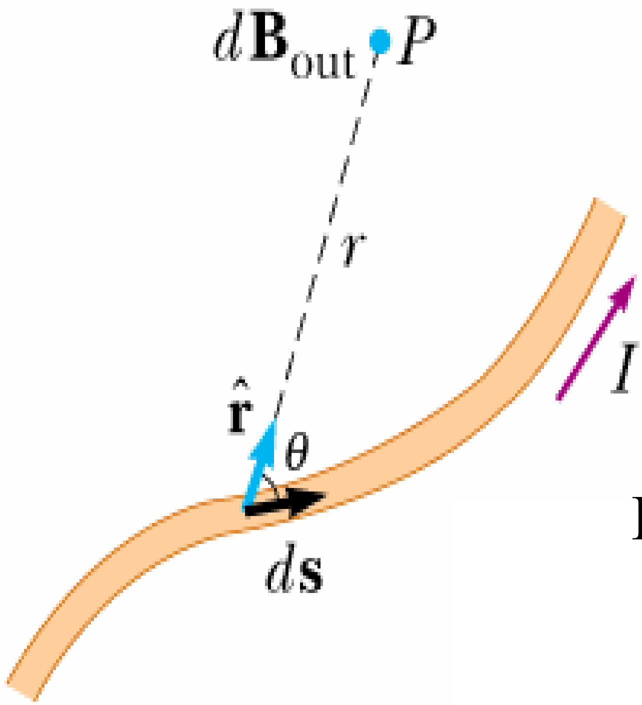
Announcements

1. Remember – Reworked exams are due now (11 AM on Monday (2/14/05)) – see me if you need an extension.
2. Assignments ≤ #15 posted (Second exam 2/25/05).
3. Class tutorial – Wednesdays at 6 PM in Olin 101 (in addition to regular tutorials by Todd Fallesen and Doug Bonessi
4. Today's topics
   a. Review magnetic fields from current sources
   b. Ampere's law
   c. Faraday's law

Sources of magnetic field – currents

Biot-Savart law



$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I d\mathbf{s}\times\hat{\mathbf{r}}}{r^2}$$

Field from a single moving charge:

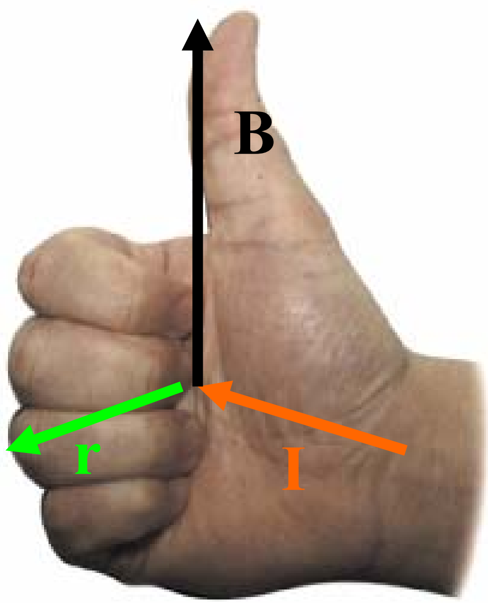
$$\mathbf{B} \approx \frac{\mu_0}{4\pi}\frac{q\mathbf{v}\times\hat{\mathbf{r}}}{r^2}$$

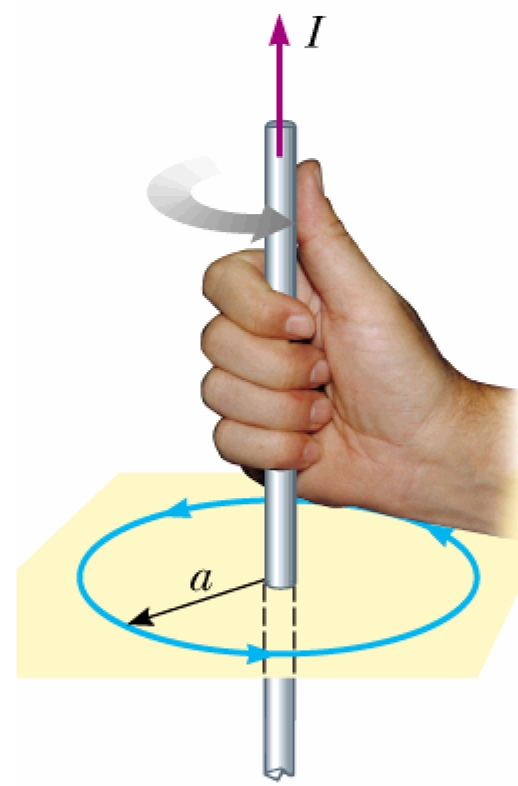
μ_0 = Permeability constant

$= 4\pi \times 10^{-7}\ \mathrm{Tm^2/A}$ (or H/m)

