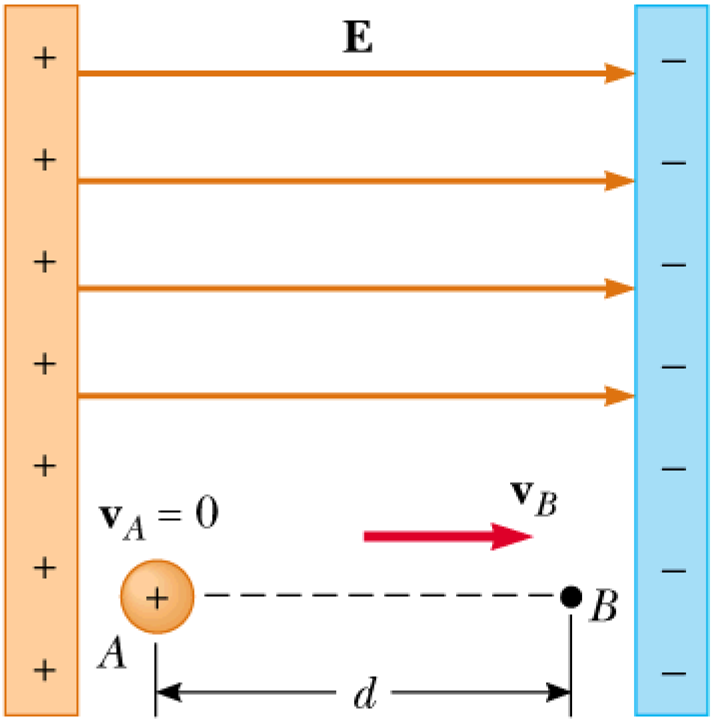
Announcements

1. Correction to Lecture 4 –
2. Problem solving session tomorrow (Tuesday) 6 PM
3. Today's topic – capacitance and dielectrics
    a. Parallel plate capacitors – relationship between charge and voltage
    b. Dielectric properties of materials and how that relates to capacitors
    c. Capacitors as components of a circuit

Work and electrostatic potential energy

$$W = \int_A^B \mathbf{F}\cdot d\mathbf{r} = q\int_A^B \mathbf{E}\cdot d\mathbf{r} = U_A - U_B$$

Because electrostatic forces are conservative.



Example:

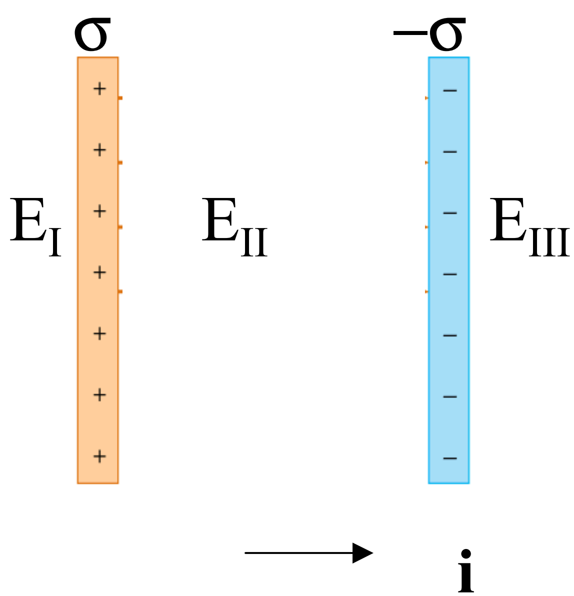
A proton (q=1.6x10^{-19}C) moves a distance d = 0.1m in the direction of a uniform electric field **E**=8000N/C.

What is its change in kinetic energy?

