February 8, 2005

PHY 114 – First Hour Test

1. (a)

$$\mathbf{E}(\mathbf{r_A}) = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i(\mathbf{r}_A - \mathbf{r}_i)}{|\mathbf{r}_A - \mathbf{r}_i|^3}. \tag{1}$$

In this case, $\mathbf{r}_A = 0$, $\mathbf{r}_1 = 0.03\hat{\mathbf{y}}$ m, and $\mathbf{r}_2 = 0.01\hat{\mathbf{x}}$ m.

$$\mathbf{E}(\mathbf{r_A}) = 8.98755\times10^9\times10^{-6}\left(+3\frac{(-0.03\hat{\mathbf{y}})}{(0.03)^3} - 2\frac{(-0.01\hat{\mathbf{x}})}{(0.01)^3}\right) = (17.9751\hat{\mathbf{x}}-2.996\hat{\mathbf{y}})\times10^7\text{N/C}. \tag{2}$$

(b)

$$V(\mathbf{r}_A) = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{|\mathbf{r}_A - \mathbf{r}_i|}. \tag{3}$$

In this case,

$$V(\mathbf{r}_A) = 8.98755\times10^9\times10^{-6}\left(\frac{(+3)}{0.03} + \frac{(-2)}{0.01}\right) = -8.988\times10^5\text{V}. \tag{4}$$

(c)

$$V(\mathbf{r}_B) = 8.98755\times10^9\times10^{-6}\left(\frac{(+3)}{0.02} + \frac{(-2)}{[(0.01)^2+(0.01)^2]^{1/2}}\right) = 0.7710\times10^5\text{V}. \tag{5}$$

$$W = q\,[V(\mathbf{r}_A) - V(\mathbf{r}_B)] = 1\times10^{-5}\text{C}\,[-8.988 - 0.7710]\times10^5\text{V} = -9.759\text{J}. \tag{6}$$

2. (a)

$$\mathbf{E} = \frac{Q/A}{2\varepsilon_0}(-\hat{\mathbf{z}}) = \frac{(1\times10^{-5})/0.3}{2\cdot8.854\times10^{-12}}(-\hat{\mathbf{z}}) = 1.8824\times10^6(-\hat{\mathbf{z}})N/C. \tag{7}$$

(b)

$$\mathbf{F} = q\mathbf{E} = -2\times10^{-8}\cdot1.8824\times10^6(-\hat{\mathbf{z}}) = 0.0377\hat{\mathbf{z}}N. \tag{8}$$

(c) In addition to the upward electrical force, the particle experiences a downward gravitational force of $-mg\hat{\mathbf{z}} = -0.0098\hat{\mathbf{z}}N$.

$$\mathbf{F}_{\text{tot}} = (0.0377 - 0.0098)\hat{\mathbf{z}}N = 0.0279\hat{\mathbf{z}}N. \tag{9}$$

The particle has a net upward force and would accelerate upward.

3. Area of each face is $A = a^2 = (1.3m)^2 = 1.69m^2$.

(a)

$$\Phi_{\text{above}} = -AE_{\text{above}} = -1.69\cdot5000Nm^2 = -8450Nm^2. \tag{10}$$

(b)

$$\Phi_{\text{below}} = AE_{\text{below}} = 1.69\cdot8000Nm^2 = 13520Nm^2. \tag{11}$$

(c) The only contributions to the flux are from the upper and lower surfaces:

$$\oint \mathbf{E} \cdot d\mathbf{A} = 1.69 \cdot (8000 - 5000) Nm^2 = 5070 Nm^2. \tag{12}$$

(d)

$$q = \varepsilon_0 \oint \mathbf{E} \cdot d\mathbf{A} = 8.854 \times 10^{-12} \cdot 5070 C = 4.49 \times 10^{-8} C. \tag{13}$$

