Announcements

1. Vote on time for extra review session for Friday afternoon (2/7/03) – 2PM

   3PM

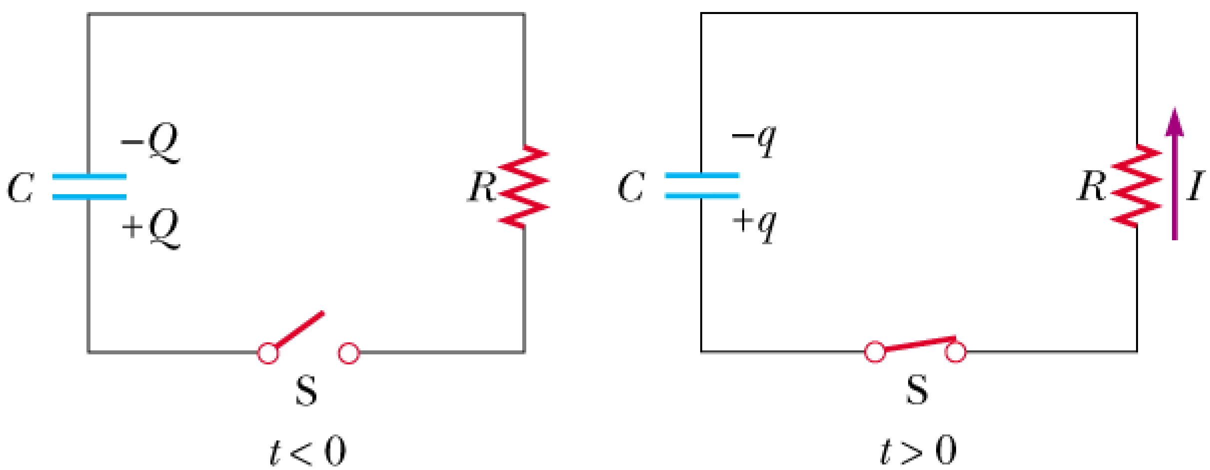
2. Today's topics –

   Continue discussion of RC circuits

   General strategies for analyzing DC circuits

3. Friday – review for first hour exam on Monday

RC circuit -- Discharging a capacitor through a resistor



initial condition:

$$q(t=0)=Q$$

after the switch is closed:

$$-\frac{q}{C} - IR = 0$$

$$\Rightarrow \quad -\frac{q}{C} - \frac{dq}{dt}R = 0$$

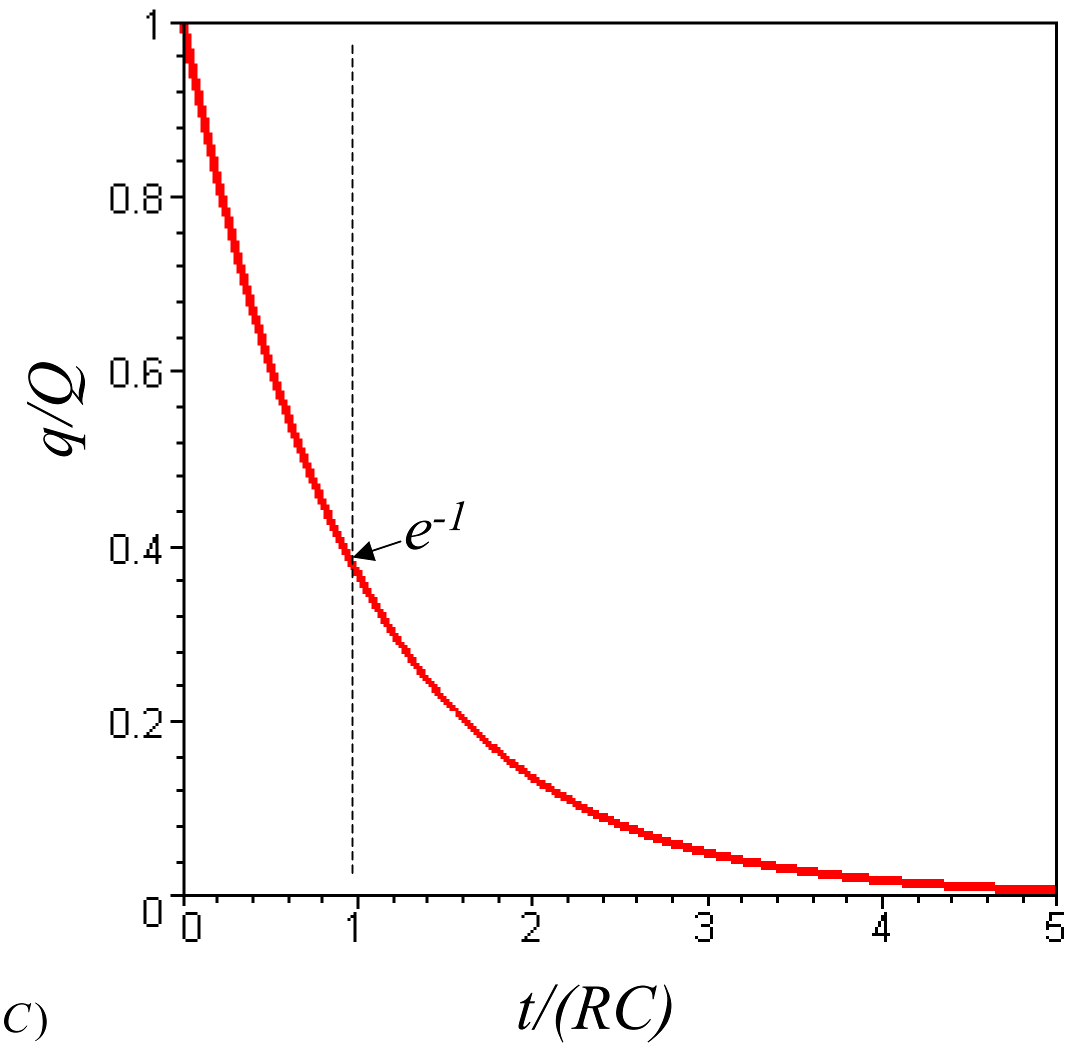
Analysis:

$$-\frac{q}{C}-\frac{dq}{dt}R=0$$

$$\frac{dq}{dt}=-\frac{1}{RC}q$$

$$\frac{dq}{q}=-\frac{1}{RC}dt$$

$$\ln\left(\frac{q(t)}{Q}\right)=-\frac{t}{RC}$$



Result :

$$q(t)=Q\left(e^{-t/(RC)}\right)$$

$$I(t)\equiv\frac{dq}{dt}=-\frac{Q}{RC}e^{-t/(RC)}$$

General method for solving first-order linear differential equation

Assume we want to find *q(t)* in terms of constants *A,B,C*.

$$A\frac{dq}{dt} + Bq + C = 0$$

Try solution form: $q(t) = Xe^{Yt} + Z$

Substitute into equation: $AX(Y)e^{Yt} + B\left(Xe^{Yt} + Z\right) + C = 0$

This must be true for all times *t*.

