Announcements

1. Second exam – Chap. 28.8-32 – Friday, Feb. 25th (practice exam now posted).
2. Problem solving session tonight -- 6 PM in Olin 101
3. Today's topics –

   Faraday's law

   Inductance

   LR circuits

Faraday's law:

Magnetic flux: (definition) $\Phi_B \equiv \int \mathbf{B} \cdot d\mathbf{A}$

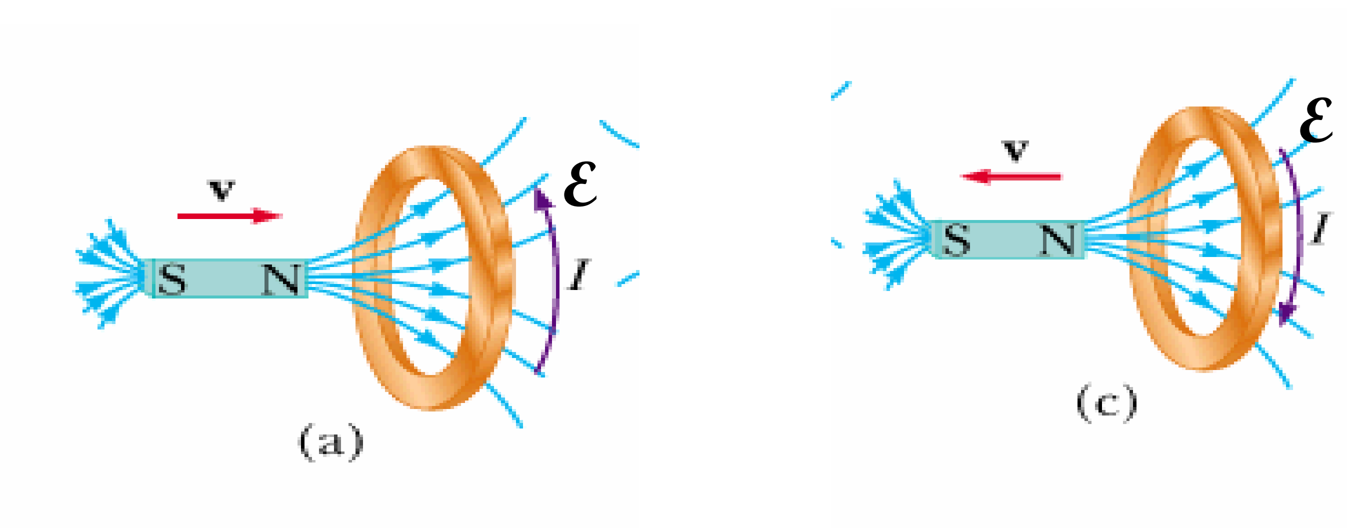
Changing flux induces an emf according to:

$$\boldsymbol{\varepsilon} = -\frac{d\Phi_B}{dt}$$

This emf has the same effects on a circuit as an emf from a battery, however it can also exist in free space (without the circuit).

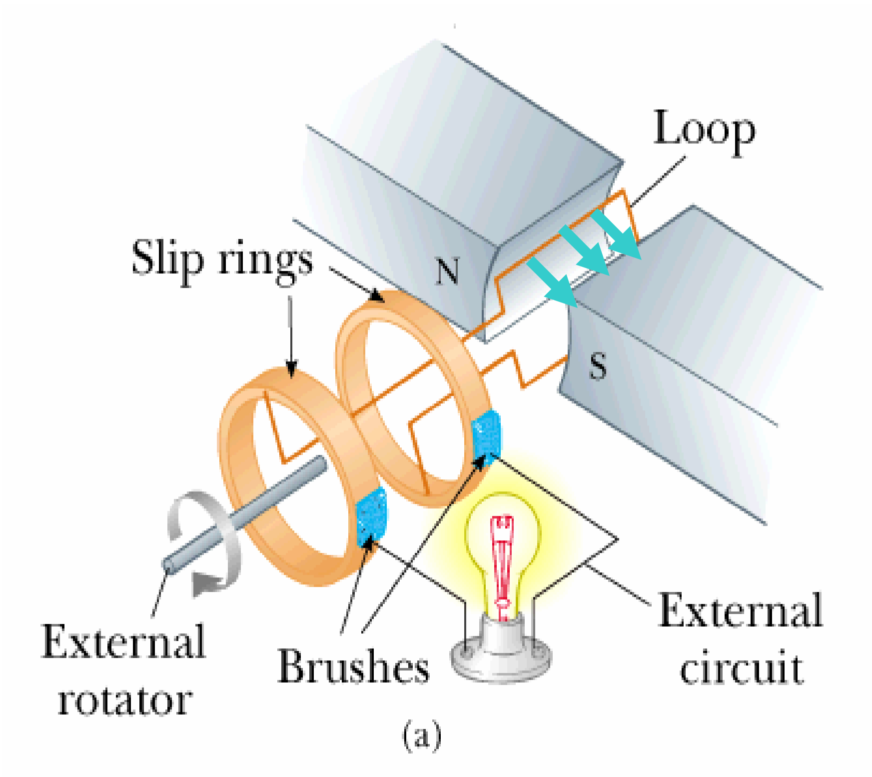
Example:

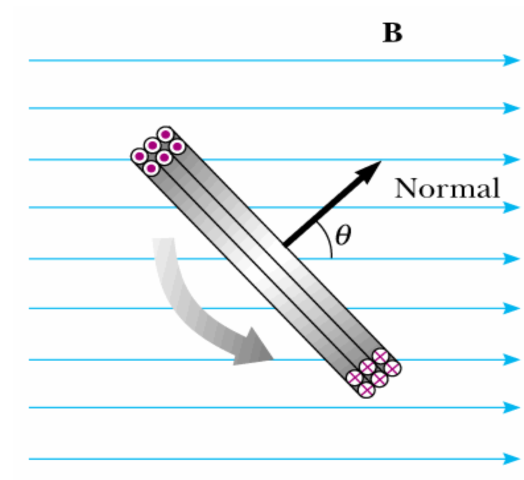
Bar magnetic moving toward (or way) from metal loop inducing emf $\mathcal{E}$ and current *I*.



An increasing flux induces a current in the direction that would produce a magnetic field opposing the field that creates it.

Example: AC generator

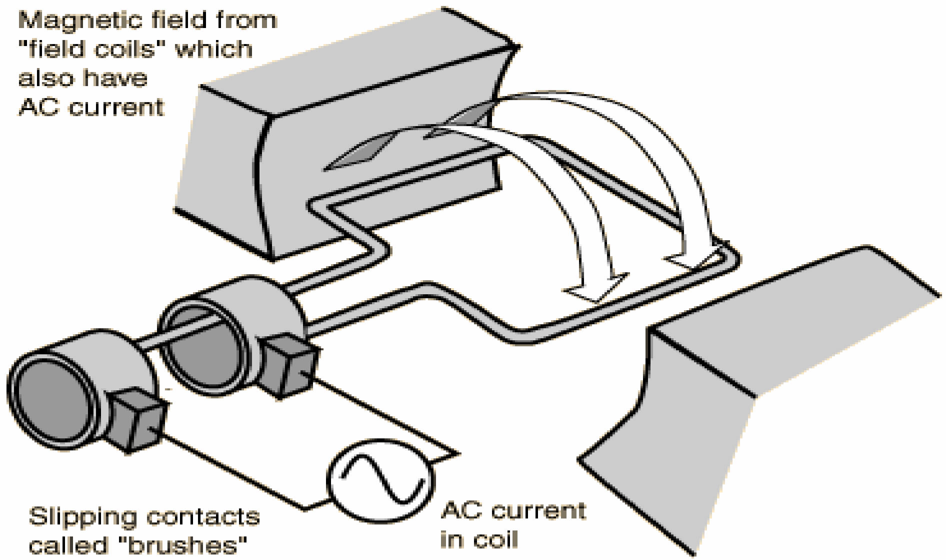




$$\frac{d}{dt}\int \mathbf{B}\cdot d\mathbf{A} = \frac{d}{dt}\left(BA\cos\omega t\right)$$
$$= -\omega BA \sin\omega t$$

$$\Rightarrow \mathcal{E} = \underbrace{\omega BA}_{\mathcal{E}_{\max}} \sin\omega t$$