K_B-K_A=U_A-U_B=1.28x10^{-16} J

Electrostatic field and voltage between two charged plates

Peer instruction question:



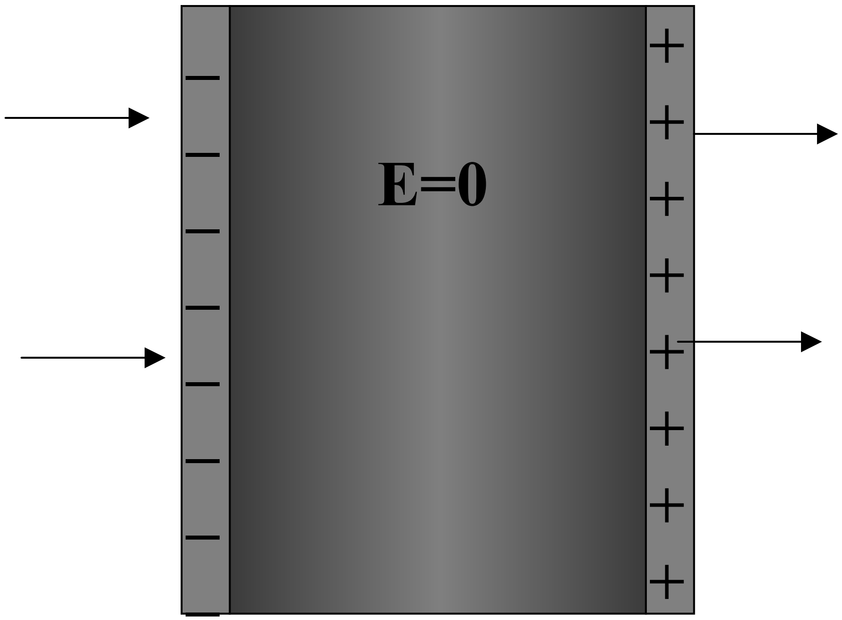
Last week we showed that the configuration of two charged sheets shown on the left correspond to the electrostatic fields $E_I = E_{III} = 0$ and $\mathbf{E}_{II} = \frac{\sigma}{\varepsilon_0}\mathbf{i}$.

Would anything change if the charged sheets were made out of a conducting material?

(A) Yes (B) No

Some special properties of conductors:

- Within conductor electrostatic field vanishes
- Excess or asymmetric charges reside only at the surfaces

