

## Announcements

1. **Pick up your re-worked exams and other extra credit work.**

2. **One week from today (Friday, Feb. 25<sup>th</sup>) –second exam – covering Chap. 28.8-32**

**Advice about studying –**

**work problems**

**work problems**

**work problems**

**formulate questions and get answers**

**(practice exam now posted).**

3. **Today's topics –**

**Review inductance and LR circuits**

**Magnetism in materials**

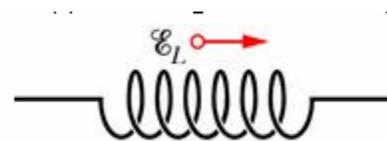
**Maxwell's equations**

Basic principle: Faraday's law

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{A}$$

Faraday's law in a solenoid:

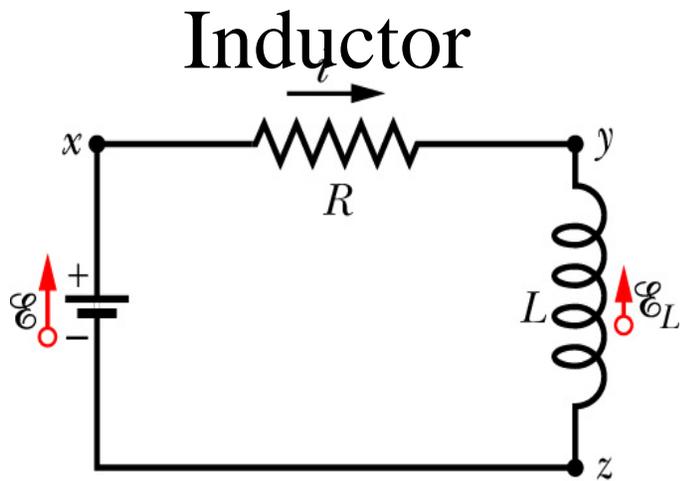
$$\mathcal{E} = -\underbrace{\mu_0 \frac{N^2 A}{\ell}}_{\text{inductance}} \frac{dI}{dt}$$



$\equiv L$  “inductance”

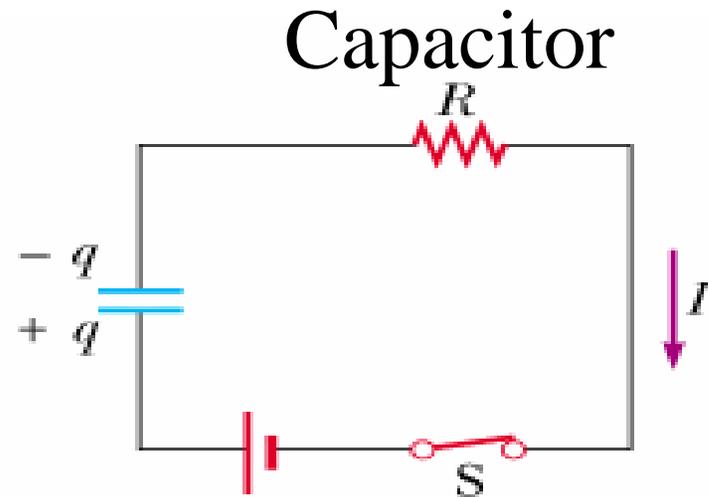
$$1 \text{ henry} \equiv \text{Volt} \cdot \text{s/A} = \text{T} \cdot \text{m}^2/\text{A}$$

## Solution of differential equations:



$$\mathcal{E} - IR - \frac{dI}{dt} L = 0$$

$$I(t) = \frac{\mathcal{E}}{R} \left( 1 - e^{-t/(L/R)} \right)$$

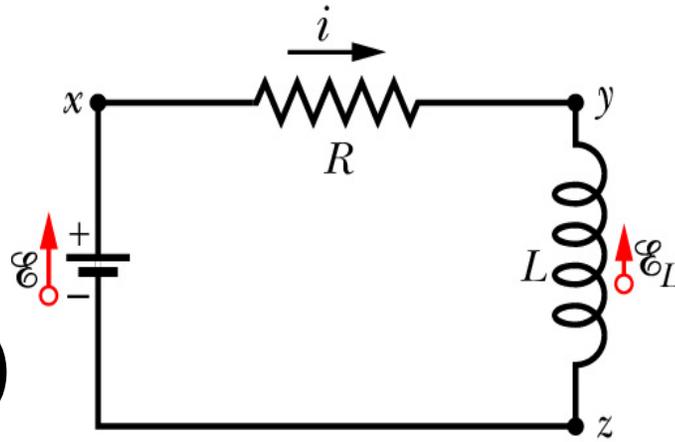


$$\mathcal{E} - \frac{q}{C} - \frac{dq}{dt} R = 0$$

$$q(t) = C\mathcal{E} \left( 1 - e^{-t/(RC)} \right)$$

More details:

$$\mathcal{E} - IR - \frac{dI}{dt}L = 0$$



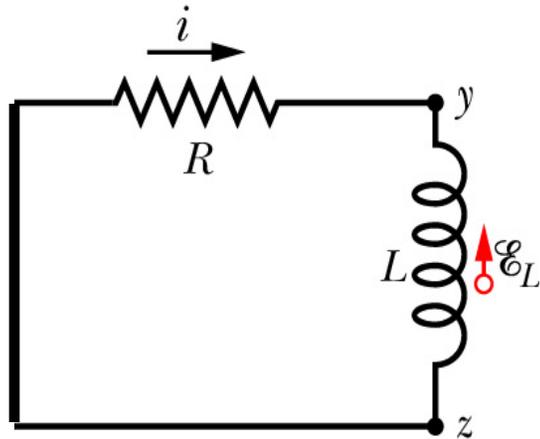
$$I(t) = \frac{\mathcal{E}}{R} \left( 1 - e^{-t/(L/R)} \right) + K e^{-t/(L/R)}$$

Extra term –depends  
on conditions

Example:

$$\text{If } I(t=0) = 0 \rightarrow K=0$$

Another example:



$$-IR - \frac{dI}{dt}L = 0$$

$$I(t) = Ke^{-t/(L/R)}$$

Constant  $K$  depends on conditions.

Example:  $I(t=0) = I_0 \rightarrow K = I_0$

Energy stored in an inductor:

$$dU = \mathcal{E}_L dq$$

$$dU = -dqL \frac{dI}{dt} = -\frac{dq}{dt} L dI = -IL dI$$

$$\left| \int_0^U dU' \right| = \int_0^I LI' dI' = \frac{1}{2} LI^2$$

Inductor energy :  $U_B = \frac{1}{2} LI^2$

Capacitor energy :  $U_E = \frac{1}{2} \frac{1}{C} Q^2$

4. [HRW6 31.P.060.] For the circuit of Fig. 31-18, assume that  $\mathcal{E} = 16.3 \text{ V}$ ,  $R = 6.15 \text{ } \Omega$ , and  $L = 5.69 \text{ H}$ . The battery is connected at time  $t = 0$ .

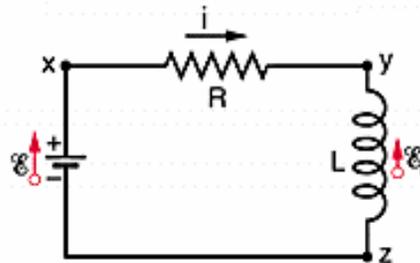


Figure 31-18

(a) How much energy is delivered by the battery during the first 2.00 s?

J

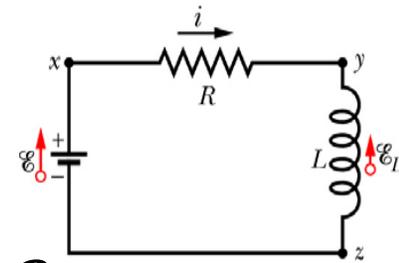
(b) How much of this energy is stored in the magnetic field of the inductor?

J

(c) How much of this energy has been dissipated in the resistor?

J

Energy delivered by battery :



$$W = \int_0^t dt' \mathcal{E} I(t'), \text{ where } I(t') = \frac{\mathcal{E}}{R} \left(1 - e^{-t'/(L/R)}\right)$$

Useful integrals (see Appendix E of text)

