Announcements

1. First hour exam – Monday, February 10, 2003 – covering Chapters 23-28.

   May bring 1 8½" x 11" sheet of paper to the exam (to be turned in with your exam papers).

   The exam will be proctored by Professor Salsbury.

   Practice exam available on website.

   Extra review session Friday afternoon (2/7/03)?

2. Today's topics – DC circuits

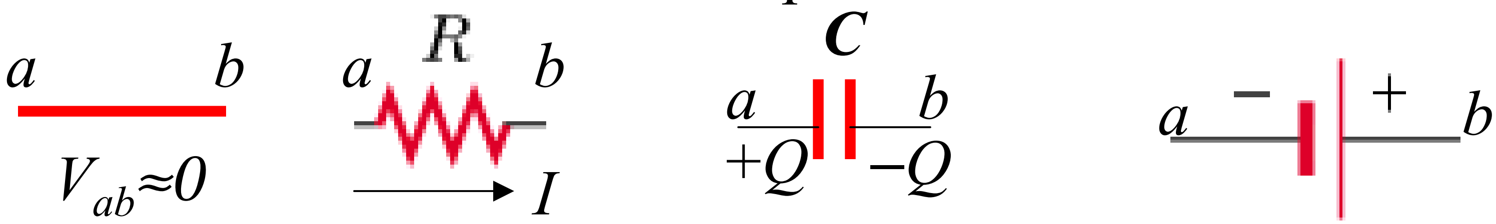
   $\mathcal{E}$, R, C, I, Q

   Kirchhoff's analysis method

Analysis of DC circuits:

Elements:

wire | resistor | capacitor | emf

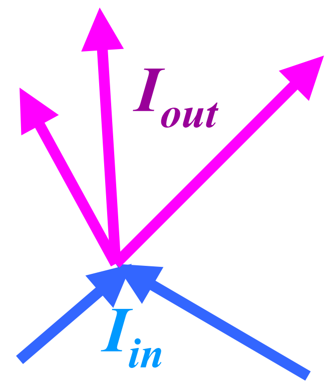


$V_{ab} \approx 0$ | $V_{ab} = -IR$ | $V_{ab} = -Q/C$ | $V_{ab} = \mathcal{E}$

The principles:

Kirchhoff's rules

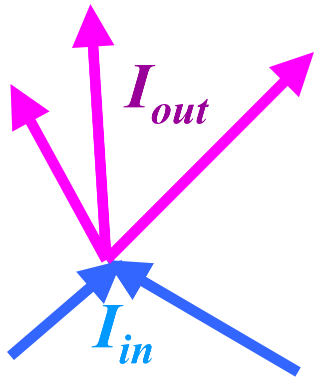
At any wire junction: $\sum I_{in} = \sum I_{out}$



For any closed wire loop: $\sum \Delta V = 0$

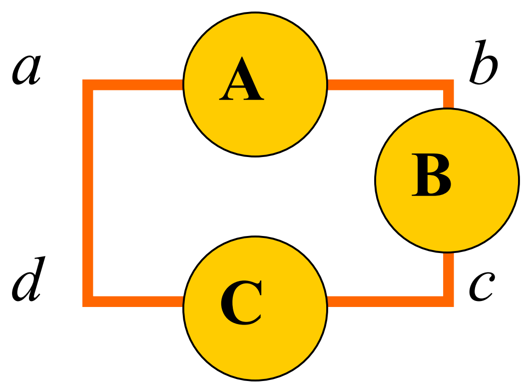
Why?

Junction rule: $\sum I_{in} = \sum I_{out}$



Consequence of conservation of charge; assumes no leakage, sparking, etc.