Online Quiz for Lecture 12
Faraday's law -- Feb. 14, 2005

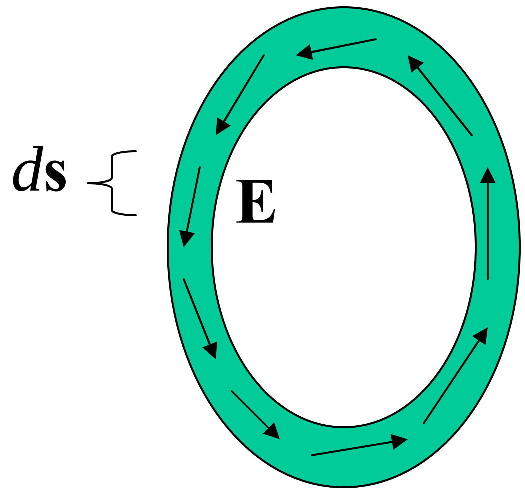


This figure was obtained from the website: http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/motorac.html . It shows a wire loop with area A turning in a a uniform magnetic field **B** so that at any given time t, the magnetic flux through the current loop is given by $\Phi(t) = \mathbf{B} \cdot \mathbf{A} = BA\cos(\Omega t)$, where Ω is the rate in radians per second that the wire turns and where (Ωt) is a measure of the angle between the area vector for the loop and the magnetic field direction. Which of the following expressions give the maximum magnitude of the induced emf in the wire loop?

A. $BA\,\Omega$
B. BA
C. BA/Ω
D. $BA\,(\Omega)^2$
E. None of these

More details about induced emf $\mathcal{E}$:

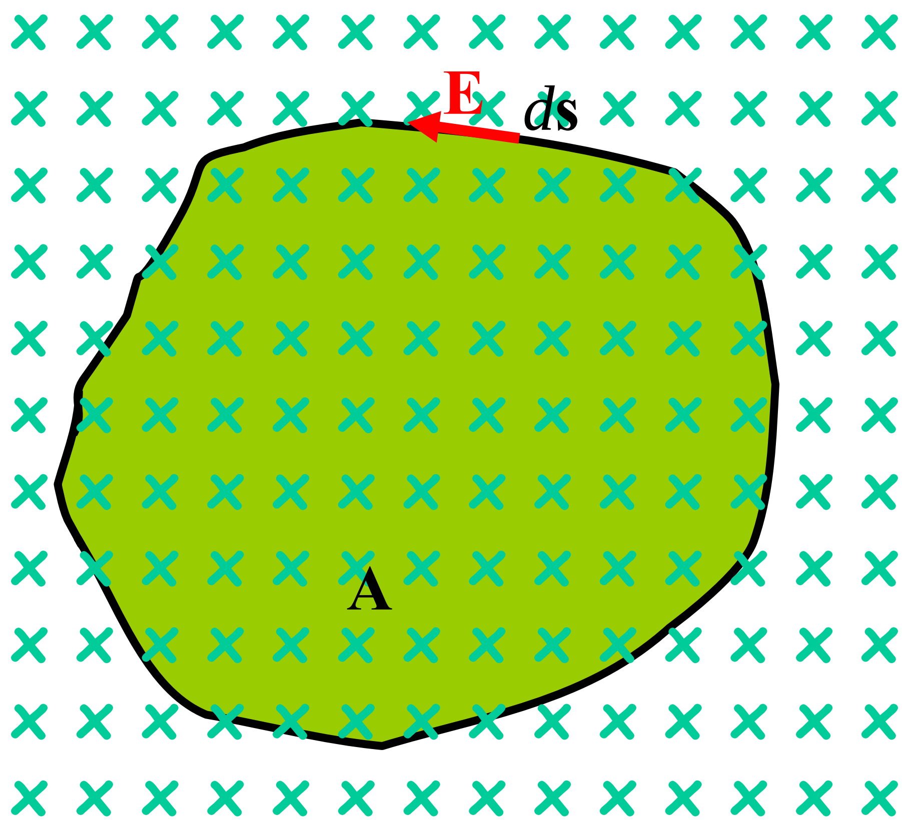
$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{s}$$



Work done on a one Coulomb test charge moving around a closed loop. (In this case, the electrical work is *not* conservative.)

