February 19, 2003

PHY 114 – First Hour Test – Solutions

1. (a)

$$\mathbf{E}(\mathbf{P}) = \frac{k_e q_1}{a^2+b^2}\left(\frac{a}{\sqrt{a^2+b^2}}\hat{\mathbf{i}} + \frac{b}{\sqrt{a^2+b^2}}\hat{\mathbf{j}}\right) - \frac{k_e|q_2|}{b^2}\hat{\mathbf{j}}. \tag{1}$$

After substituting the values in to this expression, we find:

$$\mathbf{E}(\mathbf{P}) = 6104N/C\hat{\mathbf{i}} - 40794N/C\hat{\mathbf{j}}. \tag{2}$$

(b)

$$V(\mathbf{P}) = \frac{k_e q_1}{\sqrt{a^2+b^2}} + \frac{k_e q_2}{b}. \tag{3}$$

After substituting the values in to this expression, we find:

$$V(\mathbf{P}) = -26717V. \tag{4}$$

(c)

$$\mathbf{F}_{12} = 0.29967N\hat{\mathbf{i}}. \tag{5}$$

2. (a) Diagram (C) is the correct representation of the electric field lines. We can eliminate (A) and (B) by using Gauss's law and recognizing that there is no charge inside the shell.

(b) Gauss's law tells us that outside the shell, the electric field is that of a point charge having the same total charge located at the center of the shell. The total charge is $Q = \sigma \cdot 4\pi R^2 = 6.2832 \times 10^{-7}$ C.

$$\mathbf{E}(x) = \frac{k_e Q}{x^2}\hat{\mathbf{i}} = 250980N/C\hat{\mathbf{i}}. \tag{6}$$

(c)

$$\mathbf{E}(y) = 0. \tag{7}$$

3. (a) $V = \mathcal{E} = 100$ V.

(b)

$$V = \frac{Q}{C} \quad \rightarrow Q = VC = 4 \times 10^{-4}C. \tag{8}$$

(c)

$$U = \frac{Q^2}{2C} = 0.02J. \tag{9}$$

4. (a)

$$R_{eq} = \frac{1}{\frac{1}{R_1+R_2} + \frac{1}{R_3}} = 13.636\Omega. \tag{10}$$

$$I = \frac{\mathcal{E}}{R_eq} = 7.33A. \tag{11}$$

(b) Because R_3 is a single resistor in parallel with the emf, we can conclude that

$$V_3 = \mathcal{E} = 100V = I_3 R_3. \tag{12}$$

This allows us to determine the current

$$I_3 = \frac{\mathcal{E}}{R_3} = 3.33A. \tag{13}$$

By Kirchhoff's junction rule or by conservation of current, we can conclude that the current running through the upper loop is

$$I_{12} = I - I_3 = 4A. \tag{14}$$

Therefore,

$$V_1 = I_{12}R_1 = 20V \text{ and } V_2 = I_{12}R_2 = 80V. \tag{15}$$

(c)

$$\mathcal{P} = I\mathcal{E} = 733.33Watts. \tag{16}$$

5. (a) Using Kirchhoff's rule for the outer loop, we find

$$\mathcal{E} - I_1 R_1 - QC = 0. \tag{17}$$

Expressing

$$I_1 = \frac{dQ}{dt}, \tag{18}$$

this becomes the differential equation:

$$\frac{dQ}{dt}R_1 + QC = \mathcal{E}. \tag{19}$$

The solution to this equation, noting that $Q(t=0) = 0$, is

$$Q(t) = \mathcal{E}C\left(1 - \mathrm{e}^{-t/(R_1C)}\right). \tag{20}$$

When $t = R_1C = 0.0003s$, 63% of the charging will be finished.

(b) A long time after the switch is closed, there will be no current in through R_1 so that

$$Q = \mathcal{E}C = 0.003C. \tag{21}$$

(c)

$$I_1(t) = \frac{dQ}{dt} = \frac{d}{dt}\mathcal{E}C\left(1 - \mathrm{e}^{-t/(R_1 C)}\right) = \frac{\mathcal{E}}{R_1}\mathrm{e}^{-t/(R_1 C)} = 10A\mathrm{e}^{-t/(0.0003s)}. \tag{22}$$

$$I_2 = \mathcal{E}C/R_2 = 5A. \tag{23}$$

*********Extra credit*********

When the switch is opened, the positive and negative charges on the capacitor recombine through the upper wire loop. The differential equation is

$$QC + \frac{dQ}{dt}(R_1 + R_2) = 0. \tag{24}$$

The solution to this equation with the initial condition $Q(0) = \mathcal{E}C$ is

$$Q(t) = \mathcal{E}\mathrm{e}^{-t/[(R_1+R_2)C]} = 0.003C\mathrm{e}^{-t/(0.0009s)}. \tag{25}$$