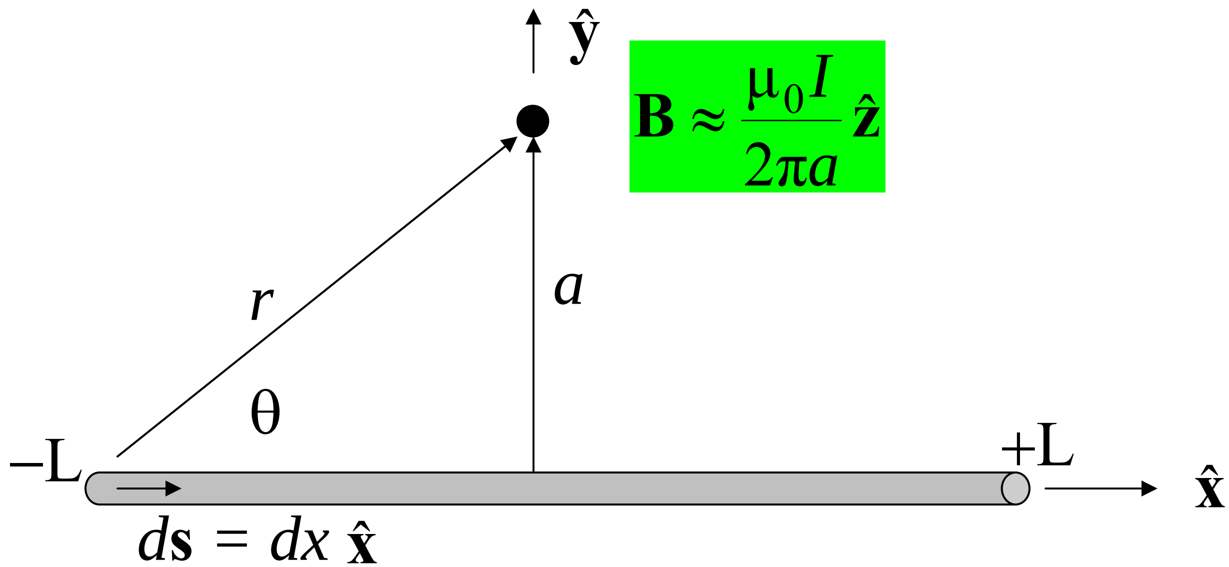
Digression on the right-hand rule:

$\mathbf{B}$	$\rightarrow$ $\mathbf{I}$	$\times$ $\mathbf{r}$
thumb	palm	fingers
palm	fingers	thumb
fingers	thumb	palm





