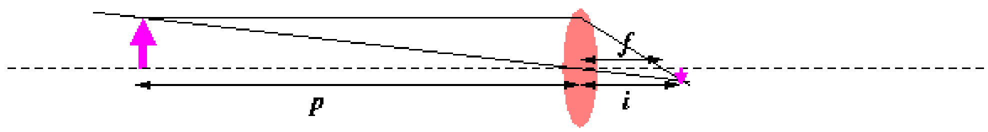
April 24, 2005

PHY 114 – Fourth Hour Exam – solutions

1. 

(a) This problem makes use of the thin lens equation

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}. \tag{1}$$

$$\frac{1}{80} + \frac{1}{2.5} = \frac{1}{f} \qquad \Rightarrow f = 2.42424\ cm. \tag{2}$$

(b)

$$\frac{1}{8} + \frac{1}{2.5} = \frac{1}{f} \qquad \Rightarrow f = 1.90476\ cm. \tag{3}$$

(c) In this case, the focal length should be 2.42424 cm as found above, but can only be 2.3 cm. The lens equation tells us that the focussed image would then be place at 2.368 cm which is to the left of the retina. In order to move the focussed image on to the retina, we need to use a diverging lens. Denote by p_l the position of an object that will be correctly focussed.

$$\frac{1}{p_l} + \frac{1}{2.5} = \frac{1}{2.3} \qquad \Rightarrow p_l = 28.75\ cm. \tag{4}$$

We need for the lens to make a virtual image at $-i_l = 28.75 - 1 = 27.75$ cm of the object at 80-1=79 cm.

$$\frac{1}{79} - \frac{1}{27.75} = \frac{1}{f_l} \qquad \Rightarrow f_l = -42.776\ cm. \tag{5}$$

(d) In this case, the focal length should be 1.90476 cm as found above, but can be no smaller than 2.0 cm. The lens equation tells us that the focussed image would then be place at 2.6667 cm which is to the right of the retina. In order to move the focussed image on to the retina, we need to use a converging lens. Denote by p_l the position of an object that will be correctly focussed.

$$\frac{1}{p_l} + \frac{1}{2.5} = \frac{1}{2.0} \qquad \Rightarrow p_l = 10\ cm. \tag{6}$$

We need for the lens to make a virtual image at $-i_l = 10 - 1 = 9$ cm of the object at 8-1=7 cm.

$$\frac{1}{7} - \frac{1}{9} = \frac{1}{f_l} \qquad \Rightarrow f_l = 31.5\ cm. \tag{7}$$

2. (a) Using the trigonometric identities:

$$E_{\text{tot}} = \{5\cos(0.0003x - 90000t) + 5\cos(0.0003x - 90000t + 1.2)\} \quad (8)$$

$$= 10\cos(0.0003x - 90000t + 0.6)\cos(0.6). \quad (9)$$

From this result, we find that the maximum electric field is $10\cos(0.6)$ N/C = 8.253356 N/C in the $\hat{\mathbf{y}}$ direction.

(b) The magnetic field is in the $\hat{\mathbf{z}}$ direction and has a magnitude of 8.25335/c T=2.7511×10^{-8} T.

(c)

$$S = \frac{1}{\mu_0}E\times B. \quad S_{\max} = \frac{1}{4\pi 10^{-7}}(8.253356)(2.7511\times10^{-8}) = 0.18W. \quad (10)$$

3. (a) From the appearance of the pattern it must be produced by a multi-slit device and the slit width is very much smaller than the distance between slits.

(b) Reading from the graph, the distance from the central peak to the first grey maximum, $\Delta y \approx 1.5$ m.

$$\lambda_1 = d\sin\theta = d\frac{y}{\sqrt{y^2+(15)^2}} = 597nm. \quad (11)$$

Reading from the graph, the distance from the central peak to the first dark maximum, $\Delta y \approx 1.7$ m.

$$\lambda_1 = d\sin\theta = d\frac{y}{\sqrt{y^2+(15)^2}} = 676nm. \quad (12)$$

4. (a) From the graph, $N/N_0 = \frac{1}{2}$ when $t = 10\mu s$.

(b) i.

$$v = \frac{h}{\Delta t_{\text{Earth frame}}} = \frac{4000m}{16.66666\times10^{-6}s} = 2.4\times10^8 m/s = 0.8c. \quad (13)$$

ii. Reading the graph at $t = 16.66667\mu s$, $N/N_0 \approx 0.3$.

iii. We infer that $\Delta t_{\text{Particle frame}} = 10\mu s$ and note that $\Delta t_{\text{Earth frame}} = 1.66667\Delta t_{\text{Particle frame}}$. We could infer that this is consistent with the Lorentz transformation, where

$$\gamma \equiv \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \quad \Rightarrow \frac{v}{c} = \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{1-\frac{1}{\gamma^2}} = 0.8. \quad (14)$$

5. (Extra problem in alternate exam.)

(a) In the following expression, $v > 0$ means that the source and detector are moving apart.

$$f' = f\sqrt{\frac{1-v/c}{1+v/c}} \quad \lambda' = \lambda\sqrt{\frac{1+v/c}{1-v/c}}. \quad (15)$$

In our example, $v < 0$ so that the source and detector are moving towards eachother.

(b)

$$\left(\frac{\lambda'}{\lambda}\right)^2 = \frac{1 + v/c}{1 - v/c}. \tag{16}$$

$$\frac{v}{c} = \frac{\left(\frac{\lambda'}{\lambda}\right)^2 - 1}{\left(\frac{\lambda'}{\lambda}\right)^2 + 1} = \frac{\left(\frac{450}{600}\right)^2 - 1}{\left(\frac{450}{600}\right)^2 + 1} = -0.28. \tag{17}$$