March 29, 2005

PHY 114 – Third Hour Test – Solutions

1. (a)

$$I_{\text{max}} = \frac{\mathcal{E}_{\text{max}}}{Z}. \tag{1}$$

$$Z = \sqrt{R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2} = 15.506\Omega. \tag{2}$$

$$I_{\text{max}} = \frac{75}{15.506} = 4.8369A. \tag{3}$$

(b)

$$i(t) = I_{\text{max}} \cos(\omega_1 t - \phi) = \frac{dq}{dt}. \tag{4}$$

It then follows that

$$q(t) = \frac{I_{\text{max}}}{\omega_1} \sin(\omega_1 t - \phi) = Q_{\text{max}} \sin(\omega_1 t - \phi). \tag{5}$$

$$Q_{\text{max}} = \frac{I_{\text{max}}}{\omega_1} = 0.01727C. \tag{6}$$

(c)

$$\mathcal{E}_L = -L\frac{dI}{dt} = L\omega_1 I_{\text{max}} \cos(\omega_1 t - \phi). \tag{7}$$

$$\mathcal{E}_L\rfloor_{\text{max}} = L\omega_1 I_{\text{max}} = 5417.33. \tag{8}$$

(d)

$$P = \frac{1}{2} I^2_{\text{max}} R = \frac{1}{2}(4.8369)^2 \cdot 15W = 175.47W. \tag{9}$$

2. (a) Gauss's law which is equivalent to Coulomb's law and which describes how electrical charges produce an electric field.

$$\oint \mathbf{E} \cdot \mathbf{dA} = Q/\epsilon_0. \tag{10}$$

(b) Gauss's law for magnetic fields which is consistent with the observation that there are no magnetic monopole source.

$$\oint \mathbf{B} \cdot \mathbf{dA} = 0. \tag{11}$$

(c) Faraday's law which represents the production of electric fields by a time varying magnetic flux.

$$\oint \mathbf{E} \cdot \mathbf{ds} = -\frac{d\Phi_B}{dt}. \tag{12}$$

(d) Ampere's and Maxwell's law which represents the production of magnetic fields by a current and by a time varying electric flux.

$$\oint \mathbf{B} \cdot \mathbf{ds} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}. \tag{13}$$

For a plane-wave form of an electromagnetic wave, with the electric field along the $\hat{\mathbf{y}}$ direction and the magnetic field along the $\hat{\mathbf{z}}$ direction, Faraday's law and Ampere-Maxwell's law reduce to

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}. \tag{14}$$

$$\frac{\partial B_z}{\partial x} = -\mu_0 \epsilon_0 \frac{\partial E_y}{\partial t}. \tag{15}$$

We can substitute these equations into each other, to find two uncoupled equations for the electric and magnetic fields:

$$\frac{\partial^2 E_y}{\partial t^2} = \frac{1}{\mu_0 \epsilon_0} \frac{\partial^2 E_y}{\partial x^2}. \tag{16}$$

$$\frac{\partial^2 B_z}{\partial t^2} = \frac{1}{\mu_0 \epsilon_0} \frac{\partial^2 B_z}{\partial x^2}. \tag{17}$$

These are both wave equations with velocity $c = \sqrt{1/(\mu_0 \epsilon_0)}$.

3. (a) Reading the vertical axis of the graph, we see that $E_{\max} = 2$ V/m.

   (b) The wave completes a cycle at $x \approx 0.6$ m. So we infer the $\lambda \approx 0.6$ m.

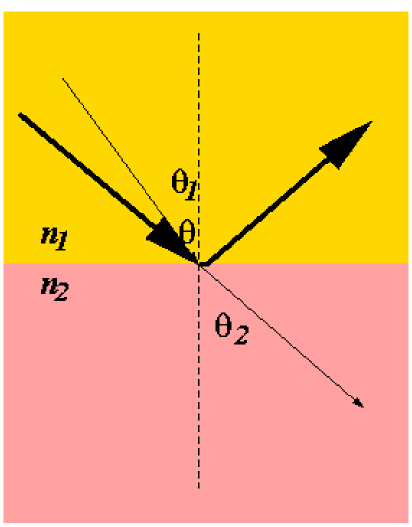
   (c)

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8 m/s}{0.6m} = 5 \times 10^8 cycles/s. \tag{18}$$

   (d) The magnetic field is oriented along the $\hat{\mathbf{z}}$ axis and has the maximum magnitude

$$B_{\max} = \frac{E_{\max}}{c} = 6.67 \times 10^{-9} T. \tag{19}$$

4.



(a) We can use Snell's law $n_1 \sin\theta_1 = n_2 \sin\theta_2$. We know the angles θ_1 and θ_2 and therefore can find the ratio of the refractive indices.

$$\frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1} = 2.138. \tag{20}$$

(b) From the previous analysis, we see the $n_1 > n_2$ so the light can be totally internally reflected within medium 1 as shown in the diagram. The smallest angle θ for which this can occur is

$$\theta \geq \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = 27.88^o. \tag{21}$$

*********************End of Exam**********************