Integrating the Biot-Savart equation

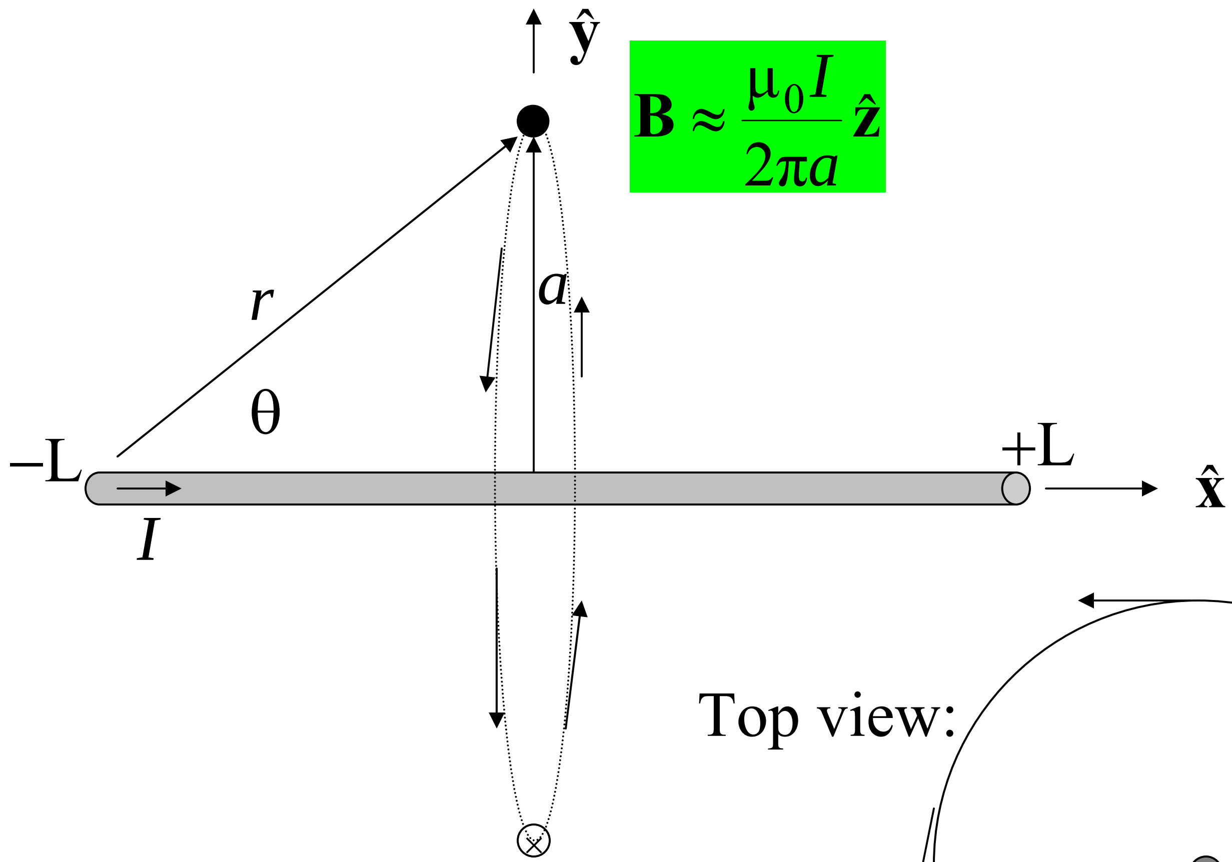


$$\boxed{\mathbf{B} \approx \frac{\mu_0 I}{2\pi a}\hat{\mathbf{z}}}$$

$$\mathbf{B} = \frac{\mu_0}{4\pi}\int \frac{I d\mathbf{s}\times\hat{\mathbf{r}}}{r^2} = \frac{\mu_0 I}{4\pi}\hat{\mathbf{z}}\int_{-L}^{+L} dx \frac{1}{x^2+a^2}\left(\underbrace{\frac{a}{\sqrt{x^2+a^2}}}_{\sin\theta}\right) = \frac{\mu_0 I}{2\pi a}\hat{\mathbf{z}}\left(\underbrace{\frac{L}{\sqrt{L^2+a^2}}}_{\approx 1}\right)$$

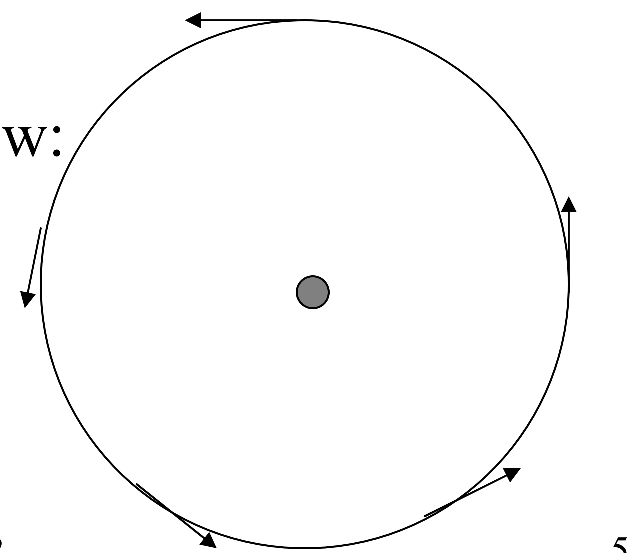
(when L → ∞)

Magnetic field from a long wire:



$$\mathbf{B} \approx \frac{\mu_0 I}{2\pi a}\hat{\mathbf{z}}$$

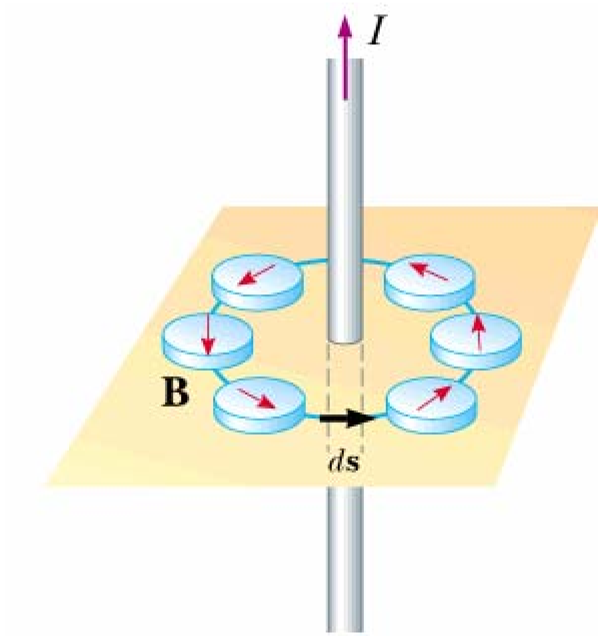
Top view:

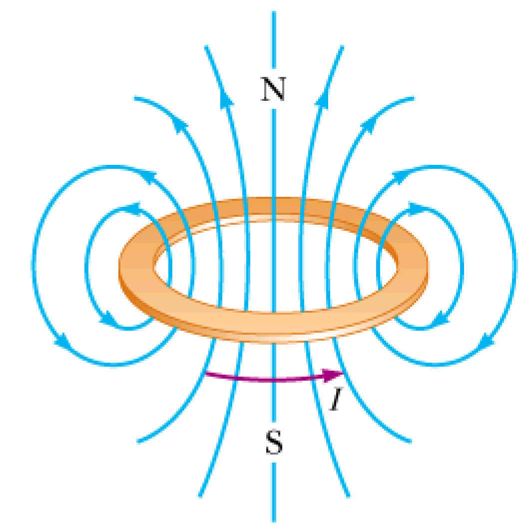


Visualizing **B** field lines using iron filings

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$$

filings move until $\tau = 0$ ➔ filings align along **B** field lines





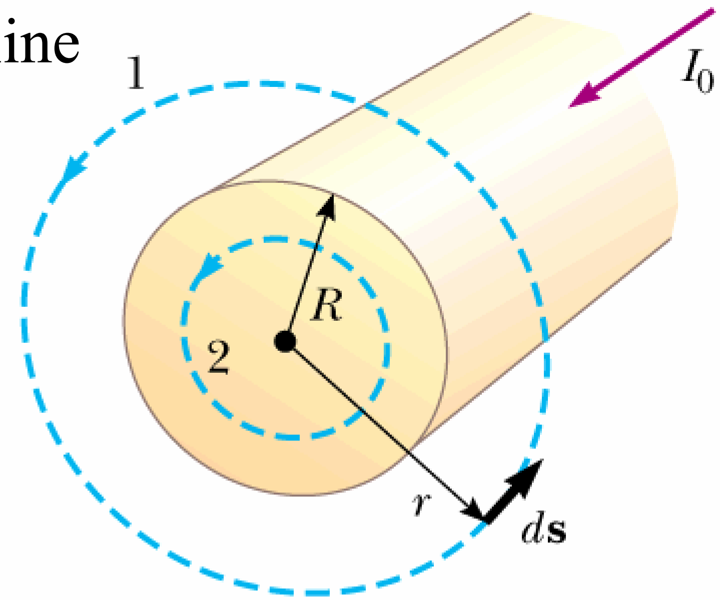
Ampere's law (Gauss's law for currents)

$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 I_{in}$$

Closed line integral

1. For $r > R$:

$$B(2\pi r) = \mu_0 I_0 \qquad \Rightarrow \quad B = \frac{\mu_0 I_0}{2\pi r}$$



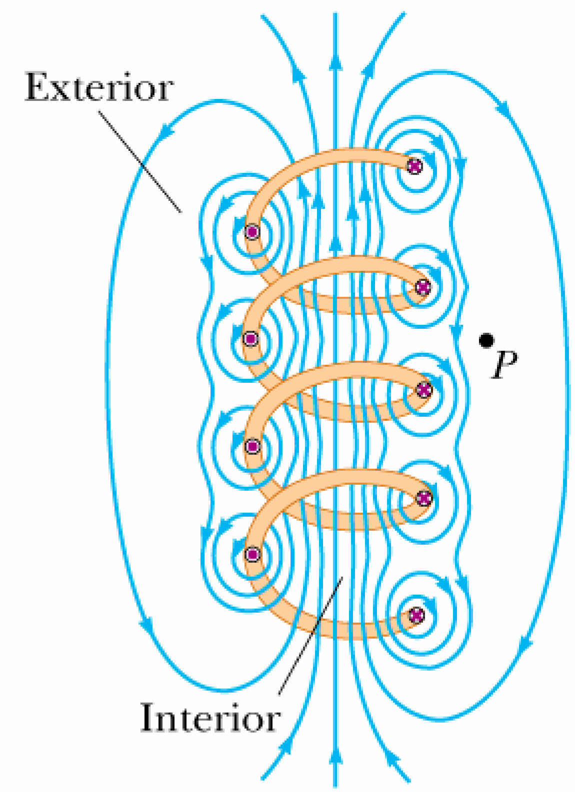
2. For $r < R$:

$$B(2\pi r) = \mu_0 I_0 \frac{r^2}{R^2} \qquad \Rightarrow \quad B = \frac{\mu_0 I_0 r}{2\pi R^2}$$