Loop rule: $\sum \Delta V = 0$

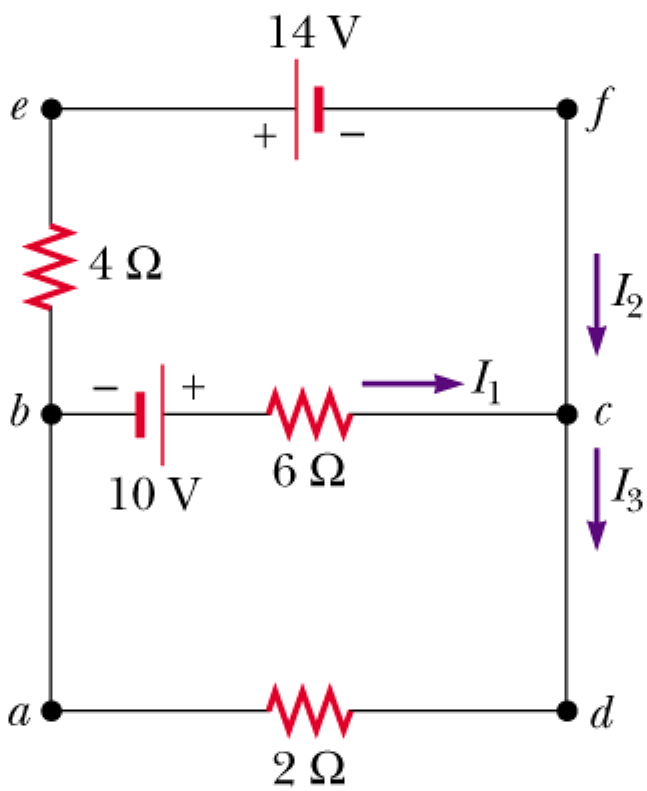


Consequence of electrostatic potential being derived from a conservative electric field

$$\sum \Delta V = (V_b - V_a) + (V_c - V_b) + (V_d - V_c) + (V_a - V_d) = 0$$

Peer instruction question

Which of the loops of the following circuit follow the "loop rule"?



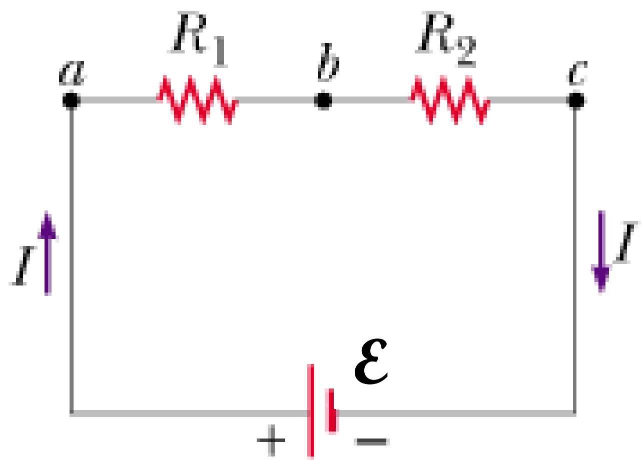
(A) abcda

(B) befcb

(C) abefcda

(D) All of these.

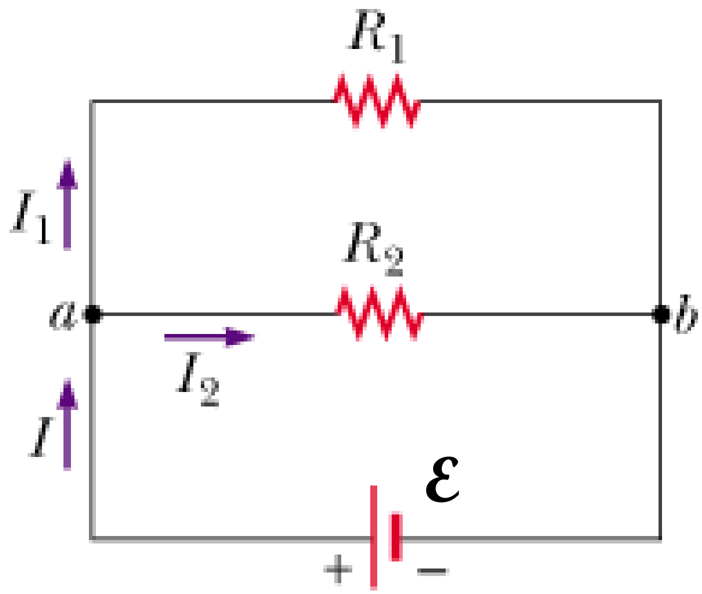
Example: resistors in series:



$$R_{eq} = \sum_i R_i$$

$$-IR_1 - IR_2 + \mathcal{E} = 0 \;\Rightarrow\; \mathcal{E} = I\,(\underbrace{R_1 + R_2}_{R_{eq}})$$