$$BZ + C = 0 \qquad \Rightarrow Z = -C/B$$

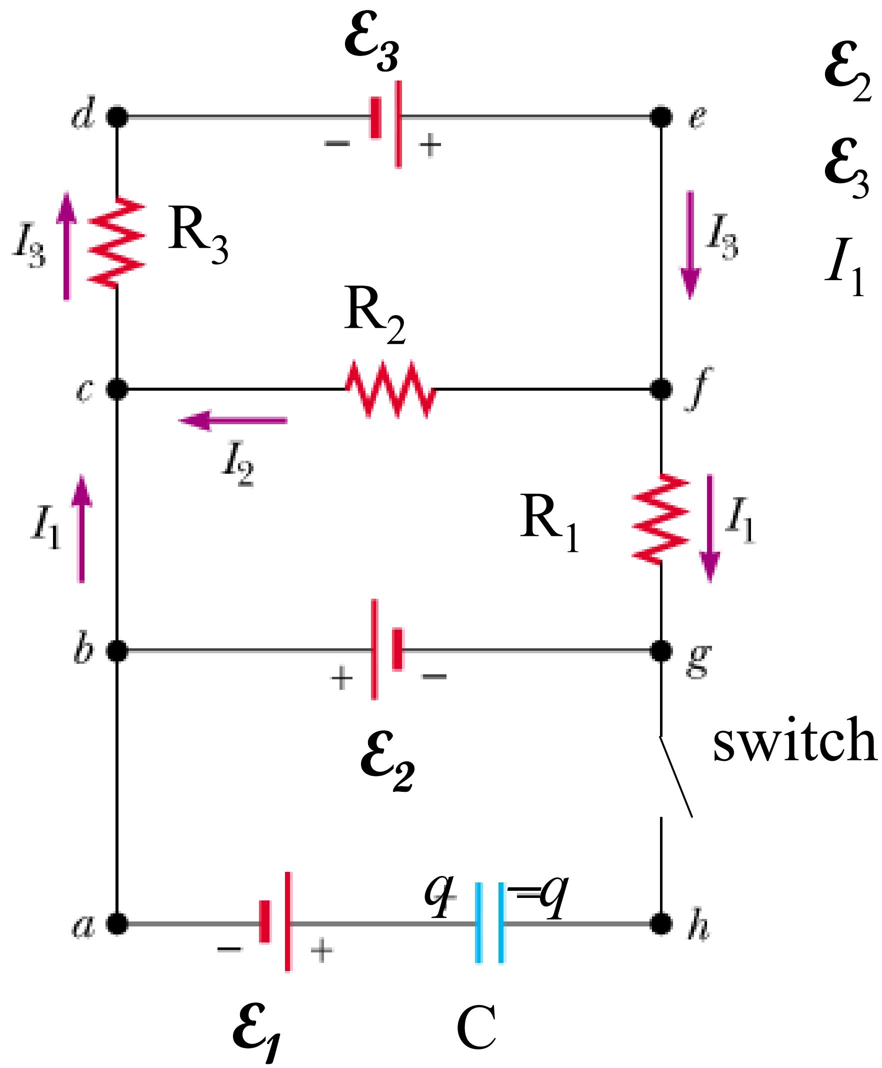
$$AXY + BX = 0 \qquad \Rightarrow Y = -B/A$$

X determined from initial conditions

If $q(t=0) = Q \qquad \Rightarrow X = Q + C/B$

$$\boxed{q(t) = Qe^{-Bt/A} + \frac{C}{B}\left(e^{-Bt/A} - 1\right)}$$

Example circuit:



Analysis with switch open :

$$\mathcal{E}_2 + I_2R_2 - I_1R_1 = 0$$

$$\mathcal{E}_3 - I_2R_2 - I_3R_3 = 0$$

$$I_1 + I_2 = I_3$$

$$\begin{aligned} -I_1R_1 + I_2R_2 \qquad\qquad &= -\mathcal{E}_2 \\ -I_2R_2 - I_3R_3 &= -\mathcal{E}_3 \\ I_1 \quad + I_2 \quad - I_3 \quad &= 0 \end{aligned}$$