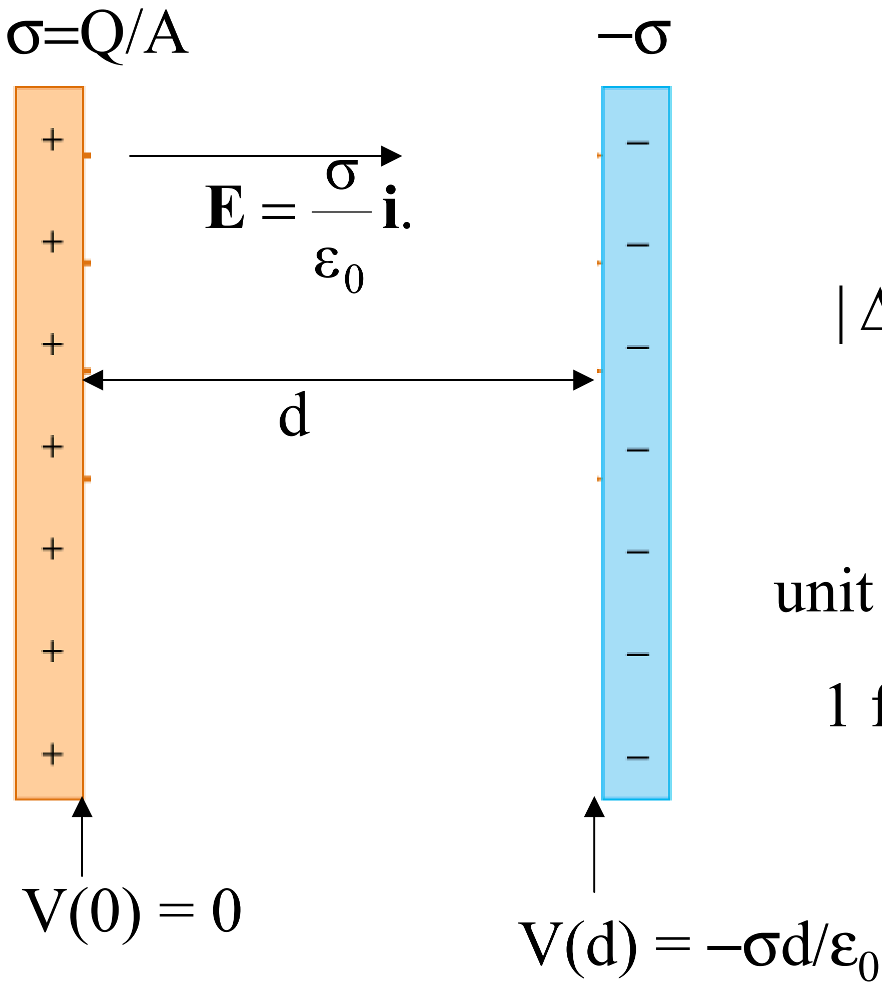


Note that $\mathbf{E} = 0$

$\Rightarrow$ V = (constant)

Thus, the interior of a metal is a region of *constant* electrostatic potential V.

Electrostatic field and voltage between two charged plates



$$|\Delta V| = \frac{\sigma d}{\varepsilon_0} = \frac{Qd}{A\varepsilon_0} \equiv \frac{Q}{C}$$

$$\text{where, } C \equiv \frac{A\varepsilon_0}{d}.$$

unit of capacitance:

1 f = 1 “farad” (named for Michael Faraday)

= 1 Volt/Coulomb

Relationship between voltage and charge:

$$|\Delta V| = \frac{Q}{C}$$ ➔ general relationship for many geometries

$$\text{for parallel plate configuration:} \quad C \equiv \frac{A\varepsilon_0}{d}.$$

