February 8, 2005

**PHY 114 – First Hour Test – alternate version – solutions**

1. (a)

$$\mathbf{E}(\mathbf{r}_A) = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i(\mathbf{r}_A - \mathbf{r}_i)}{|\mathbf{r}_A - \mathbf{r}_i|^3}. \tag{1}$$

In this case $\mathbf{r}_A = (0.01\hat{\mathbf{x}} + 0.02\hat{\mathbf{y}})$ m, $\mathbf{r}_1 = 0$, and $\mathbf{r}_2 = (0.01\hat{\mathbf{x}})$ m.

$$\mathbf{E}(\mathbf{r}_A) = 8.98755 \times 10^9 \cdot 10^{-6}\left(\frac{+3(0.01\hat{\mathbf{x}} + 0.02\hat{\mathbf{y}})}{[(0.01)^2 + (0.02)^2]^{3/2}} + \frac{(-2)0.02\hat{\mathbf{y}}}{(0.02)^3}\right) N/C. \tag{2}$$

$$\mathbf{E}(\mathbf{r}_A) = (24.116\hat{\mathbf{x}} + 3.295\hat{\mathbf{y}})10^6 N. \tag{3}$$

(b)

$$V(\mathbf{r}_A) = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{|\mathbf{r}_A - \mathbf{r}_i|}. \tag{4}$$

$$V(\mathbf{r}_A) = 8.98755 \times 10^9 \cdot 10^{-6}\left(\frac{+3}{[(0.01)^2 + (0.02)^2]^{1/2}} + \frac{(-2)}{0.02}\right) V = 3.07051 \times 10^5 V. \tag{5}$$

(c)

$$V(\mathbf{r}_B) = 8.98755 \times 10^9 \cdot 10^{-6}\left(\frac{+3}{0.01} + \frac{(-2)}{\sqrt{2}(0.01)}\right) V == 1.425233 \times 10^6 V. \tag{6}$$

$$W = q\left(V(\mathbf{r}_A) - V(\mathbf{r}_B)\right) = 1 \times 10^{-5}(14.25233 - 3.07051) \times 10^5 = 11.1818 J. \tag{7}$$

2. (a)

$$\mathbf{E} = \frac{Q/A}{2\varepsilon_0}(-\hat{\mathbf{z}}) = \frac{(1 \times 10^{-5})/0.3}{2 \cdot 8.854 \times 10^{-12}}(-\hat{\mathbf{z}}) = 1.8824 \times 10^6(-\hat{\mathbf{z}})N/C. \tag{8}$$

(b)

$$\mathbf{F} = q\mathbf{E} = -2 \times 10^{-8} \cdot 1.8824 \times 10^6(-\hat{\mathbf{z}}) = 0.0377\hat{\mathbf{z}}N. \tag{9}$$

(c) In addition to the upward electrical force, the particle experiences a downward gravitational force of $-mg\hat{\mathbf{z}} = -0.0098\hat{\mathbf{z}}N$.

$$\mathbf{F}_{\text{tot}} = (0.0377 - 0.0098)\hat{\mathbf{z}}N = 0.0279\hat{\mathbf{z}}N. \tag{10}$$

The particle has a net upward force and would accelerate upward.

3. Area of each face is $A = a^2 = (1.3m)^2 = 1.69m^2$.

(a)

$$\Phi_{\text{above}} = -AE_{\text{above}} = -1.69 \cdot 5000 Nm^2 = -8450 Nm^2. \tag{11}$$

(b)

$$\Phi_{\text{below}} = AE_{\text{below}} = 1.69 \cdot 8000 Nm^2 = 13520 Nm^2. \tag{12}$$

(c) The only contributions to the flux are from the upper and lower surfaces:

$$\oint \mathbf{E} \cdot d\mathbf{A} = 1.69 \cdot (8000 - 5000) Nm^2 = 5070 Nm^2. \tag{13}$$

(d)

$$q = \varepsilon_0 \oint \mathbf{E} \cdot d\mathbf{A} = 8.854 \times 10^{-12} \cdot 5070 C = 4.49 \times 10^{-8} C. \tag{14}$$

4. (a) For series configuration:

$$Q_1 = Q_2. \tag{15}$$

$$\Delta V = Q_1 \left( \frac{1}{C_1} + \frac{1}{C_2} \right). \tag{16}$$

$$Q_1 = \frac{\Delta V}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{100}{\frac{10^6}{7} + \frac{10^6}{2}} = 1.5555 \times 10^{-4} C. \tag{17}$$

$$V_1 = \frac{Q_1}{C_1} = 22.2222 V. \tag{18}$$

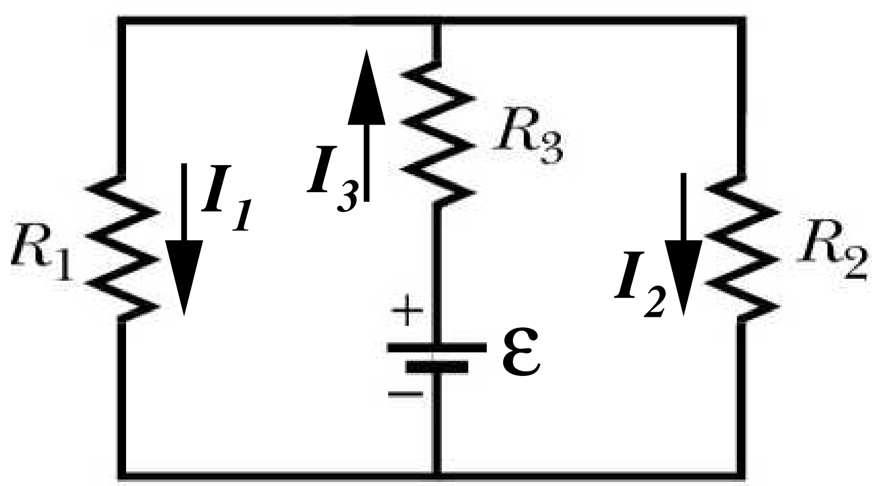
$$V_2 = \frac{Q_2}{C_2} = 77.7778 V. \tag{19}$$

(b)

$$V_1 = V_2 = 100 V. \tag{20}$$

$$Q_1 = C_1 V_1 = 7 \times 10^{-6} \cdot 100 C = 7 \times 10^{-4} C. \tag{21}$$

$$Q_2 = C_2 V_2 = 2 \times 10^{-6} \cdot 100 C = 2 \times 10^{-4} C. \tag{22}$$




5.

In preparation for solving this problem, it is helpful to define the currents $I_1$, $I_2$, and $I_3$ as shown.

We can solve this problem using Kirchhoff's rules.

$$I_3 = I_1 + I_2. \tag{23}$$

$$\mathcal{E} - I_1 R_1 - I_3 R_3 = 0. \tag{24}$$

$$I_2 R_2 - I_1 R_1 = 0. \tag{25}$$

Solving these three equations we get the following answers for the currents. $I_1 = 11.864$ A, $I_2 = 3.390$ A, and $I_3 = 15.254$ A.. We can deduce the voltages from

$$V_i = I_i R_i. \tag{26}$$

$V_1 = V_2 = 23.729$ V and $V_3 = 76.271$ V.