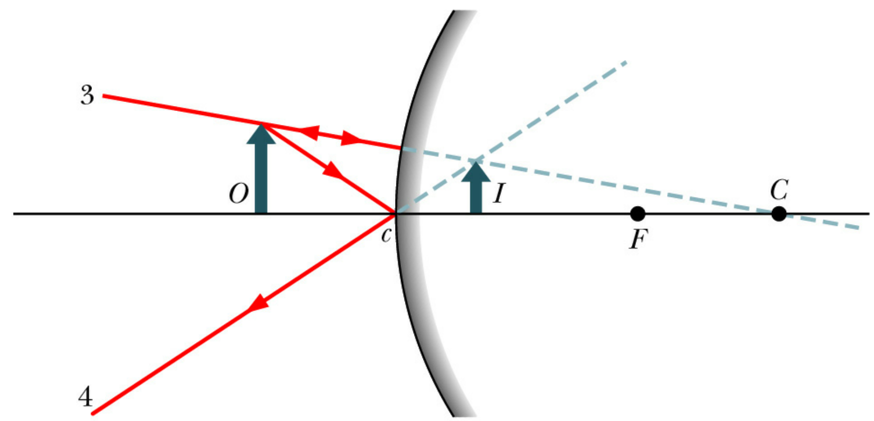

(a)




(b)



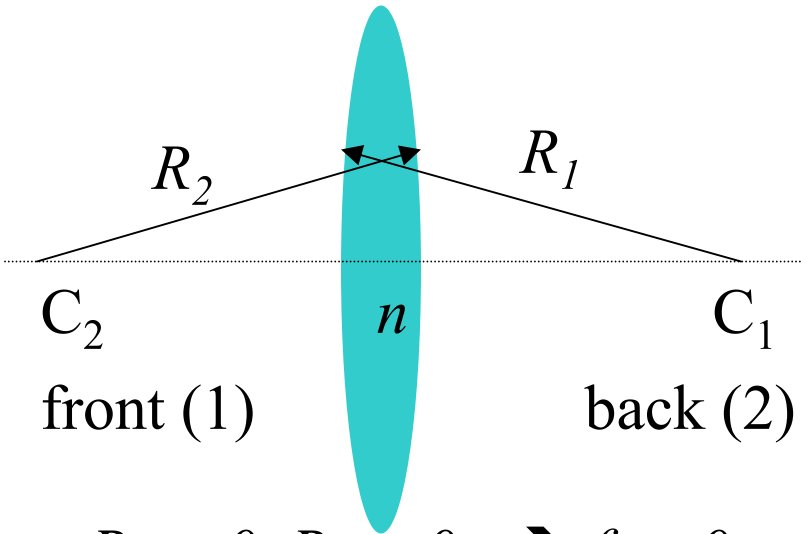

(c)




(d)

Lens makers' equation:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

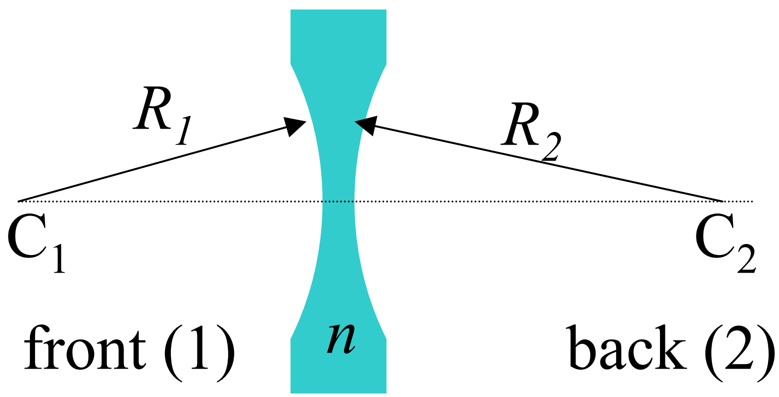



Sign convention:

$R_i$ is positive if $C_i$ is in "back" of len

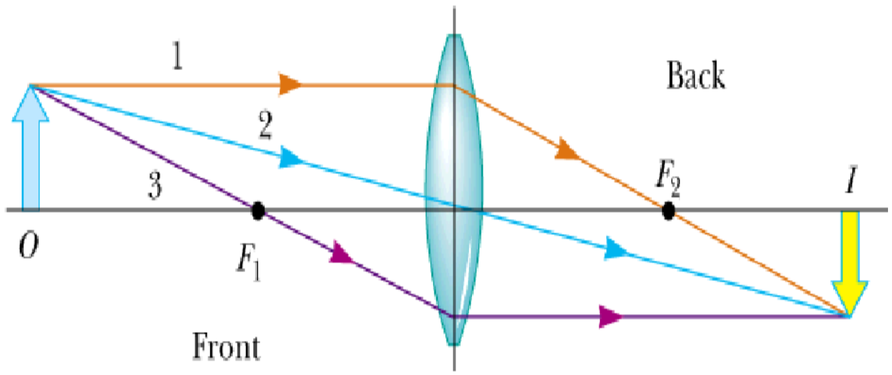
$R_i$ is negative if $C_i$ is in "front" of le

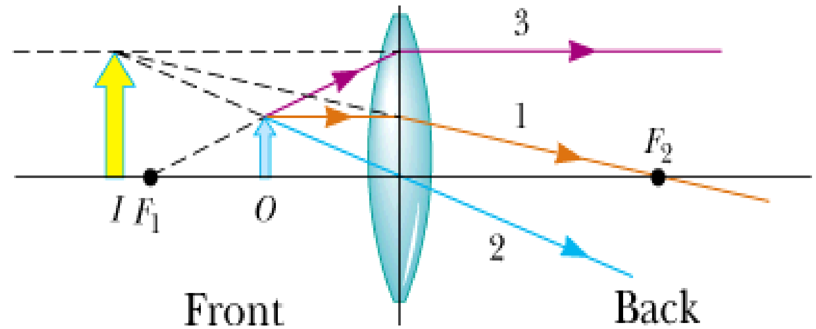
$R_1 > 0,\ R_2 < 0$ ➔ $f > 0$ ➔ converging lens