Example: $A = 0.02\ \text{m}^2$, $d = 0.003$ m $V = 100$ V

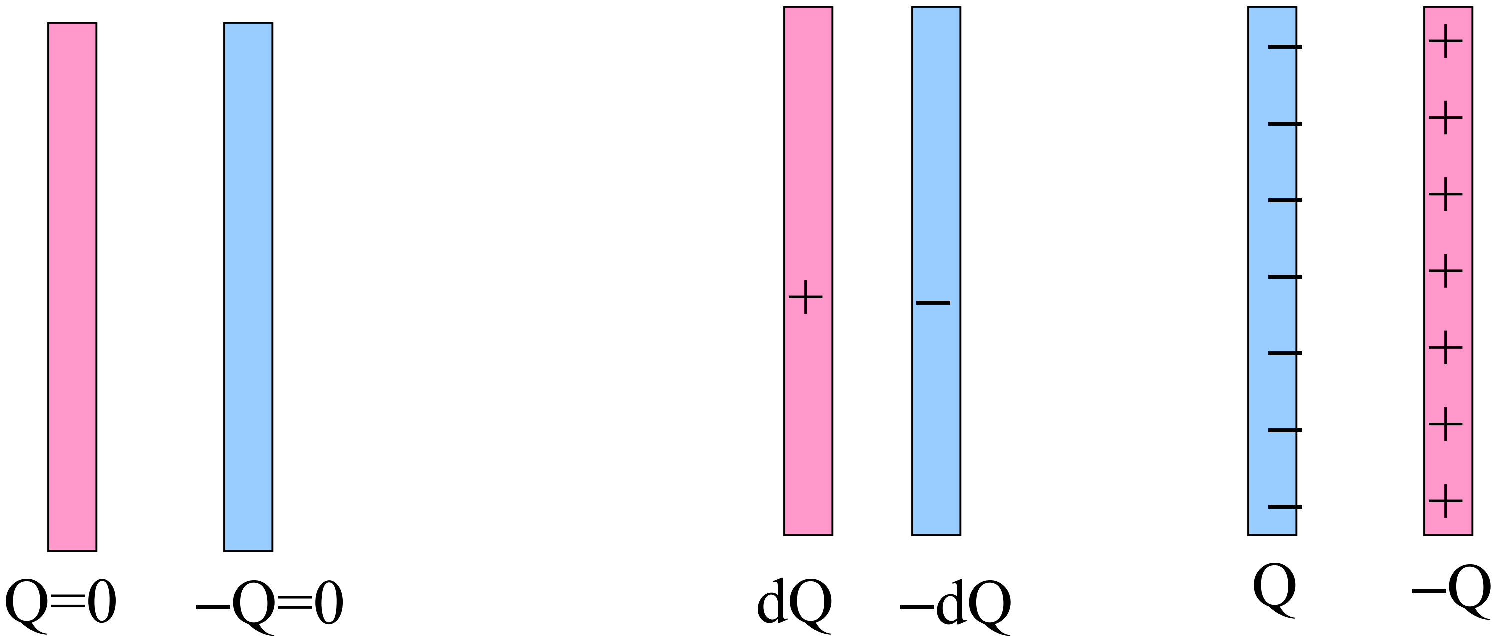
➔ $C = 5.9 \times 10^{-11}$ f

➔ $Q = 5.9 \times 10^{-9}$ C

A capacitor can be used to store charge and energy.

Energy stored in a capacitor.

Electrical work without motion--



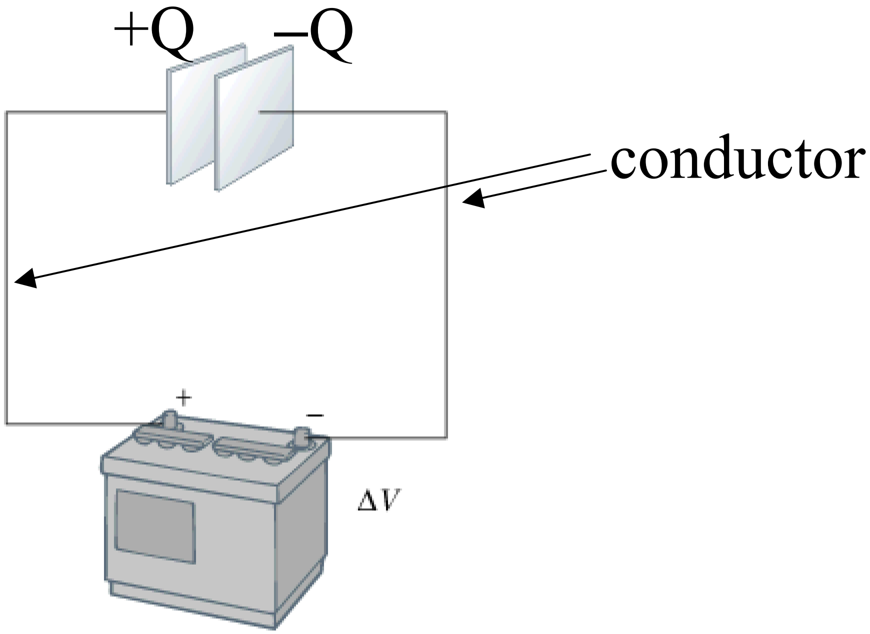
$$dU = dq\, V$$

$$U = \int_0^U dU = \int_0^Q dq\, V = \int_0^Q dq\, \frac{q}{C}$$

$$\boxed{\Rightarrow U(Q) = \frac{Q^2}{2C}}$$

Capacitors in a circuit:

$$Q = \Delta V\, C$$

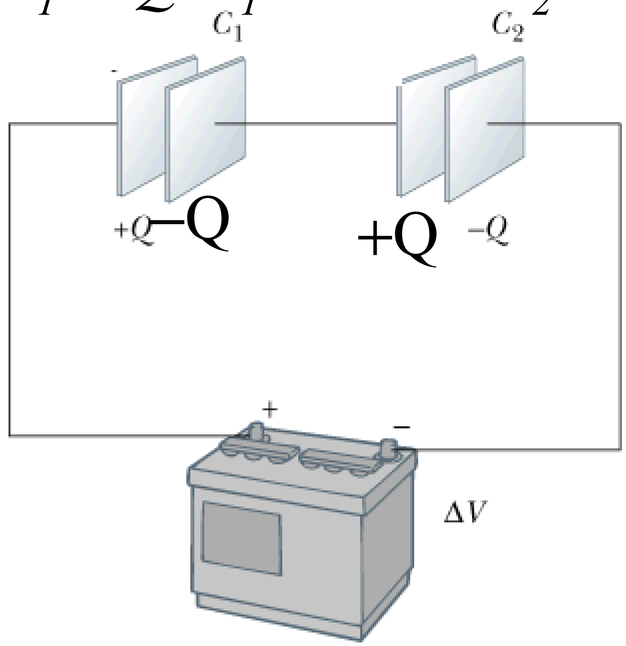


battery – source of charge (thanks to chemistry)

Two capacitors in a circuit –

Consider the following configuration:

$\Delta V_1 = Q/C_1$ $\qquad$ $\Delta V_2 = Q/C_2$



$$\Delta V = \Delta V_1 + \Delta V_2$$

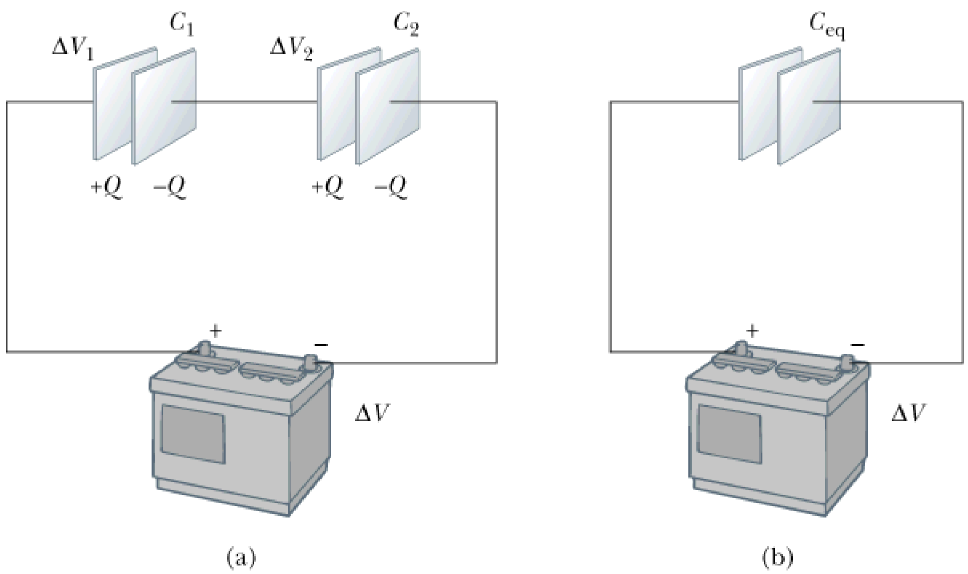
$$= \frac{Q}{C_1} + \frac{Q}{C_2} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

Capacitors connected in *series* are equivalent to C_{eq}:

$$\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i}$$

Peer instruction question

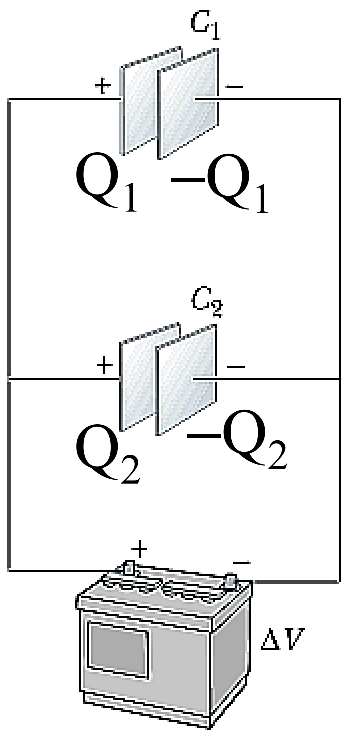
Consider a circuit with two capacitors $C_1 = C_2 = 2$ pf as shown. If these were replace by a single capacitance C_{eq}, what would be its value?