Expressing Faraday's law in terms of induced electric field

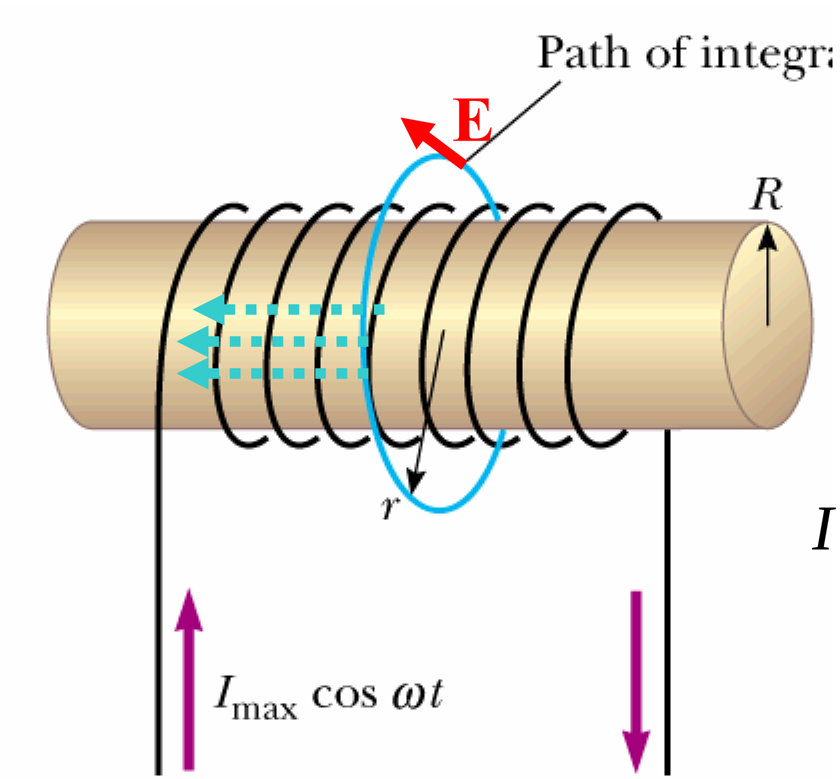
$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt}\int \mathbf{B} \cdot d\mathbf{A}$$



B pointing into screen and changing with time.

Example:

Changing **B** produced by changing I in a solenoid geometry:



$$\oint \mathbf{E} \cdot d\mathbf{s} = E(2\pi r)$$

$$\mathbf{B} = \mu_0 \, n \, I$$

$$\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{A} = \frac{d}{dt}\left(\mu_0 n I \;\; \pi R^2\right)$$

$$= \mu_0 n \frac{dI}{dt} \;\; \pi R^2$$

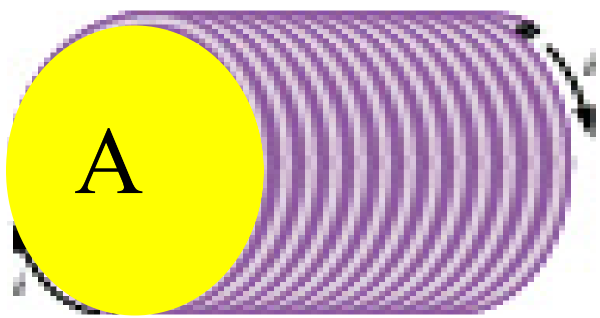
$$I = I_{max} \cos(\omega t) \;\; \Rightarrow \;\; \frac{dI}{dt} = -\omega I_{max} \sin(\omega t)$$