Example: resistors in parallel



$$\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$$

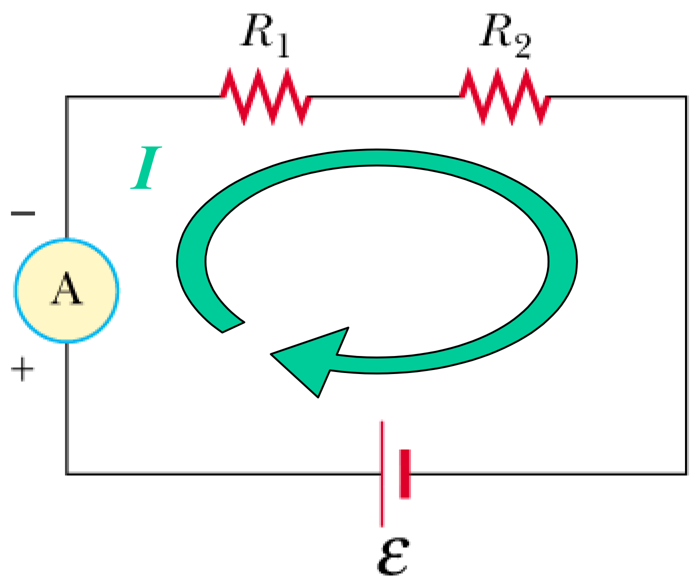
$$\text{upper loop}: \; -I_1R_1 + I_2R_2 = 0$$

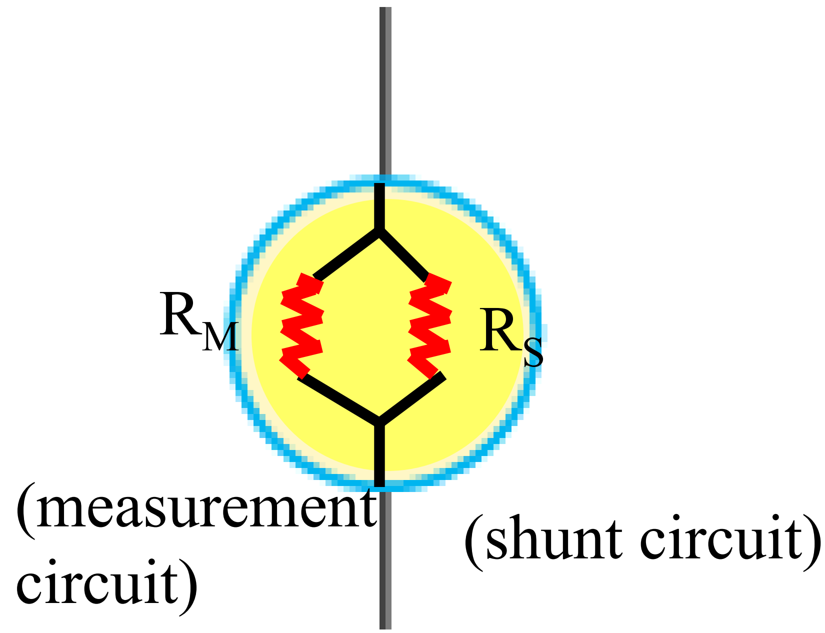
$$\text{lower loop}: \; -I_2R_2 + \mathcal{E} = 0$$

$$\text{junction}: \; I = I_1 + I_2 \quad \Rightarrow I = \frac{\mathcal{E}}{R_{eq}} = \mathcal{E}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Practical circuits:

Ammeter



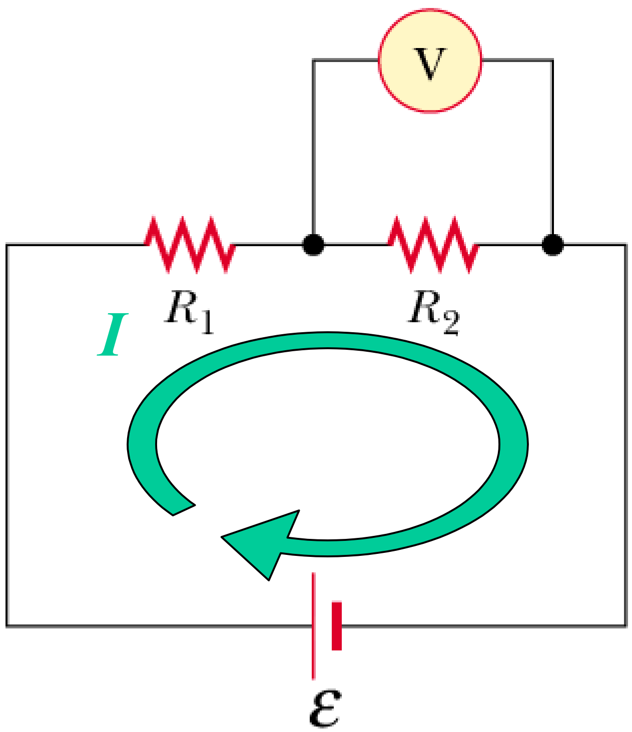


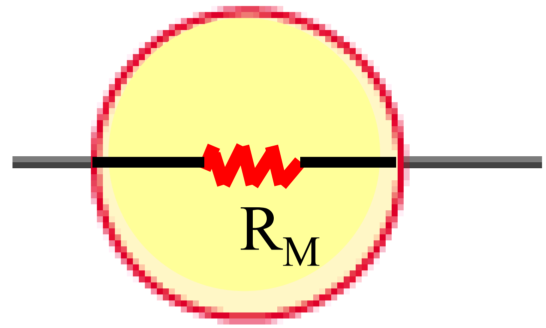
$R_S \ll R_1$, R_2

$R_M \gg R_S$

Practical circuits:

Voltmeter

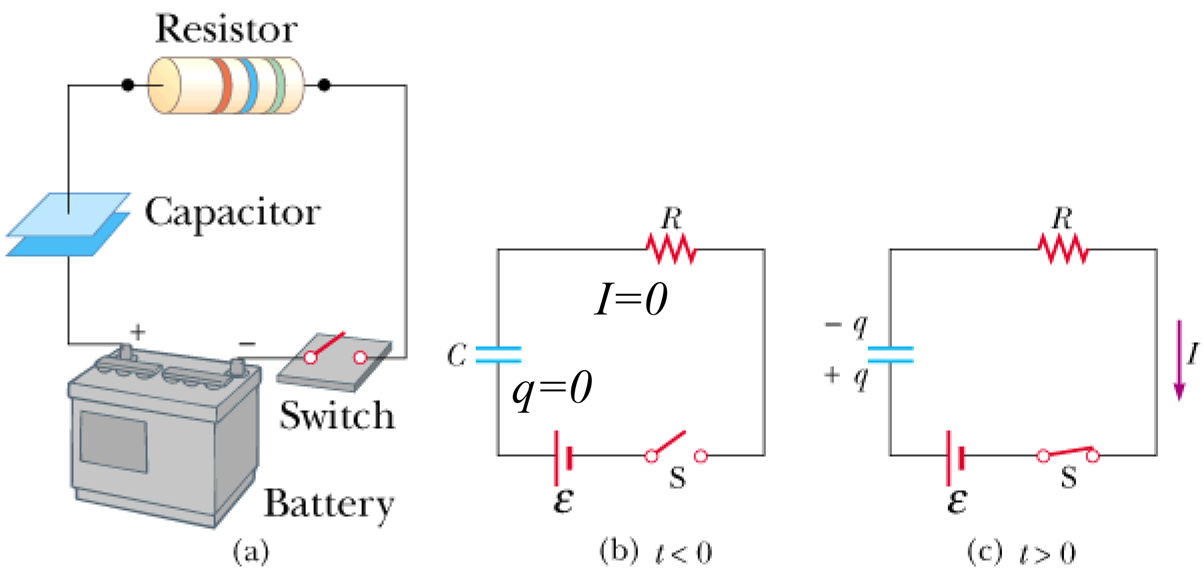




$R_M >> R_1$, R_2

Circuit containing capacitor and resistor --

Charging a capacitor:



$$\mathcal{E} - \frac{q}{C} - IR = 0 \quad \Rightarrow \quad \mathcal{E} - \frac{q}{C} - \frac{dq}{dt}R = 0$$

First order differential equation for charge $q(t)$ and current $I(t)=\frac{dq}{dt}$

$$\mathcal{E}-\frac{q}{C}-\frac{dq}{dt}R=0$$

initial condition : $q(t=0)=0$

solution :

$$q(t)=C\mathcal{E}\left(1-e^{-t/(RC)}\right)$$

$$I(t)\equiv\frac{dq}{dt}=\frac{\mathcal{E}}{R}e^{-t/(RC)}$$

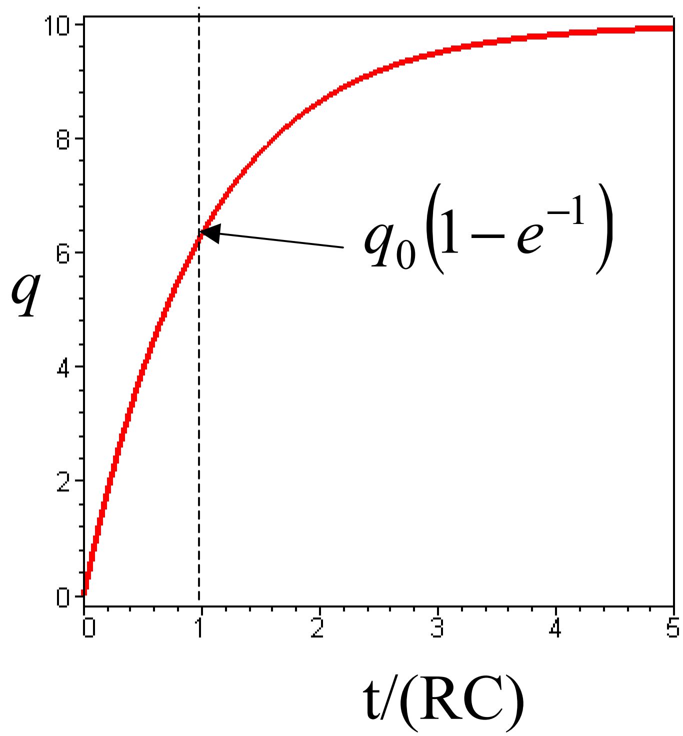
characteristic time constant for RC circuit