$R_1=R_3=5\Omega,\ R_2=3\Omega$

$\mathcal{E}_1=3V,\ \ \mathcal{E}_2=8V,\ \mathcal{E}_3=4V$

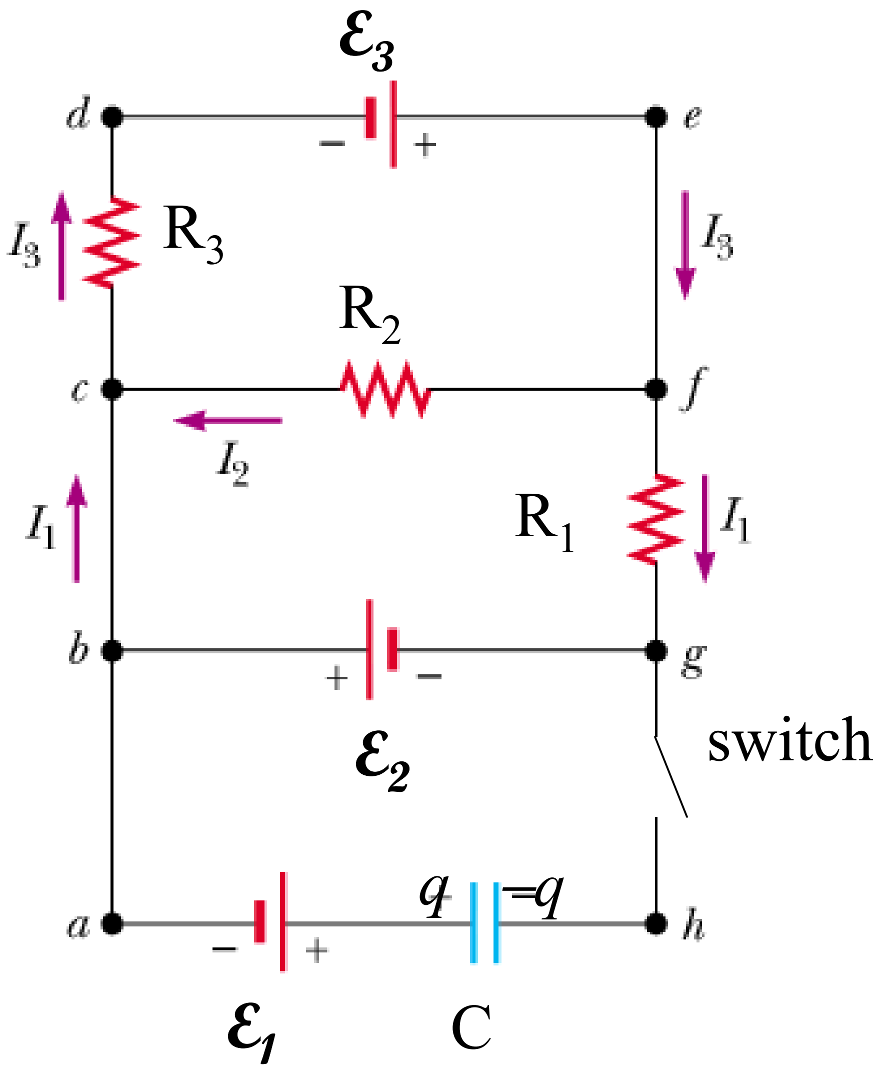
$C = 6x10^{-6}F$

Solution strategy:

$$
\begin{aligned}
-I_1R_1 + I_2R_2 \qquad\qquad &= -\mathcal{E}_2 \\
-I_2R_2 - I_3R_3 &= -\mathcal{E}_3 \\
I_1 \quad + I_2 \quad - I_3 \quad &= 0
\end{aligned}
$$

$$
\begin{aligned}
aX + bY + cZ &= A \\
dX + eY + fZ &= B \\
gX + hY + iZ &= C
\end{aligned}
\quad \Rightarrow \quad
\begin{aligned}
a'X \qquad\qquad &= A' \\
d'X + e'Y \qquad &= B' \\
gX + hY + iZ &= C
\end{aligned}
$$

Example circuit:



$R_1=R_3=5\Omega$, $R_2=3\Omega$

$\mathcal{E}_1=2V$, $\mathcal{E}_2=8V$, $\mathcal{E}_3=4V$

$C = 6x10^{-6}F$

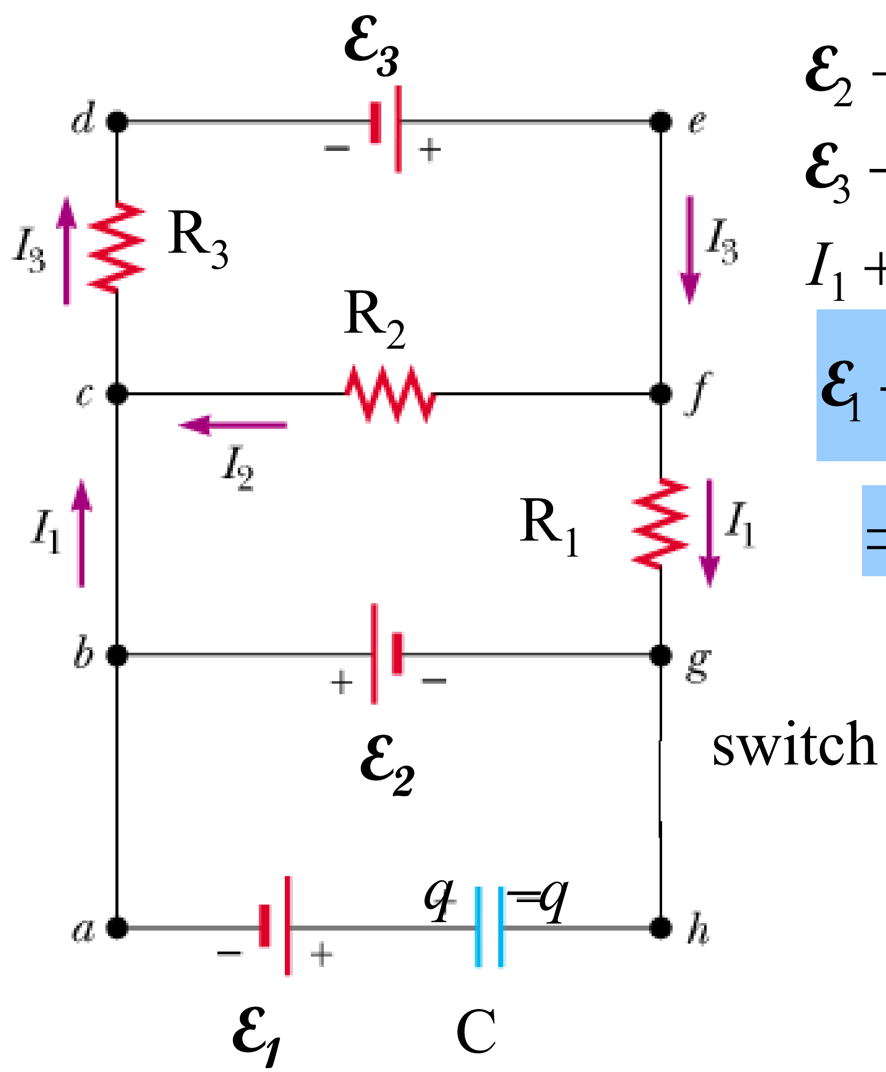
Result with open switch:

I_1= 1.381818 A

I_2=-0.363636 A

I_3= 1.018182 A

Example circuit:



Analysis with switch closed:

$$\mathcal{E}_2 + I_2R_2 - I_1R_1 = 0$$

$$\mathcal{E}_3 - I_2R_2 - I_3R_3 = 0$$

$$I_1 + I_2 = I_3$$

$$\mathcal{E}_1 - \frac{q}{C} + \mathcal{E}_2 = 0$$

$$\Rightarrow q = (\mathcal{E}_1 + \mathcal{E}_2)C = 6.6\times10^{-5}\,C$$

$R_1=R_3=5\Omega$, $R_2=3\Omega$

$\mathcal{E}_1=3V$, $\mathcal{E}_2=8V$, $\mathcal{E}_3=4V$

$C = 6x10^{-6}F$

Shortcuts – using equivalent circuits

$$R_{eq} = \frac{1}{\frac{1}{7\Omega} + \frac{1}{R}}$$

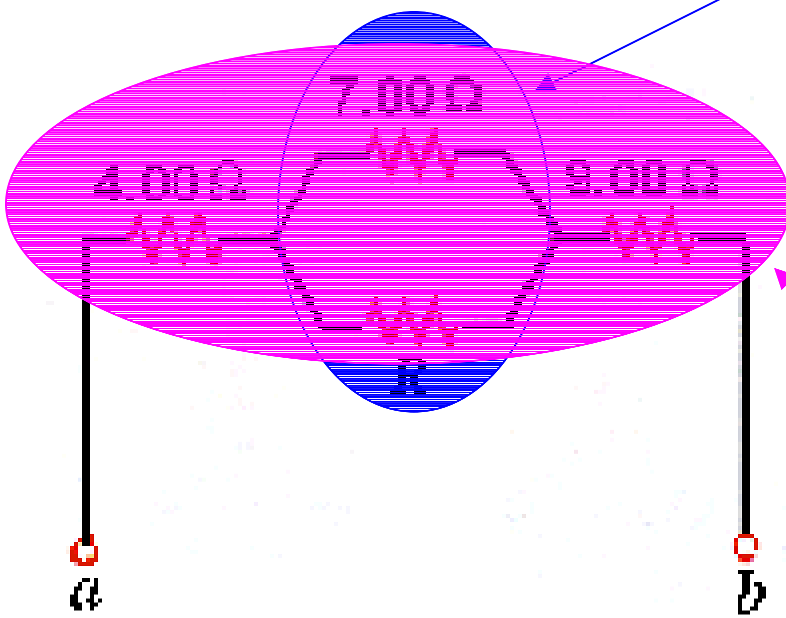
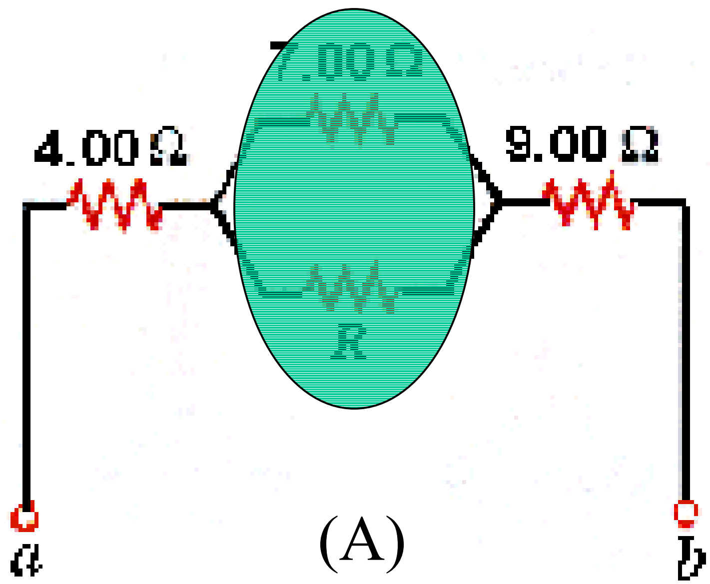