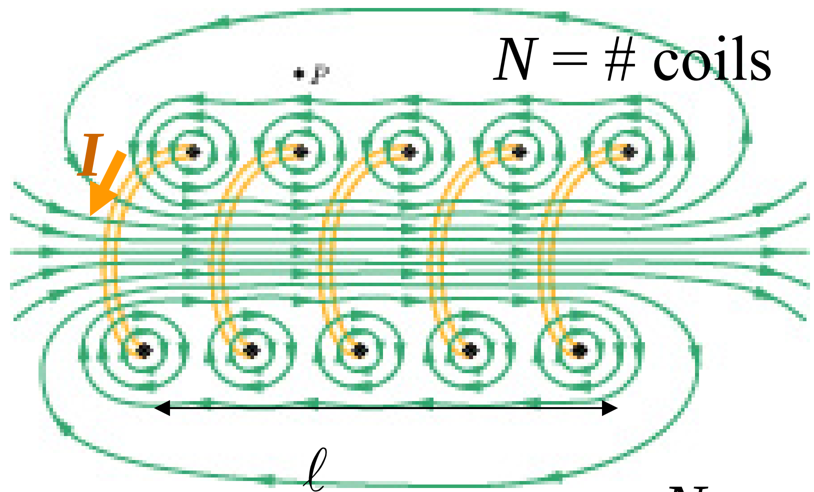
$$\boxed{E = \left(\frac{\mu_0 n I_{max} \omega R^2}{2r}\right) \sin \omega t}$$

Faraday's law ➔ an electric field can be produced a magnetic field which changes as a function of time in a wire and also in a free space.

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{A}$$

Faraday's law in a wire in the solenoid geometry





$N = \#$ coils

$$B = \mu_0 \frac{N}{\ell} I$$

$$\int \mathbf{B} \cdot d\mathbf{A} = \mu_0 \frac{N}{\ell} I(NA)$$

$$\mathcal{E} = -\frac{d}{dt}\int \mathbf{B} \cdot d\mathbf{A} = -\mu_0 \frac{N^2 A}{\ell}\frac{dI}{dt}$$

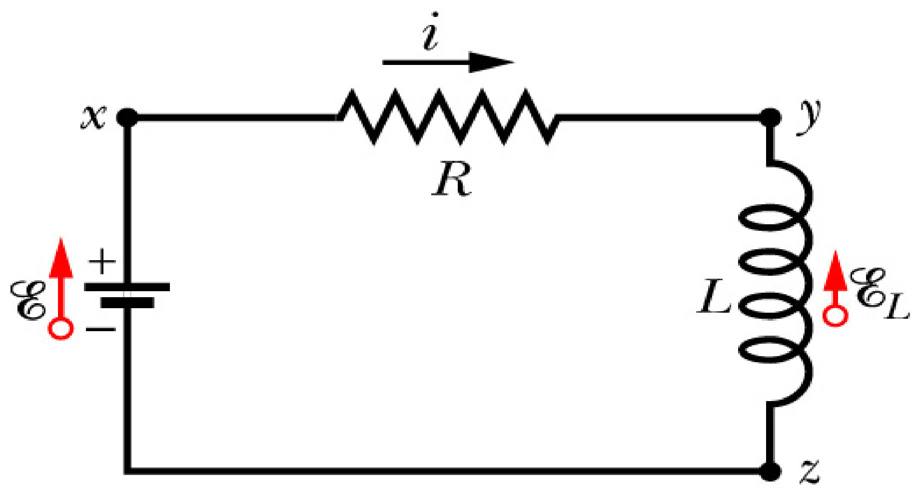
➔ In this geometry, a coil can induce an emf in itself!

Faraday's law in a solenoid:

$$\mathcal{E} = -\underbrace{\mu_0 \frac{N^2 A}{\ell}}_{\equiv L \text{ ``inductance''}} \frac{dI}{dt}$$

1 henry ≡ Volt · s/A = T · m^2/A

The LR circuit

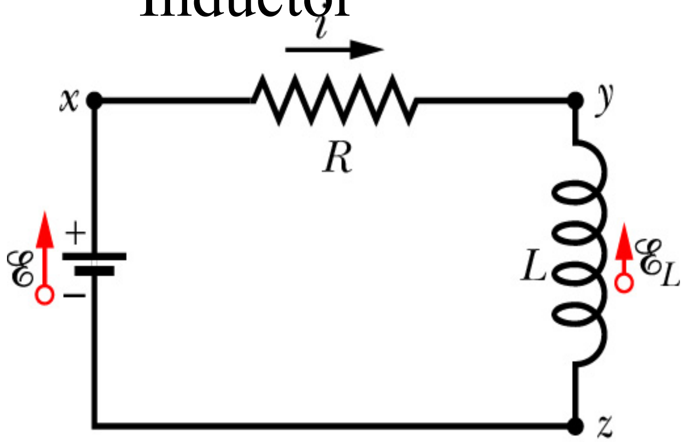


Kirchhoff's loop analysis:

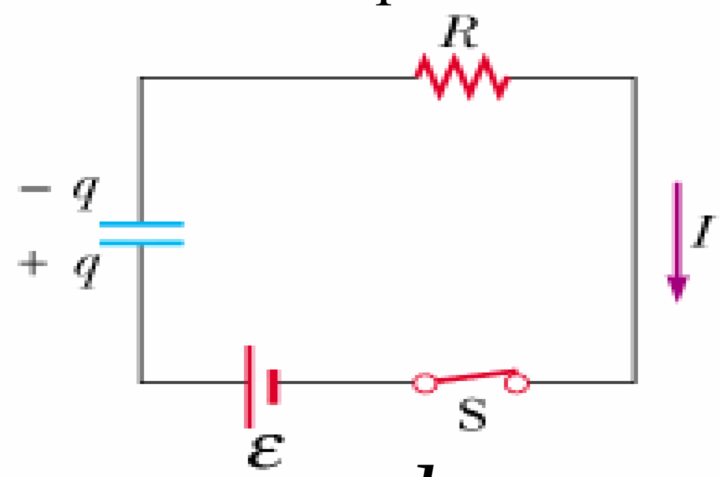
$$\mathcal{E} - iR - \frac{di}{dt}L = 0$$

Solution of differential equations:

Inductor



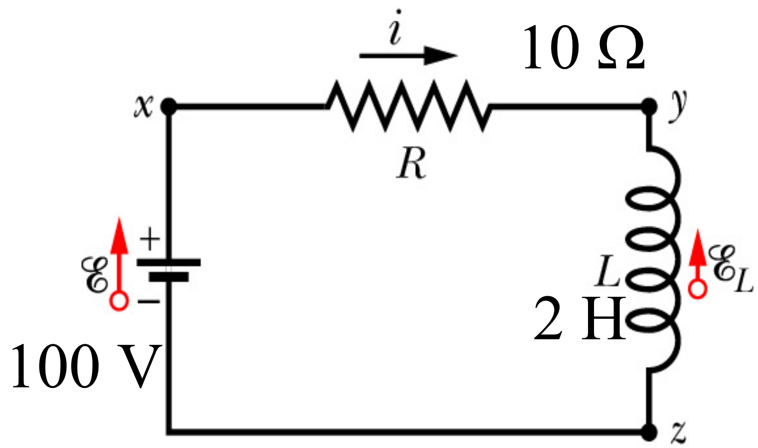
Capacitor



$$\mathcal{E} - iR - \frac{di}{dt}L = 0 \qquad \mathcal{E} - \frac{q}{C} - \frac{dq}{dt}R = 0$$

$$I(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/(L/R)}\right) \qquad q(t) = C\mathcal{E}\left(1 - e^{-t/(RC)}\right)$$

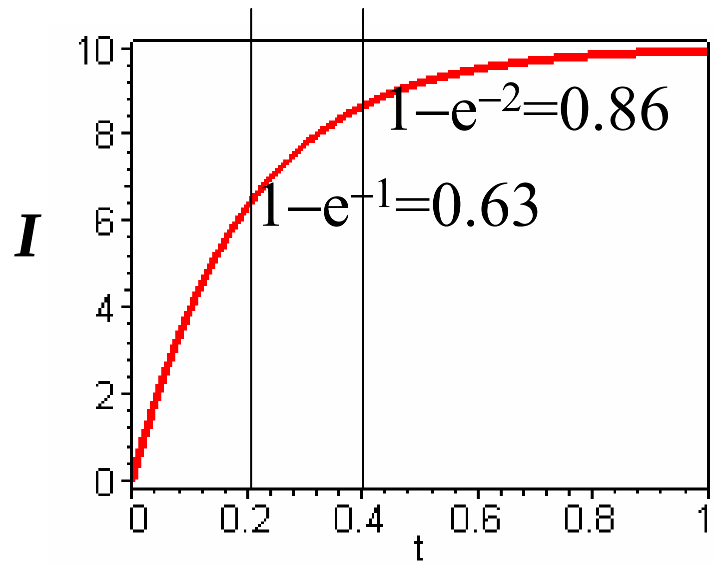
Example:
(assume switch closed at t=0)