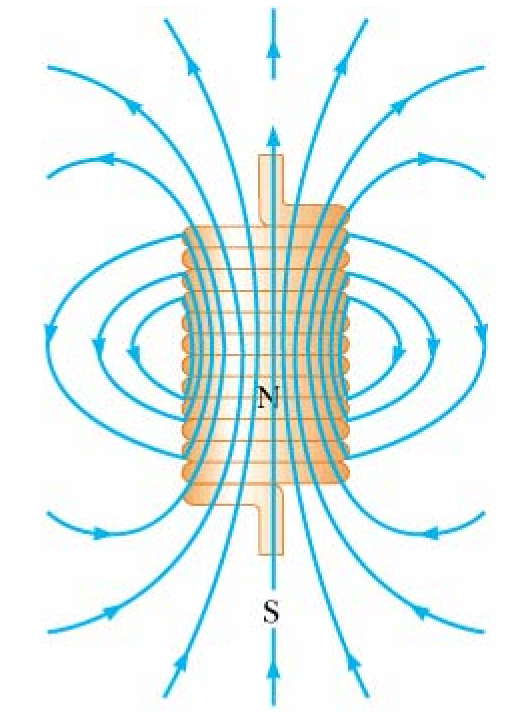
Magnetic field in the solenoid geometry

$$B_{interior} = \mu_0 n I$$

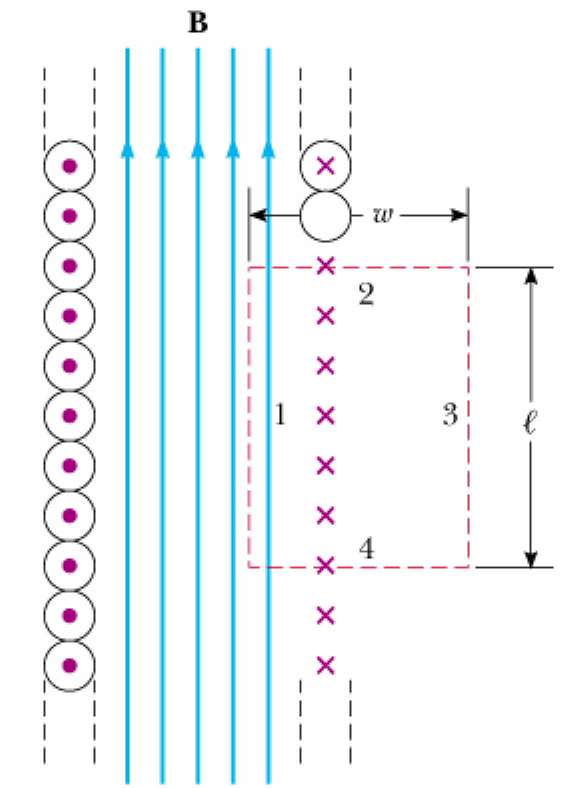
number of coils/unit length



Helical form

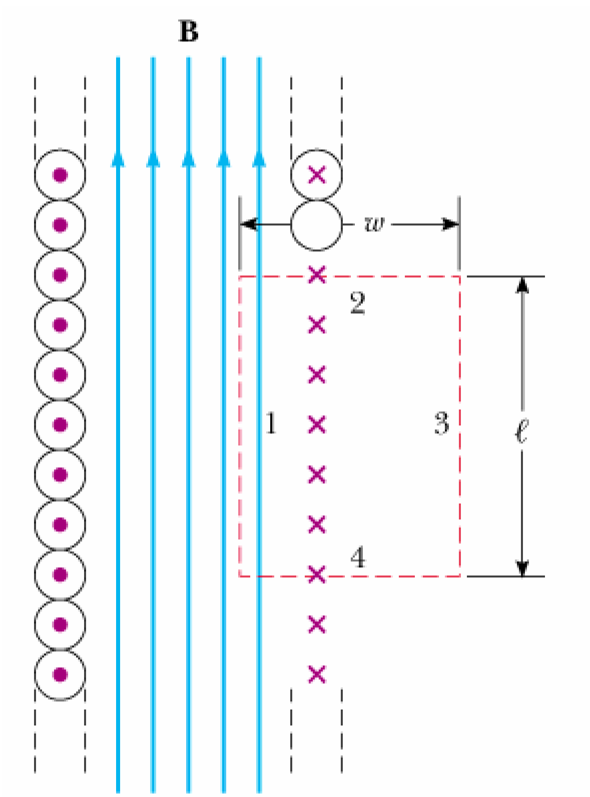


Tight coil form



Ideal form

Details of the solenoid field:



Ampere's law:

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{in}$$

$$B_{in} l + 0w + 0l = \mu_0 N I$$

$$B_{in} = \frac{\mu_0 N I}{l} = \mu_0 n I$$

$$n \equiv N/l$$

Summary of magnetic fields from currents:

Ampere's law: $\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{in}$

Closed line integral

Field from a single long wire having current *I*:

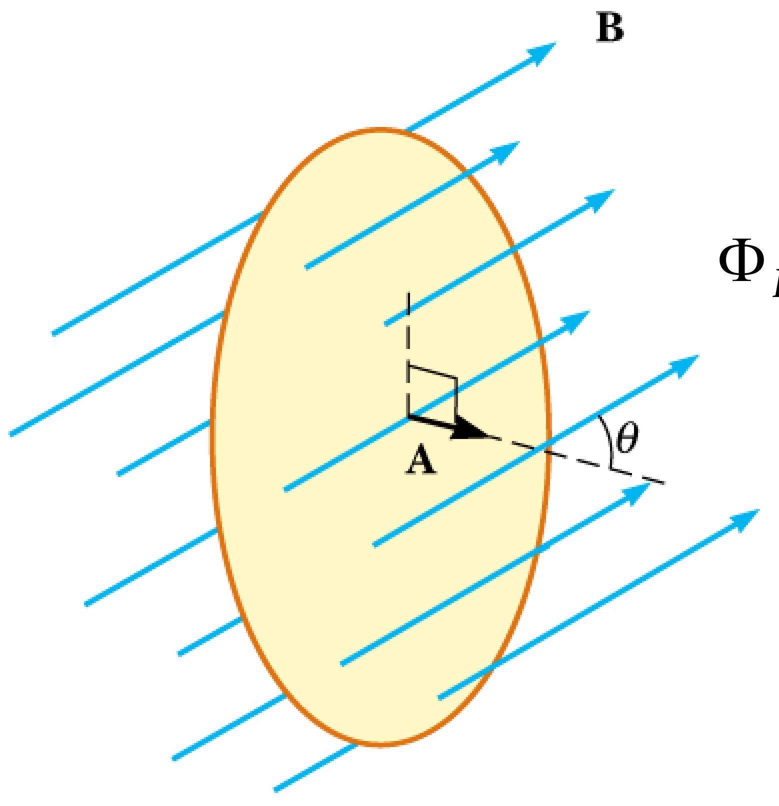
$$\boxed{\mathbf{B} = \frac{\mu_0 I}{2\pi r}}$$

➔ at distance r from wire

direction tangent to circle

Field inside a long solenoid having current *I* and loops/length *n*: $\boxed{\mathbf{B} = \mu_0 n I}$

Up to now, most of our discussion has concerned quantities which are constant in time. New physics is introduced when fields and sources change with time:



$$\Phi_B \equiv \int \mathbf{B} \cdot d\mathbf{A} = BA\cos\theta$$

Faraday's law:

$$\boxed{\boldsymbol{\varepsilon} = -\frac{d\Phi_B}{dt}}$$

➔A changing magnetic flux produces an emf!

Faraday's law:

$$\frac{d\Phi_B}{dt} = \frac{d\int \mathbf{B}\cdot d\mathbf{A}}{dt} = \frac{d\left(BA\cos\theta\right)}{dt} = -\boldsymbol{\varepsilon}$$

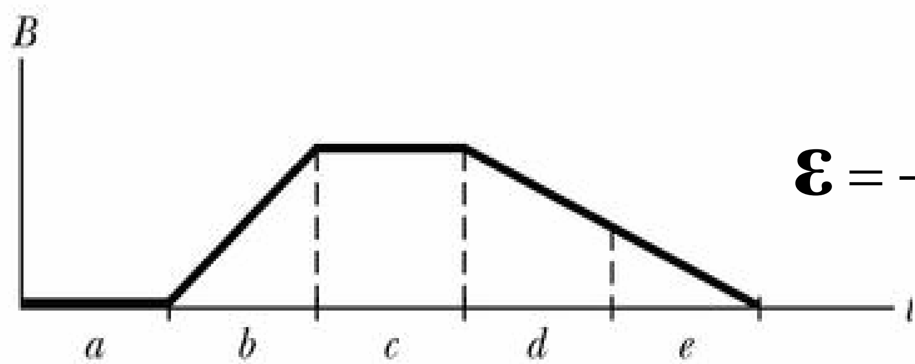
$$\frac{dB}{dt}A\cos\theta + B\frac{dA}{dt}\cos\theta - BA\sin\theta\frac{d\theta}{dt} = -\boldsymbol{\varepsilon}$$