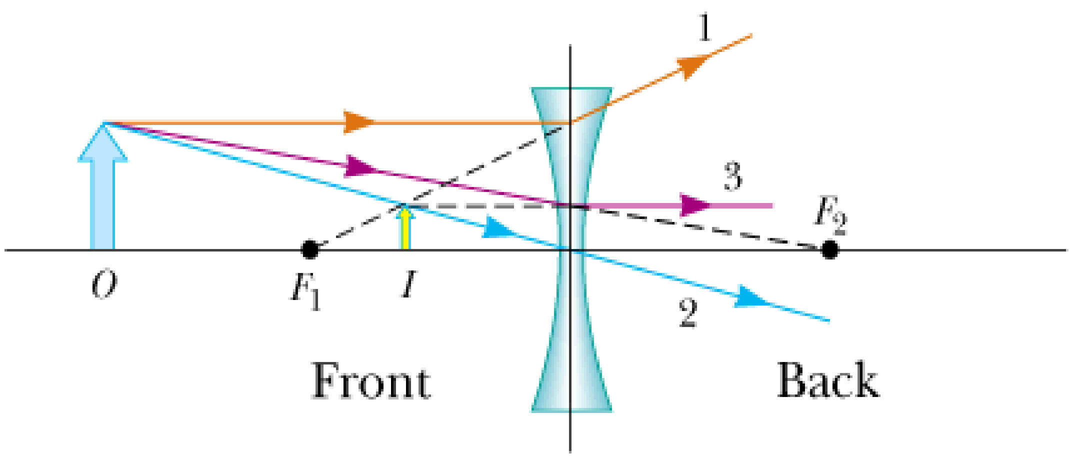



$R_1 < 0,\ R_2 > 0$ ➔ $f < 0$ ➔ diverging lens

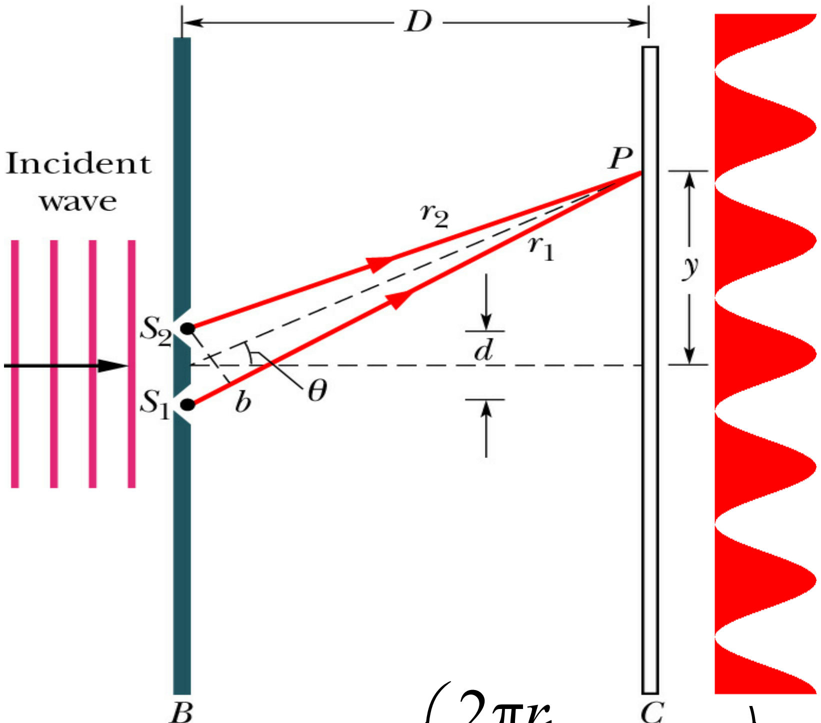
# Examples of  array diagrams for thin lenses:

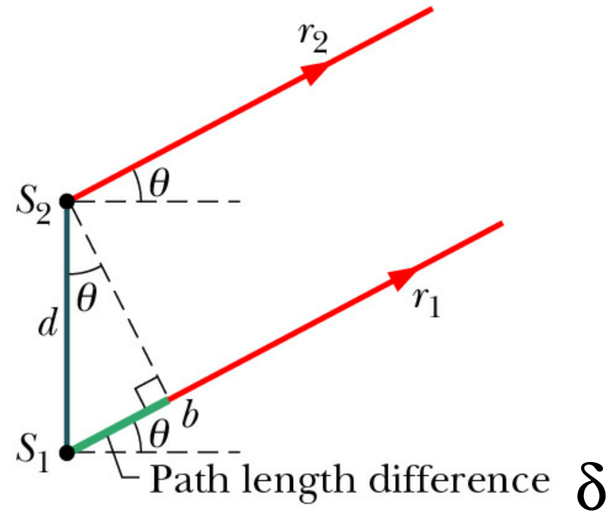


# Diffraction pattern from a plane wave incident on a double slit:




$$r_1 - r_2 = \delta = d \sin \theta$$

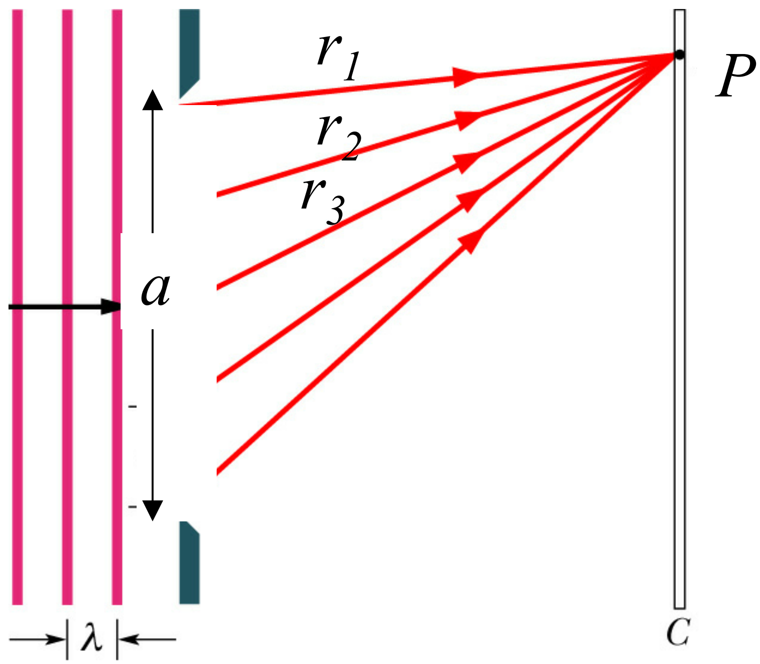
$$E(P,t) = E_{\max} \sin\left(\frac{2\pi r_1}{\lambda} - 2\pi f t\right) + E_{\max} \sin\left(\frac{2\pi r_2}{\lambda} - 2\pi f t\right)$$

$$= 2E_{\max} \sin\left(\frac{\pi(r_1 + r_2)}{\lambda} - 2\pi f t\right) \cos\left(\frac{\pi(r_1 - r_2)}{\lambda}\right)$$