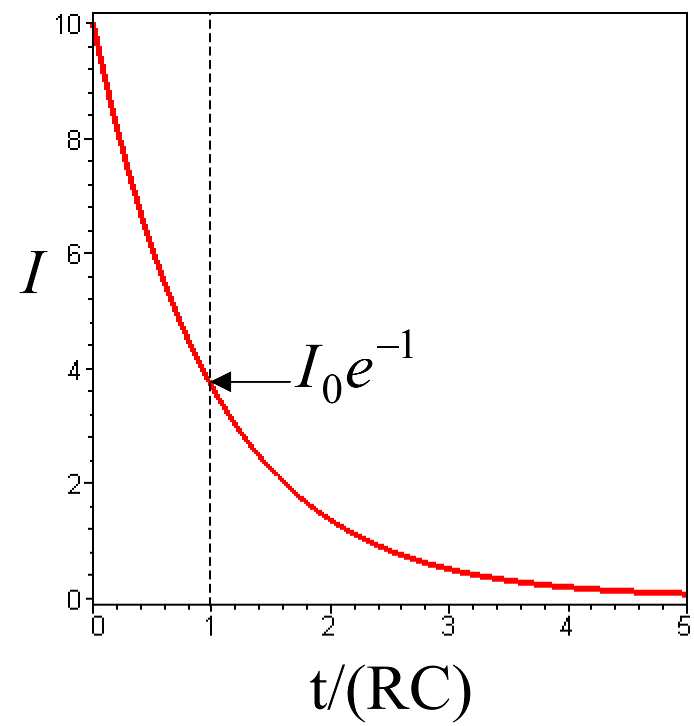
Charge

$$q(t) = C\mathcal{E}\left(1 - e^{-t/(RC)}\right)$$

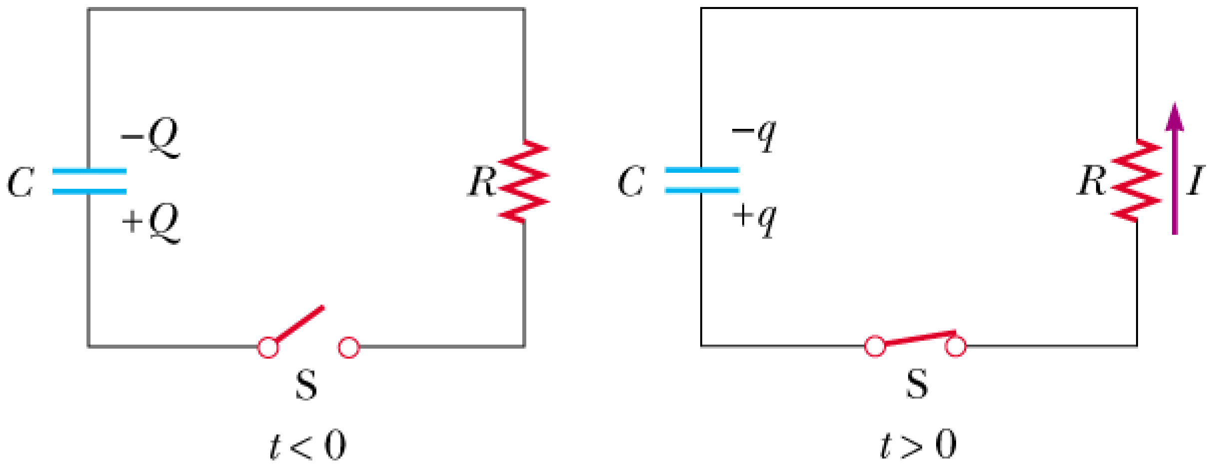
Current

$$I(t) \equiv \frac{dq}{dt} = \frac{\mathcal{E}}{R} e^{-t/(RC)}$$



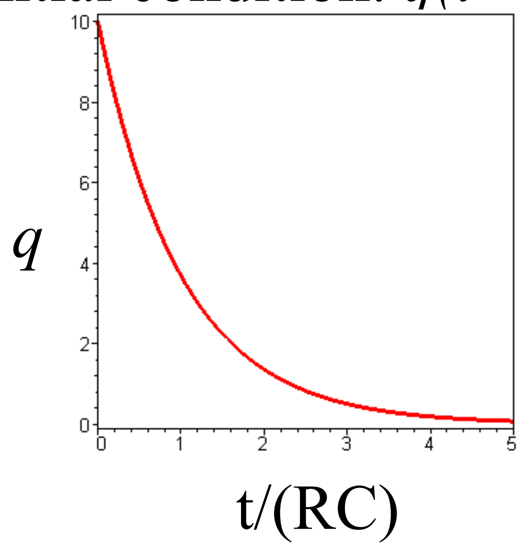


Discharging a capacitor



initial condition: $q(t=0)=Q$

$$-\frac{q}{C}-IR=0 \qquad \Rightarrow \qquad -\frac{q}{C}-\frac{dq}{dt}R=0$$



solution :

$$q(t)=Q\left(e^{-t/(RC)}\right)$$

$$I(t)\equiv\frac{dq}{dt}=-\frac{Q}{RC}e^{-t/(RC)}$$