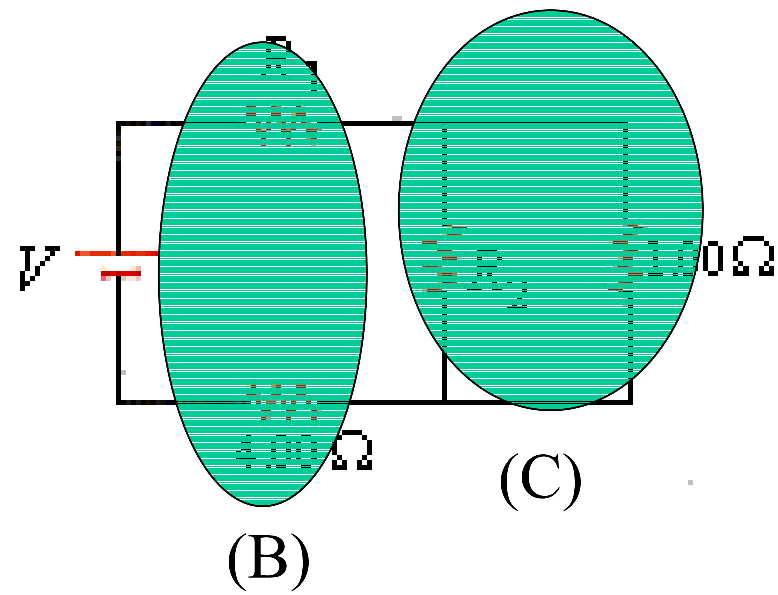
Figure P28.6.

$$R'_{eq} = 4\Omega + R_{eq} + 9\Omega$$

Peer instruction question:

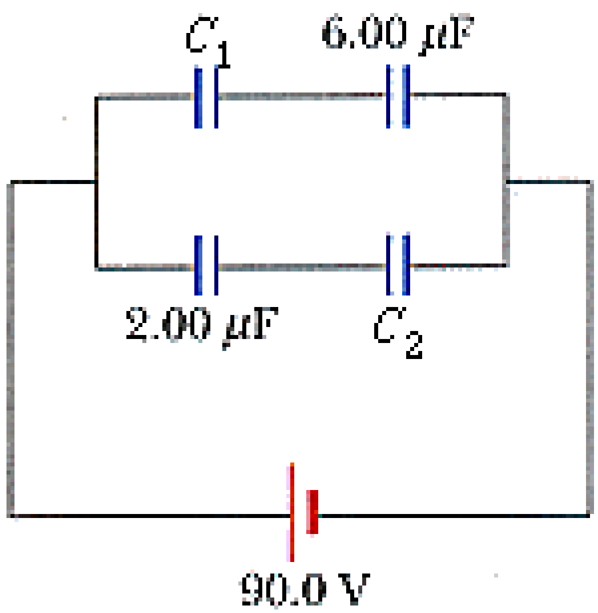
Which of these are NOT in parallel?





Peer instruction question:

Which of these capacitors are in parallel?



(A) C_1 & 6μF

(B) C_1 & 2μF

(C) C_1 + 6μF & C_2 + 2μF

(D) None these answers

Homework examples:

1. [SB5 28.P.18. (47538)] The ammeter shown in Figure P28.18 reads 2.20 A. Find I_1, I_2, and ε.

I_1 = [0.06666666667] [0.571] A

I_2 = [0.06666666667] [1.63] A

ε = [0.06666666667] [14.3] V

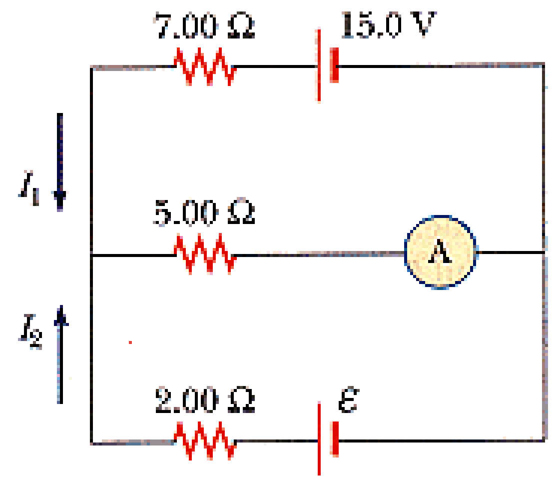


Figure P28.18.