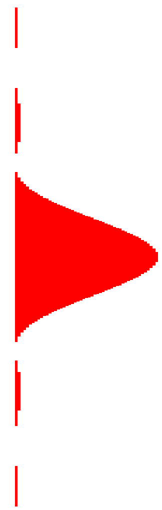
➔ intensity maxima occur for $\frac{\pi(r_1 - r_2)}{\lambda} = m\pi \quad \Rightarrow d \sin \theta = m\lambda$

Single slit intensity pattern:

$$\langle I \rangle_{av} = I_{\max} \left\{ \frac{\sin\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\left(\frac{\pi a \sin\theta}{\lambda}\right)} \right\}^2$$

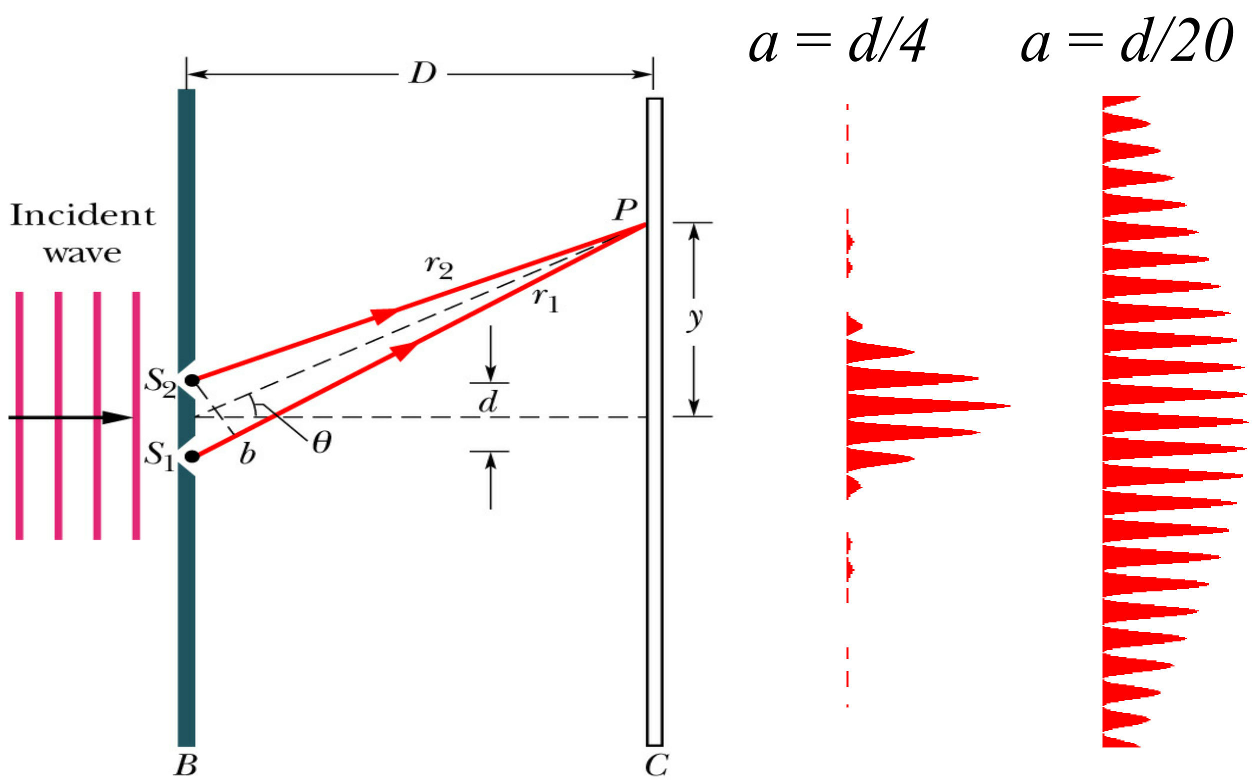



small $a$

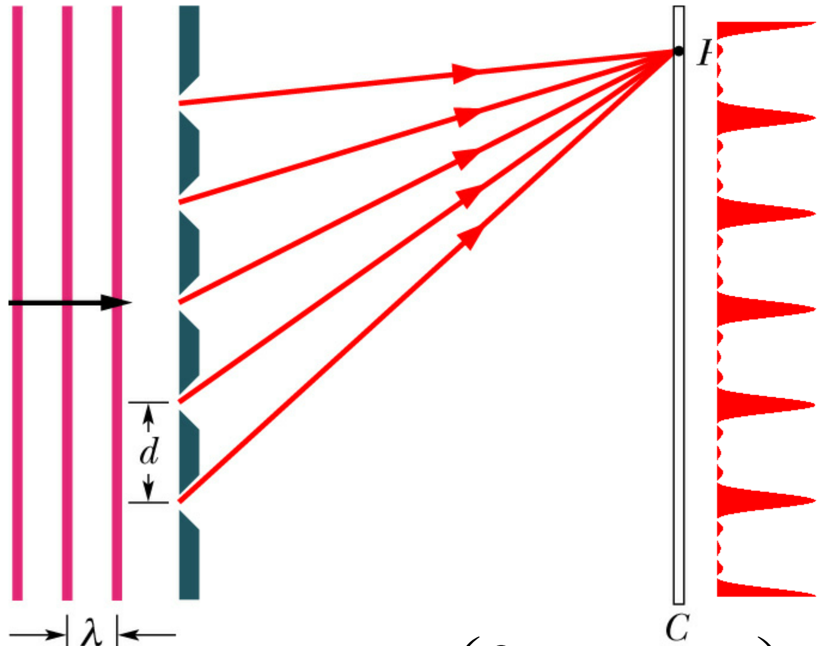


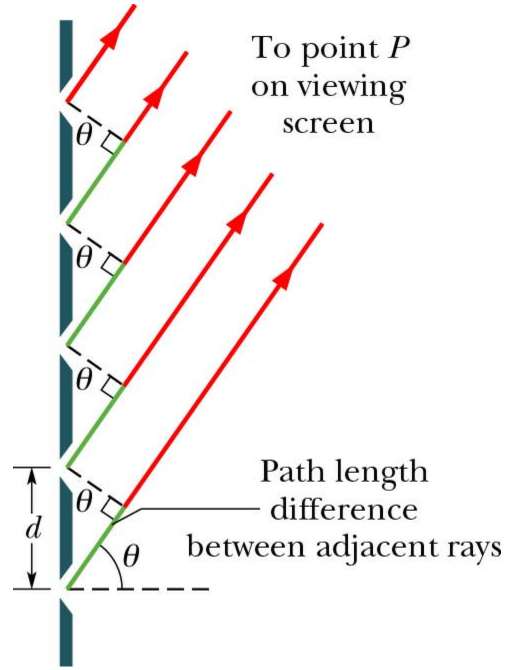
smaller $a$

Effect of slit size on double slit pattern



# Diffraction pattern for *N* slits – diffraction grating



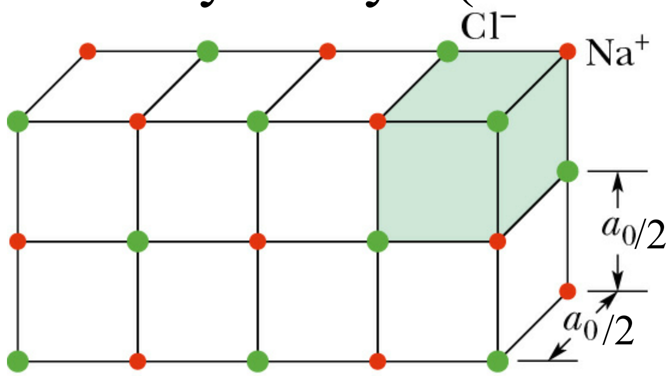




$$E(P,t) = \sum_i E_{\max} \sin\left(\frac{2\pi r_i}{\lambda} - 2\pi f t\right)$$