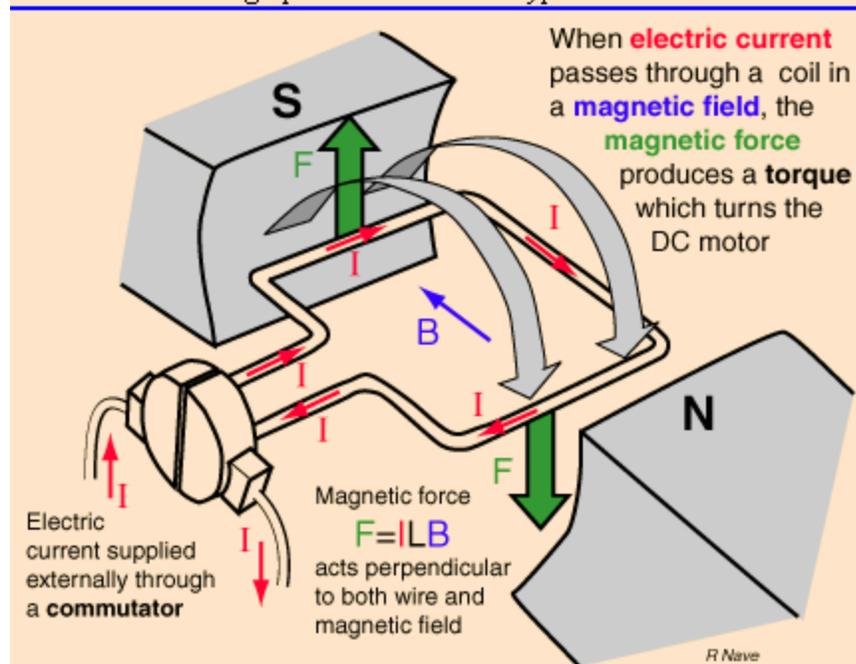
$$\int_a^b t^n dt = \frac{b^{n+1}}{n+1} - \frac{a^{n+1}}{n+1} \quad (n \neq -1)$$

$$\int_a^b dt = b - a$$

$$\int_a^b e^{at} dt = \frac{1}{a} e^{at} \Big|_a^b = \frac{1}{a} e^{bt} - \frac{1}{a} e^{at}$$

# Peer instruction question

<http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/motdc.html#c1>

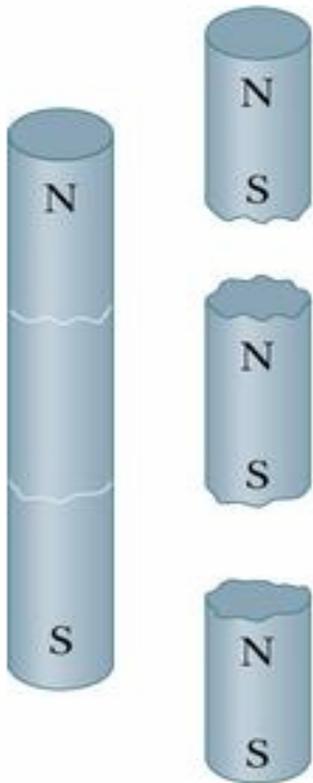


We have discussed that fact that the set up for a motor and a generator are very similar, with the magnetic force law and Faraday's law operational, respectively. What happens to Faraday's law when we use the motor configuration?

- (A) It has no effect.
- (B) It makes the motor run faster.
- (C) It makes the motor run slower.

# Sources of magnetic fields (in addition to currents)

- ▶ The simplest magnetic structure that can exist is a magnetic dipole. Magnetic monopoles do not exist (as far as we know).



**Fig. 32-2** If you break a magnet, each fragment becomes a separate magnet, with its own north and south poles.

Online Quiz for Lecture 14  
Magnetic materials -- Feb. 18, 2005

Gauss's law says that the total electric flux through any closed surface is proportional to the the electric charge inside the volume which the surface encloses. Now consider the total magnetic flux through any closed surface. What is that equal to? Write your answer in the box to the left.

## Gauss's law for electric fields

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{enclosed}}{\epsilon_0}$$

## Gauss's law for magnetic fields

$$\Phi_B = \oint \mathbf{B} \cdot d\mathbf{A} = 0 \quad \rightarrow \text{no magnetic "monopole"}$$

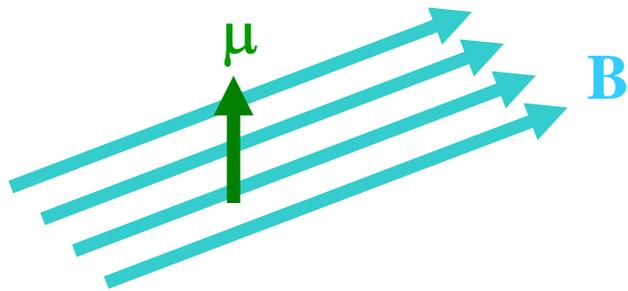
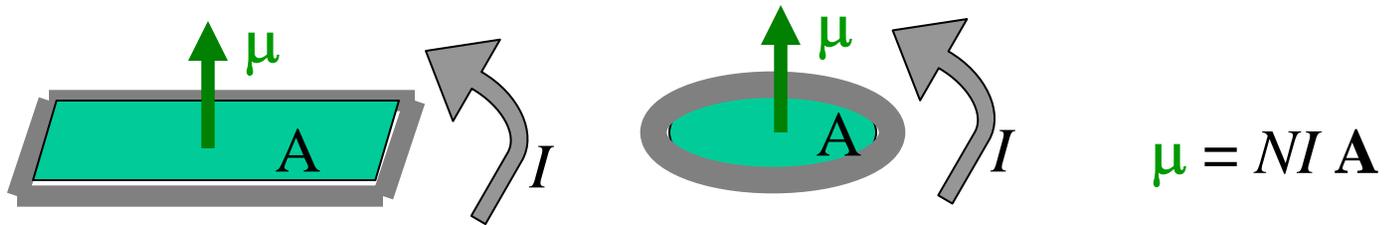
Magnetic materials –