generator

metal detector

inductors

Online Quiz for Lecture 12
Faraday's law -- Feb. 14, 2005



$$\boldsymbol{\varepsilon} = -\frac{d\Phi_B}{dt} = -\frac{d\mathbf{B}}{dt}A$$

The graph gives the magnitude B(t) of a uniform magnetic field that exists throughout a conducting loop, perpendicular to the plane of the loop, plotted as a function of time t. For each of the following questions, find the region of the graph (a,b,c,d,e) which best represents the answer.

1. In which interval is the induced emf the largest? B
2. In which interval(s) is the induced emf 0? A,C

1. [HRW6 31.P.004.] The magnetic field through a single loop of wire, 12 cm in radius and of 8.6 Ω resistance, changes with time as shown in Fig. 31-34. Calculate the emf in the loop as a function of time for each of the following time intervals. The (uniform) magnetic field is perpendicular to the plane of the loop.

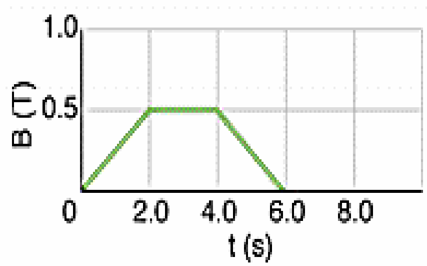


Figure 31-34

(a) $t = 0$ to $t = 2.0$ s

[ ] V

(b) $t = 2.0$ s to $t = 4.0$ s

[ ] V

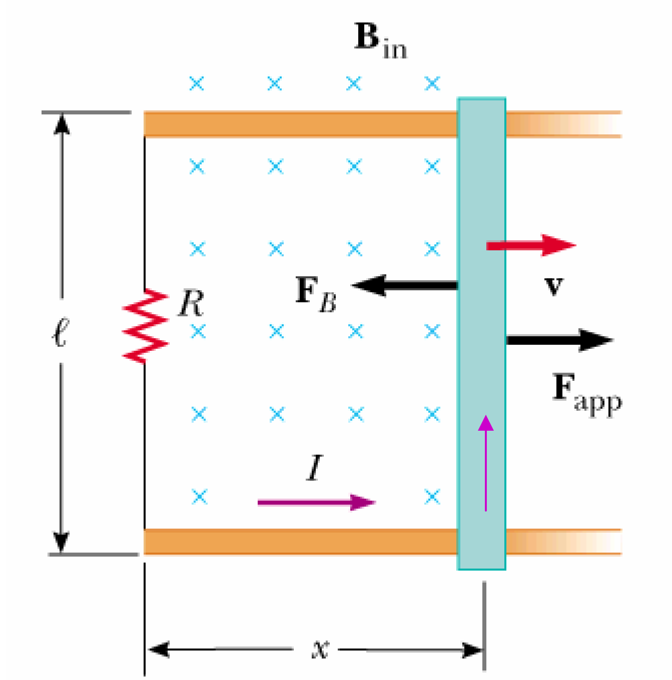
(c) $t = 4.0$ s to $t = 6.0$ s

[ ] V

$$\boldsymbol{\varepsilon} = -\frac{d\Phi_B}{dt} = -\frac{d\mathbf{B}}{dt} A$$

$$\frac{d\mathbf{B}}{dt} \approx \frac{\mathbf{B}(t=2) - \mathbf{B}(t=0)}{2-0} \quad \text{for} \quad 0 \le t \le 2s$$

Simple example:



$$\Phi_B = Blx$$

$$\frac{d\Phi_B}{dt} = Blv = -\boldsymbol{\mathcal{E}}$$

(induced current creates magnetic field opposing B_{in})