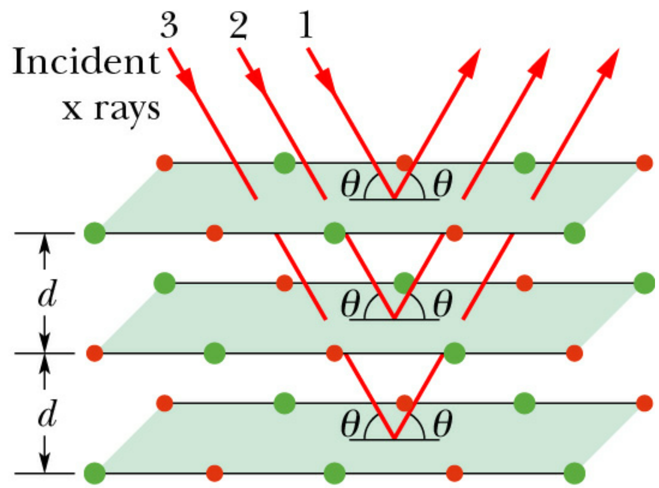
$$= E_{\max} \sin\left(\frac{2\pi r_{av}}{\lambda} - 2\pi f t\right) \frac{\sin\left(\frac{N\pi d \sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi d \sin\theta}{\lambda}\right)}$$

Intensity maxima at $d \sin\theta = m\lambda$

# Diffraction by X-rays ($\lambda \approx 0.1$ nm)

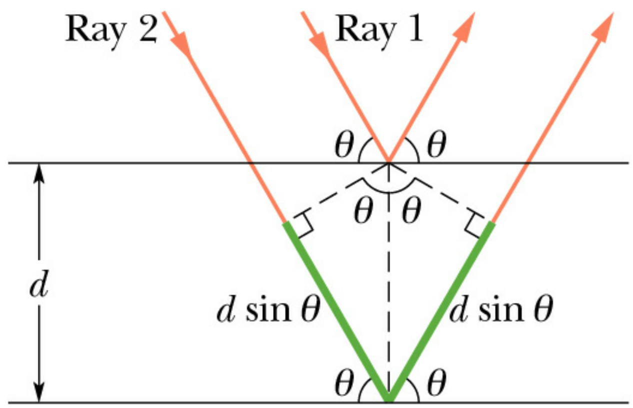



(*a*)

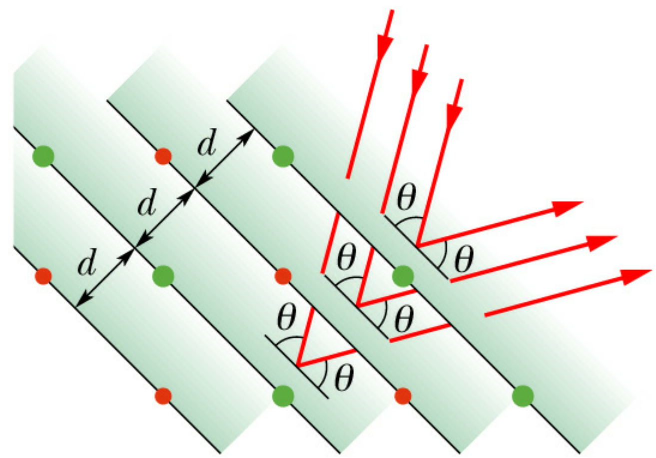



(*b*)

NaCl $a_0 \approx 0.56$ nm




Bragg condition: (*c*)




(*d*)

$$2d \sin\theta = m\lambda$$

Modern physics

Special theory of relativity:

$$\gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\mathbf{p} = \gamma m \mathbf{u} \qquad E = \gamma mc^2$$

$$E^2 = p^2c^2 + m^2c^4$$

Kinetic energy: $K = E - mc^2 \approx \frac{p^2}{2m}$ if $v << c$

Quantum physics:

Photon $E=hf$, $p=h/\lambda$

Electron $p=h/\lambda$

Heisenberg's hypotheses: $\Delta x \Delta p \geq \frac{\hbar}{2} \quad \Delta t \Delta E \geq \frac{\hbar}{2}$