Diamagnetic – respond to an applied magnetic field by producing “eddy currents” to oppose it. A superconductor is an ideal diamagnet.

Paramagnetic – respond to an applied magnetic field with induced magnetic dipoles

Ferromagnet – magnetic dipoles exist even without a magnetic field (Fe)

Magnetic moment associated with current loop:



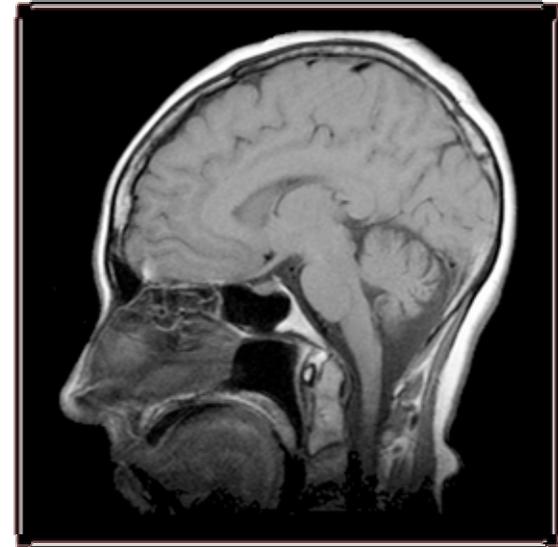
**Torque:**

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$$

Potential energy:

$$U = -\boldsymbol{\mu} \cdot \mathbf{B}$$

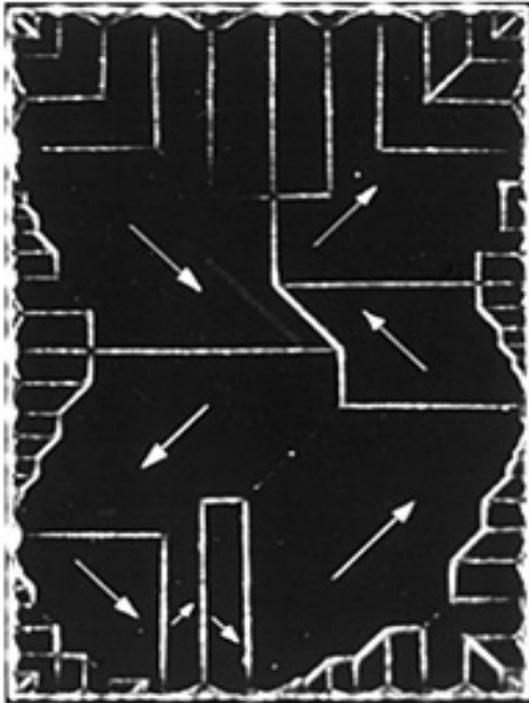
# MRI – Magnetic resonance imaging



$$\tau = \mu \times \mathbf{B}$$

$U = -\mu \cdot \mathbf{B}$  plus time varying  $B_1 \rightarrow$  signal from  $\mu$  of protons

Ref: <http://www.cis.rit.edu/htbooks/mri/inside.htm>



Courtesy Ralph W.  
DeBlois

Fig. 32-12 A photograph of domain patterns within a single crystal of nickel; white lines reveal the boundaries of the domains. The white arrows superimposed on the photograph show the orientations of the **magnetic** dipoles within the domains and thus the orientations of the net magnetic dipoles of the domains. The crystal as a whole is unmagnetized if the net **magnetic field** (the vector sum over all the domains) is zero.