Summary -- Faraday's law: Integral over area

Define magnetic flux: $\Phi_B \equiv \int \mathbf{B} \cdot d\mathbf{A}$

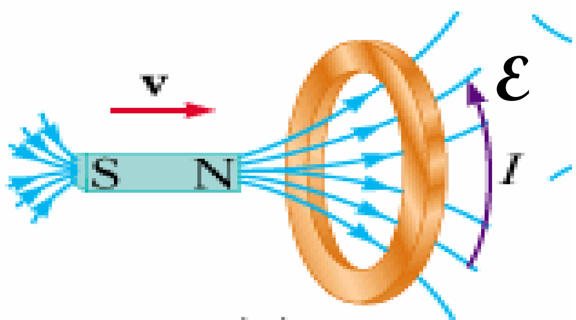
Changing flux induces an emf according to:

$$\boldsymbol{\varepsilon} = -\frac{d\Phi_B}{dt}$$

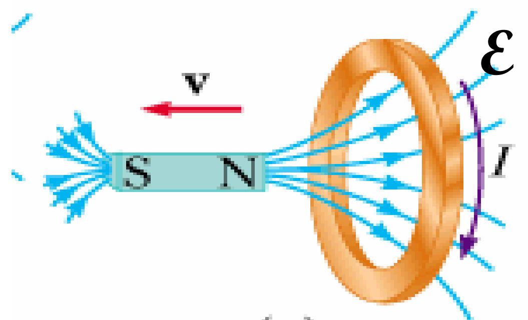
This emf has the same effects on a circuit as an emf from a battery, however, it can also exist in free space (without the circuit).

Example:

Bar magnetic moving toward (or way) from metal   loop inducing emf $\mathcal{E}$ and current I.

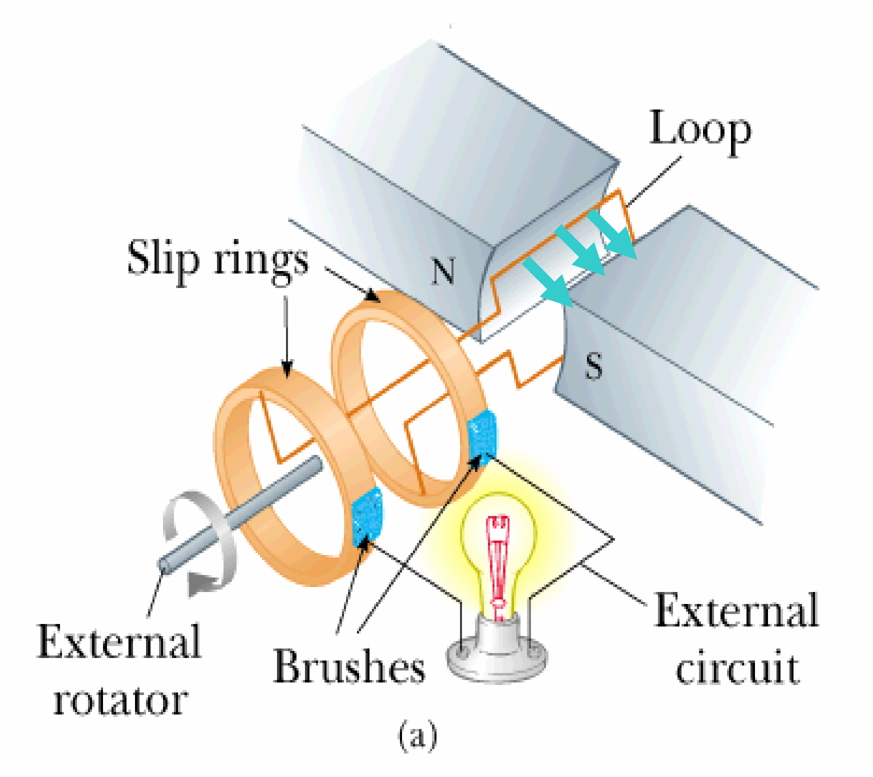


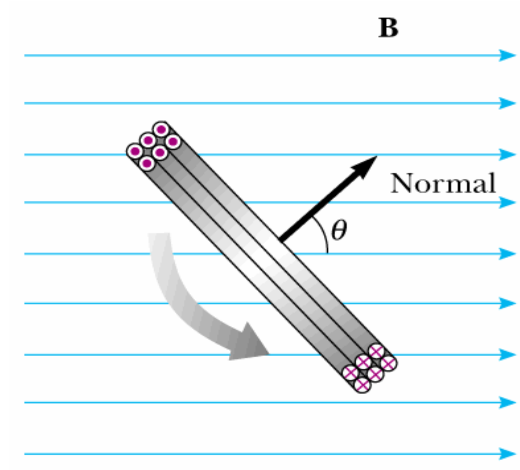
(a)



(c)

Example: AC generator





$$\frac{d}{dt}\int \mathbf{B}\cdot d\mathbf{A} = \frac{d}{dt}\left(BA\cos\omega t\right)$$

$$= -\omega BA\sin\omega t$$

$$\Rightarrow \mathcal{E} = \underbrace{\omega BA}_{\mathcal{E}_{\max}}\sin\omega t$$