(A) 1 pf (B) 2 pf (C) 3 pf (D) 4 pf

Two capacitors in a circuit –

Consider the following configuration:



$$\Delta V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

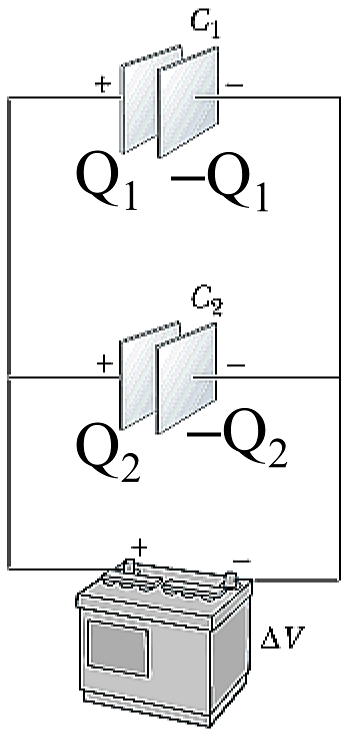
$$Q = Q_1 + Q_2 = C_1 \Delta V + C_2 \Delta V$$
$$= (C_1 + C_2)\Delta V$$

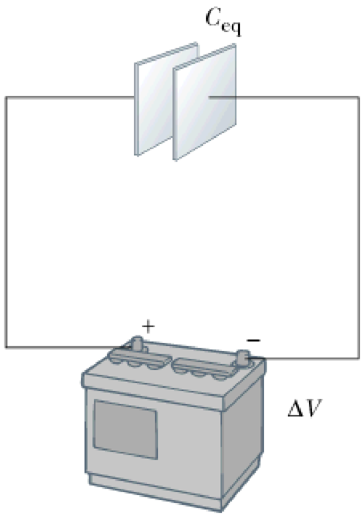
Capacitors connected in *parallel* are equivalent to C_{eq}:

$$C_{eq} = \sum_i C_i$$

Peer instruction question

Consider a circuit with two capacitors $C_1 = C_2 = 2$ pf as shown. If these were replace by a single capacitance C_{eq}, what would be its value?





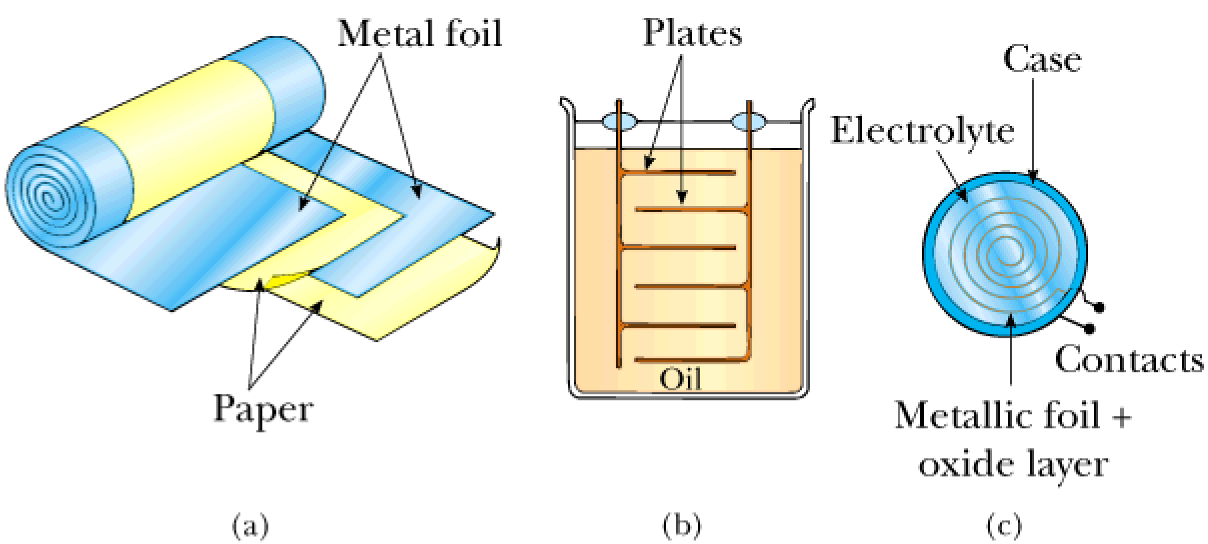
(b)