$$\text{Torque: } \tau = \mu \times \mathbf{B}$$

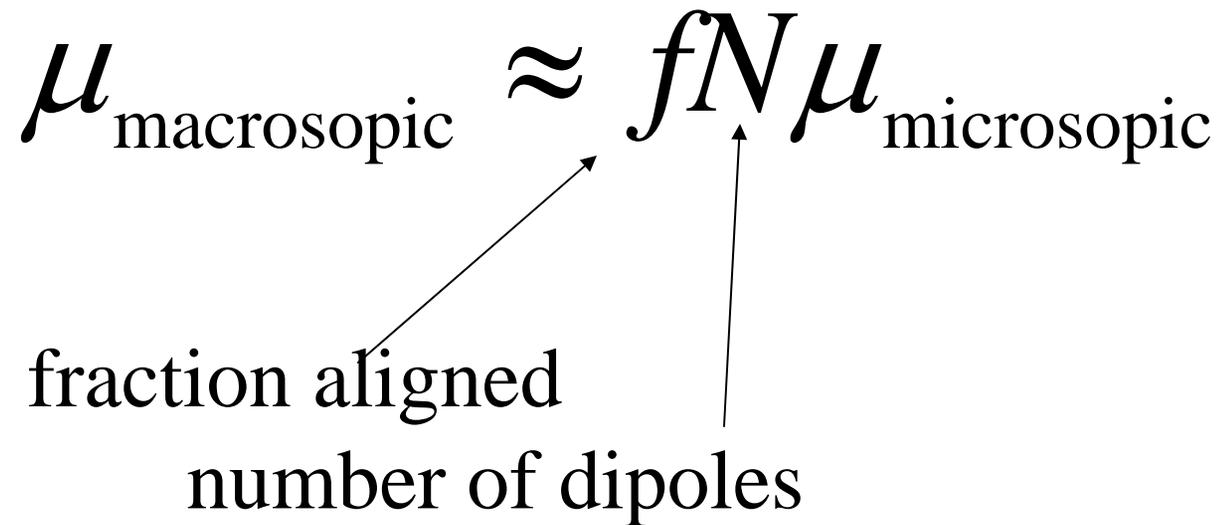
$$\text{Energy: } U = -\mu \cdot \mathbf{B}$$

Relationship between microscopic  
and macroscopic magnetic dipoles:

$$\mu_{\text{macroscopic}} \approx fN \mu_{\text{microscopic}}$$

fraction aligned

number of dipoles



3. [HRW6 32.P.022] The dipole moment associated with an atom of iron in an iron bar is  $2.1 \times 10^{-23}$  J/T. Assume that all the atoms in the bar, which is 5.7 cm long and has a cross-sectional area of  $1.0 \text{ cm}^2$ , have their dipole moments aligned.

(a) What is the dipole moment of the bar?

$\text{A} \cdot \text{m}^2$

(b) What torque must be exerted to hold this magnet perpendicular to an external field of 1.1 T? The density of iron is  $7.9 \text{ g/cm}^3$ .

$\text{N} \cdot \text{m}$

$$\mu_{\text{macroscopic}} \approx fN\mu_{\text{microscopic}}$$

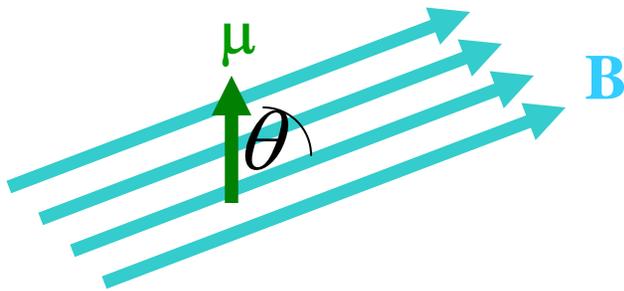
$$N = N_A \frac{\text{density} \cdot \text{volume}}{\text{molar mass}}$$

Note: Appendix F lists molar masses for all the elements in units of grams/mole

Calculation of the torque:

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$$

$$|\boldsymbol{\tau}| = |\boldsymbol{\mu}| |\mathbf{B}| \sin \theta$$



Summary of the  
basic laws of electricity and magnetism

**\*\*\*Maxwell's Equations\*\*\***

Coulomb's & Gauss's law :  $\oiint \mathbf{E} \cdot d\mathbf{A} = q_{\text{enclosed}} / \epsilon_0$

Magnetic Gauss's law :  $\oiint \mathbf{B} \cdot d\mathbf{A} = 0$

Ampere's law :  $\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{enclosed}} + \boxed{\phantom{\text{ }}}$

Faraday's law :  $\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt} \iint \mathbf{B} \cdot d\mathbf{A}$