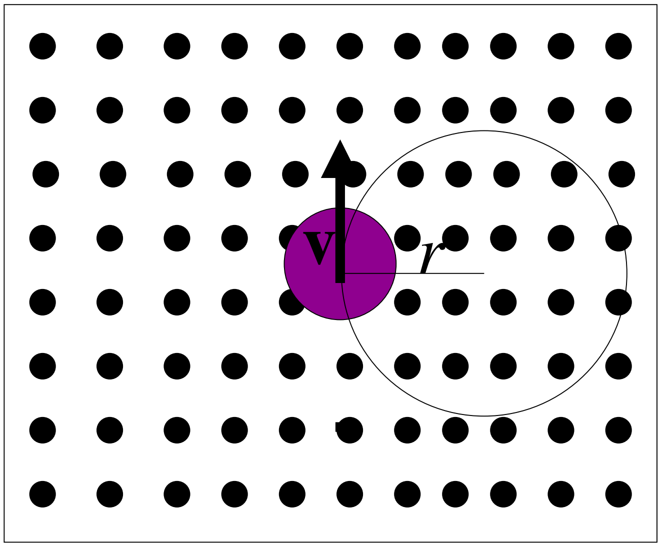
February 28, 2005

PHY 114 – Second Hour Test – solutions

1. 

(a) Using the right hand rule, we find that the force on the proton is pointing to the right of the diagram. The magnitude of the force is

$$F = qvB = 1.6 \times 10^{-19} \cdot 1 \times 10^{6} \cdot 0.5 = 8 \times 10^{-14} N. \quad (1)$$

(b) The proton will move in a circular trajectory as shown.

$$F = ma \quad \Rightarrow \quad qvB = m\frac{v^2}{r}. \quad (2)$$

Solving for the radius of the circle, we find

$$r = \frac{mv}{qB} = \frac{1.67 \times 10^{-27} \cdot 1 \times 10^{6}}{1.6 \times 10^{-19} \cdot 0.5} = 0.0209m. \quad (3)$$

2. (a)

$$\Phi_B = \mathbf{B} \cdot \mathbf{A} = B \cdot \pi r^2 = 2 \cdot \pi \cdot (0.01)^2 T m^2 = 6.283 \times 10^{-4} T m^2. \quad (4)$$

(b) i. When the ring is in the solenoid the flux is constant and $\mathcal{E} = 0$ and $i = 0$.

ii. When the ring is in the region of the fringing field,

$$\frac{d\Phi_B}{dt} = \frac{dB}{dx}\frac{dx}{dt} A = 4 \cdot (-0.1) \cdot \pi \cdot (0.01)^2 = 1.257 \times 10^{-4} = -\mathcal{E}. \quad (5)$$

$$i = \frac{\mathcal{E}}{R} = \frac{1.257 \times 10^{-4}}{5 \times 10^{-3}} = 0.025A. \quad (6)$$

3. (a)

$$\tau = \frac{L}{R} = \frac{2}{5}s = 0.4s. \quad (7)$$

(b) i. Expressing current in units of A:

$$i(t) = \frac{\mathcal{E}}{R}\left(1 - \mathrm{e}^{-t/\tau}\right) + i_0\mathrm{e}^{-t/\tau} = 1 - 0.8\mathrm{e}^{-t/0.4}. \tag{8}$$

ii.

$$i(t = 0.2s) = 1 - 0.8\mathrm{e}^{-0.2/0.4} = 0.5148A. \tag{9}$$

4. (a)

$$\tau = RC = 6 \times 10^{-8}s. \tag{10}$$

(b)

$$q(t) = q_0\mathrm{e}^{-t/RC}. \tag{11}$$

$$i(t) = \frac{dq}{dt} = -\frac{q_0}{RC}\mathrm{e}^{-t/RC}. \tag{12}$$

$$i(0) = -\frac{q_0}{RC} = -\frac{1 \times 10^{-6}}{6 \times 10^{-8}}A = -16.667A. \tag{13}$$

(c)

$$B_A = \frac{\mu_0 i(t)}{2\pi a} = \frac{4\pi \times 10^{-7} \cdot 16.667}{2\pi(0.001)} = 0.00333T. \tag{14}$$

(d) The electric field between the capacitor plates is

$$E = \frac{q(t)/A}{\epsilon_0}. \tag{15}$$

The electrical flux between the capacitor plates at the radius a is

$$\Phi_E = E \cdot \pi a^2 = \frac{q(t)\pi a^2}{A\epsilon_0}. \tag{16}$$

The time rate of change of this flux is

$$\frac{d\Phi_E}{dt} = \frac{\pi a^2}{A\epsilon_0}\frac{dq}{dt} = \frac{\pi a^2}{A\epsilon_0}i(t). \tag{17}$$

Therefore, we see that

$$B_B = \frac{\mu_0 a}{2A}i(0) == 3.089 \times 10^{-8}T. \tag{18}$$