March 18, 2003

PHY 114 – Second exam – Solutions

1. (a) i. According to the right hand rule, the force is initially vertically upward in the plane of the page and has the magnitude $F = qvB = 1.6 \times 10^{-19} \cdot 1 \times 10^{6} \cdot 2 = 3.2 \times 10^{-13}$ N.

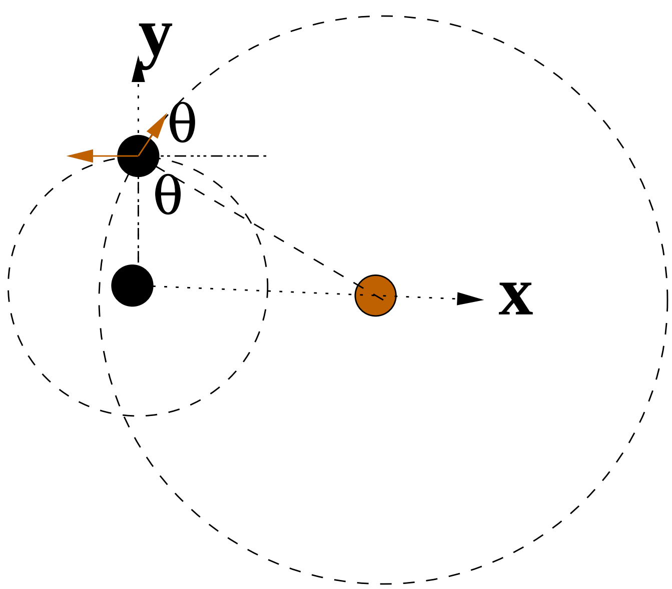
   ii. The particle will have a circular trajectory with radius

$$r = \frac{mv}{qB} = 0.0053\ m. \tag{1}$$

   (b) i. The force is pointing into the page of the paper and has the magnitude $F = qvB\sin\theta = 1.6 \times 10^{-19} \cdot 1 \times 10^{6} \cdot 2\sin(20^{o}) = 1.09 \times 10^{-13}$ N.

   ii. The particle will have a helical trajectory, with a constant velocity of $v\cos\theta = 0.64 \times 10^{6}$ m/s along the field direction and projected helical radius of

$$r = \frac{mv}{qB}\sin\theta = 0.0018\ m. \tag{2}$$



2. It was intended that the problem could be solved using the long wire approximation. In fact, since wire lengths are comparable to their separation, there would be "fringing" effects. The solution below assumes that the wires are much longer than 0.5 m. The length of the wire comes in explicitly only in calculating the force.

   (a) We can use Ampere's law to make the construction shown above.

$$\mathbf{B} = \frac{\mu_0}{2\pi}\left(\frac{I_2}{r_2}(-\hat{\mathbf{x}}) + \frac{I_3}{r_3}(\cos\theta\hat{\mathbf{x}} + \sin\theta\hat{\mathbf{y}})\right), \tag{3}$$

   where $\theta = \tan^{-1} a/b = 63.435^{o}$, $r_2 = b = 0.2$ m, and $r_3 = \sqrt{a^2+b^2} = 0.4472$ m. $\mathbf{B} = (-1.4\hat{\mathbf{x}} + 1.2\hat{\mathbf{y}}) \times 10^{-6}$ T.

(b) The force will be pointing in the page and perpendicular to the magnetic field direction. Noting that the current is in the $\hat{\mathbf{z}}$ direction and that $\hat{\mathbf{z}} \times \hat{\mathbf{x}} = \hat{\mathbf{y}}$ and $\hat{\mathbf{z}} \times \hat{\mathbf{y}} = -\hat{\mathbf{x}}$, we have

$$\mathbf{F} = lI_1\hat{\mathbf{z}} \times \mathbf{B} = (-1.4\hat{\mathbf{y}} - 1.2\hat{\mathbf{x}}) \times 10^{-6}N. \tag{4}$$

3. (a) When the magnetic field is constant there is no electric field and therefore there is no force on the charge.

(b) In the time interval $0 \leq t \leq \tau$, there is a changing flux

$$\frac{\Phi_B}{dt} = \frac{dB}{dt}\pi R^2 = -18.3783T \cdot m^2/s. \tag{5}$$

i. According to Faraday's law

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{\Phi_B}{dt}. \tag{6}$$

$$E \cdot 2\pi r = -\frac{d\Phi_B}{dt} \quad \to |E| = \frac{18.3783}{2\pi 0.7} = 4.17857N/C. \tag{7}$$

The direction of the field will be horizontally to the left in the page of the diagram.

ii. $F = qE = 8.36 \times 10^{-3}$ N, horizontally to the left.

(c) For $t > \tau$ the force is zero.

4. (a)

$$\mathcal{E}(t) = -N\frac{d\Phi_B}{dt} = -N\frac{dBA\cos(\omega t)}{dt} = NBA\omega\sin(\omega t) = 2964V\sin(380t). \tag{8}$$

(b)

$$I(t) = \frac{\mathcal{E}(t)}{R} = 4234.286A\sin(380t). \tag{9}$$

(c) The current loop has a magnetic moment $\mu = NIA = 25405.8Am^2\sin(380t)$. The torque due to this moment is $\tau = \mu \times \mathbf{B}$.

$$\tau = 33030\sin^2(380t)Nm. \tag{10}$$

(d) It opposes the generator torque.

5. (a) $I_L = I_R = \mathcal{E}/R = 30$ A.

i. $\omega = 1/\sqrt{LC} = 40$ rad/s.

ii.

$$U = \frac{1}{2}LI^2 = \frac{1}{2}6.25(30)^2 = 2812.5J. \tag{11}$$

iii. The solution of the differential equation for the charge $q(t)$ is $q(t) = Q_0\sin(\omega t)$. Initially,

$$I(0) = \left.\frac{dq}{dt}\right|_0 = \omega Q_0 = 30A. \tag{12}$$

Therefore $Q_0 = 30/40C = 0.75C$ and $V_{max} = Q_0/C = 7500$ V.

6. (a)

$$-RI - L\frac{dI}{dt} - \frac{q}{C} + \mathcal{E} = 0. \tag{13}$$

(b)

$$I(t) = \frac{\mathcal{E}_m ax}{Z} \cos(\omega t - \phi). \tag{14}$$

In our case, $Z = \sqrt{(100)^2 + (700 \cdot 0.006 - 1/(700 \cdot 1 \times 10^{-5}))^2} = 170.96\Omega$
$\phi = \tan^{-1}\left(700 \cdot 0.006 - 1/(700 \cdot 1 \times 10^{-5})\right)/100 = -0.94598$ rad.

$$I(t) = 1.7548A \cos(700t + 0.94598). \tag{15}$$

(c)

$$q(t) = \frac{1.7548A}{700rad/s} \sin(700t + 0.94598) = 0.002507C \sin(700t + 0.94598). \tag{16}$$