(A) 1 pf (B) 2 pf (C) 3 pf (D) 4 pf

Practical capacitor design

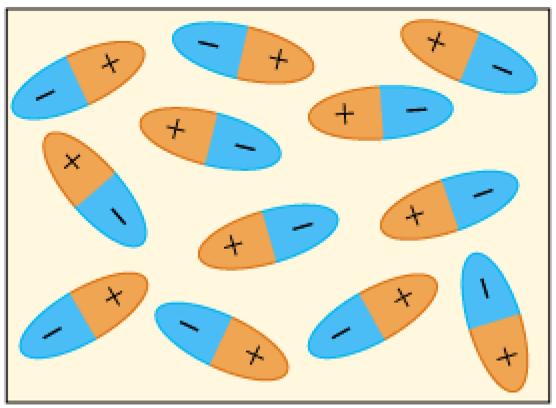
•Compact geometries

•Dielectric media $\varepsilon_0 \rightarrow \varepsilon = \kappa\,\varepsilon_0$ $C_\kappa = \kappa\, C_0$

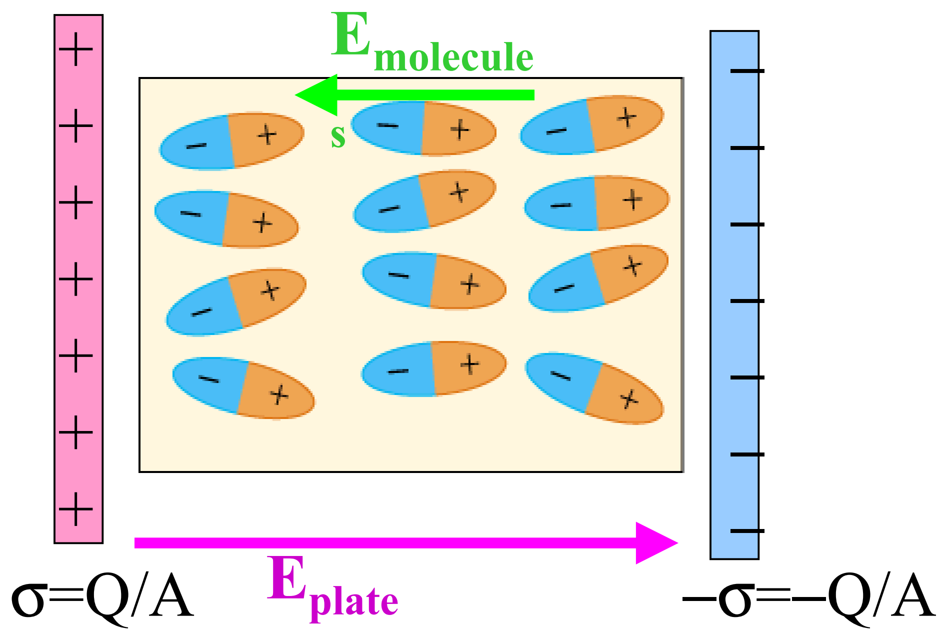


How dielectrics work:

Polar molecules in the absense of external forces:



Polar molecules in the aligned between two charged plates:



$$\mathbf{E}_{\mathbf{total}} = \mathbf{E}_{\mathbf{plate}} + \mathbf{E}_{\mathbf{molecules}} = \mathbf{E}_{\mathbf{plate}} / \kappa$$

$$V_{total} = V_{plate} + V_{molecules} = V_{plate} / \kappa$$

$$V_{total} = \frac{Qd}{A\kappa\varepsilon_0} \Rightarrow C(\kappa) = \frac{A\kappa\varepsilon_0}{d}$$

Some values of dielectric constants --

Material	κ
air	1
paper	3.7
water	80.0