4. (a) For series configuration:

$$Q_1 = Q_2. \tag{14}$$

$$\Delta V = Q_1 \left(\frac{1}{C_1} + \frac{1}{C_2} \right). \tag{15}$$

$$Q_1 = \frac{\Delta V}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{100}{\frac{10^6}{7} + \frac{10^6}{2}} = 1.5555 \times 10^{-4} C. \tag{16}$$

$$V_1 = \frac{Q_1}{C_1} = 22.2222 V. \tag{17}$$

$$V_2 = \frac{Q_2}{C_2} = 77.7778 V. \tag{18}$$

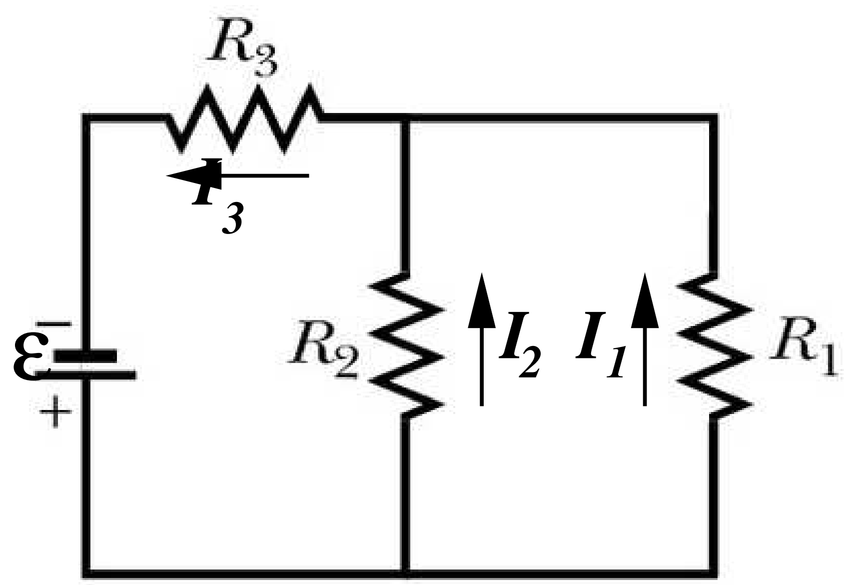
(b)

$$V_1 = V_2 = 100 V. \tag{19}$$

$$Q_1 = C_1 V_1 = 7 \times 10^{-6} \cdot 100 C = 7 \times 10^{-4} C. \tag{20}$$

$$Q_2 = C_2 V_2 = 2 \times 10^{-6} \cdot 100 C = 2 \times 10^{-4} C. \tag{21}$$

5.



In preparation for solving this problem, it is helpful to define the currents I_1, I_2, and I_3 as shown. In this case, we can use an equivalent circuit with $\mathcal{E}$, I_3 and

$$R_{\text{eq}} = R_3 + \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = 6.5555 \Omega. \tag{22}$$

$$I_3 = \frac{E}{R_{\text{eq}}} = \frac{100V}{6.55555\Omega} = 15.25424A. \tag{23}$$

Using the notion of parallel circuits,

$$V_2 = V_1 = \mathcal{E} - I_3 R_3 = 100V - 15.25424 \cdot 5V = 23.7288V. \tag{24}$$

$$V_3 = I_3 R_3 = 15.25424 \cdot 5V = 76.2712V. \tag{25}$$

$$I_1 = \frac{V_1}{R_1} = \frac{23.7288V}{2\Omega} = 11.8644A. \tag{26}$$

$$I_2 = \frac{V_2}{R_2} = \frac{23.7288V}{7\Omega} = 3.3898A. \tag{27}$$

Alternatively, we can solve this problem using Kirchhoff's rules.

$$I_3 = I_1 + I_2. \tag{28}$$

$$\mathcal{E} - I_2 R_2 - I_3 R_3 = 0. \tag{29}$$

$$I_2 R_2 - I_1 R_1 = 0. \tag{30}$$

Solving these three equations we get the same answers as above for the currents and then we can deduce the voltages from

$$V_i = I_i R_i. \tag{31}$$