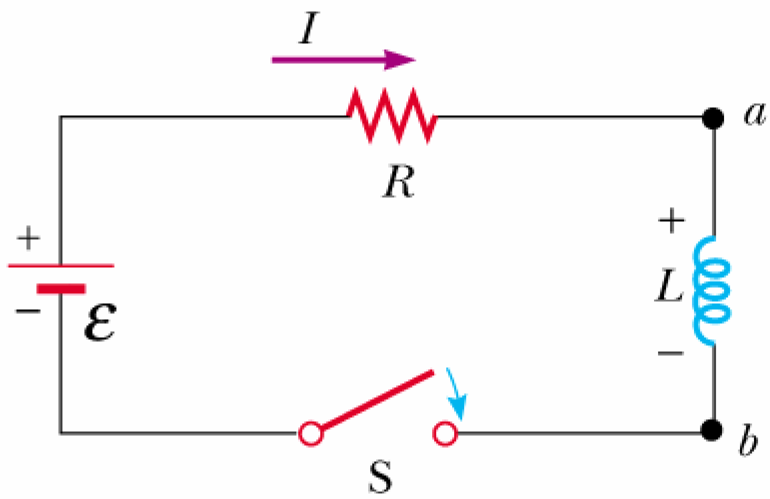
$$I(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/(L/R)}\right)$$

$$= \frac{100}{10}\left(1 - e^{-t/(2/10)}\right)$$

$$= 10\left(1 - e^{-t/0.2}\right)A$$



Summary: Inductors in a circuit:



$$\mathcal{E}_{\text{battery}} - IR - L\frac{dI}{dt} = 0$$

solution for $I(t)$ assuming $I(t=0)=0$:

$$I(t) = \frac{\mathcal{E}_{\text{battery}}}{R}\left(1 - e^{-t/(L/R)}\right)$$

emf across inductor:

$$\mathcal{E}_L = -L\frac{dI}{dt} = -\mathcal{E}_{\text{battery}} e^{-t/(L/R)}$$

emf across resistor:

$$\mathcal{E}_R = -RI = -\mathcal{E}_{\text{battery}}\left(1 - e^{-t/(L/R)}\right)$$

[HRW6 31.P.053.] In Fig. 31-55. $\mathcal{E}$ = 100 V, R_1 = 10.0 Ω, R_2 = 21.0 Ω, R_3 = 30.0 Ω, and L = 1.70 H.

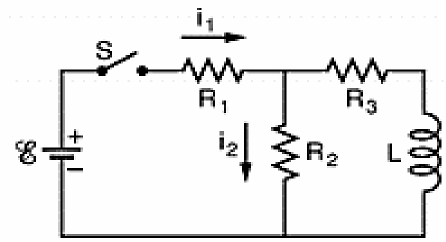


Fig 31-55

(a) Find the values of i_1 and i_2 immediately after the closing of switch S

_____ A (i_1)

_____ A (i_2)

(b) Find the values of i_1 and i_2 a long time later

_____ A (i_1)

_____ A (i_2)

(c) Find the values of i_1 and i_2 immediately after the reopening of switch S

_____ A (i_1)

_____ A (i_2)

(d) Find the values of i_1 and i_2 a long time after the reopening

_____ A (i_1)

_____ A (i_2)

Energy stored in an inductor:

$$dU = \mathcal{E}_L \, dq$$

$$dU = -dqL\frac{dI}{dt} = -\frac{dq}{dt}LdI = -ILdI$$

$$\left|\int_0^U dU'\right| = \int_0^I LI'dI' = \frac{1}{2}LI^2$$