Announcements

1. **Schedule for next few weeks –**

2. **Today's topics – Maxwell's equations "displacement" current**

Mon	Wed	Fri
2/21 Maxwell's Eq.	2/23 Review	2/25 2^{nd} Exam
2/28 AC circuits	3/2 AC circuits	3/4 EM waves
3/7 Spring break	3/9 Spring break	3/11 Spring break
3/14 EM waves	3/16 EM waves	3/18 EM waves
3/21 Review	3/23 3^{rd} Exam	3/25 Good Friday
3/28	3/30	4/1

Basic laws of electricity and magnetism

Lorentz force law:

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$$

Maxwell's Equations

Coulomb's & Gauss's law : $\oiint \mathbf{E} \cdot d\mathbf{A} = q_{\text{enclosed}} / \varepsilon_0$

Magnetic Gauss's law : $\oiint \mathbf{B} \cdot d\mathbf{A} = 0$

Ampere's law : $\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{enclosed}} + \boxed{}$

Faraday's law : $\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt} \iint \mathbf{B} \cdot d\mathbf{A}$

Complete Maxwell's Equations

$$\oiint \mathbf{E} \cdot d\mathbf{A} = q_{\text{enclosed}} / \varepsilon_0$$

$$\oiint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{enclosed}} + \mu_0 \varepsilon_0 \frac{d}{dt} \iint \mathbf{E} \cdot d\mathbf{A}$$

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt} \iint \mathbf{B} \cdot d\mathbf{A}$$

Why?

- **Experimental evidence**
- **Mathematical consistency of the 4 Maxwell equations**
- **Symmetry between E and B (Einstein's theory of relativity)**

Online Quiz for Lecture 15
Magnetic materials -- Feb. 21, 2005

The constant $\mu_0\varepsilon_0$ appears in Maxwell's induction law.

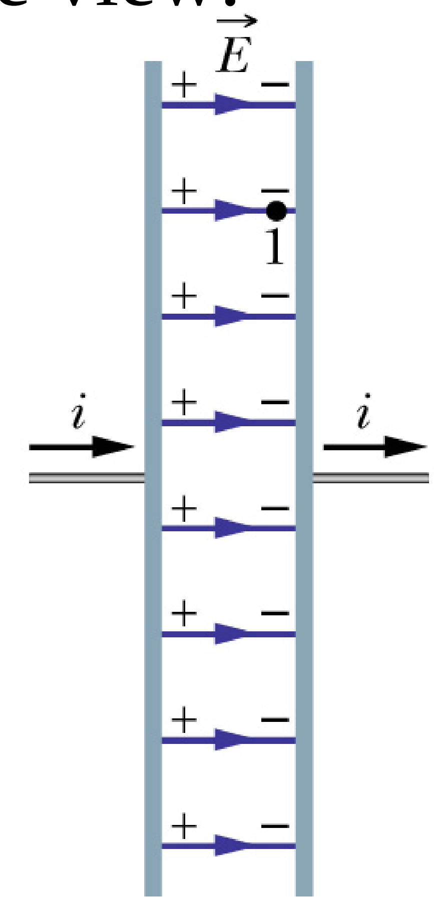
1. Plug in the values of the permeability and permittivity constants into this product and find the square root of the inverse. Compare the value you get with the values of the physical constants listed on the back jacket of your text.
2. What physical constant has this value?
3. Do the units make sense?

$$\mu_0\varepsilon_0 = 12.566\times10^{-7}\cdot 8.85\times10^{-12}\,\frac{s^2}{m^2} = \frac{1}{\left(3\times10^{8}\,m/s\right)^2}$$

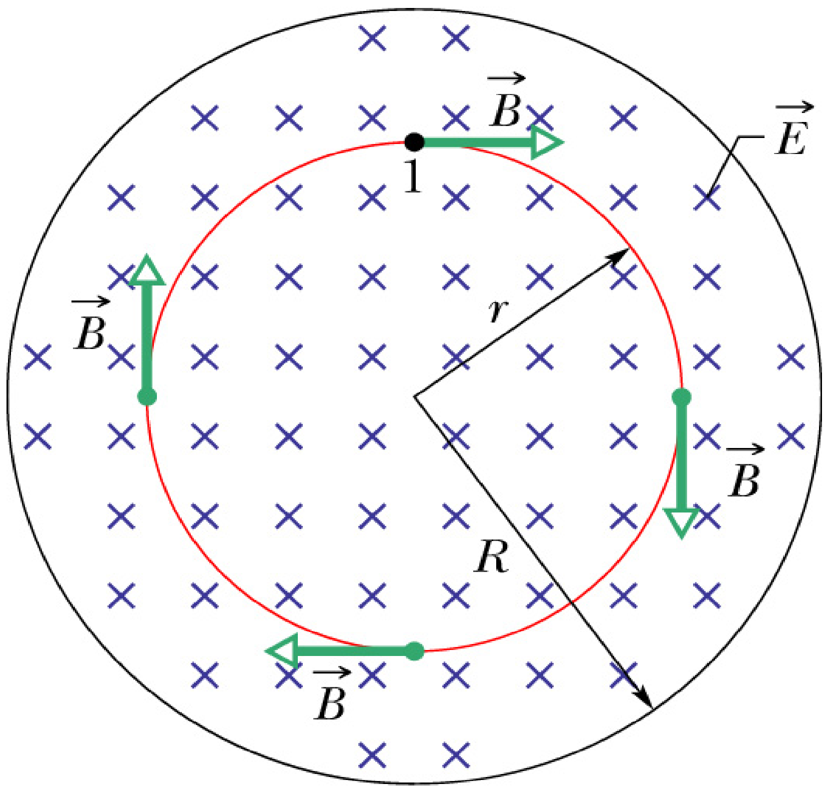
Example of time-varying electric flux between two capacitor plates:

$$\mathbf{E} = \frac{\sigma}{\varepsilon_0} = \frac{q(t)/A}{\varepsilon_0}$$

Side view:

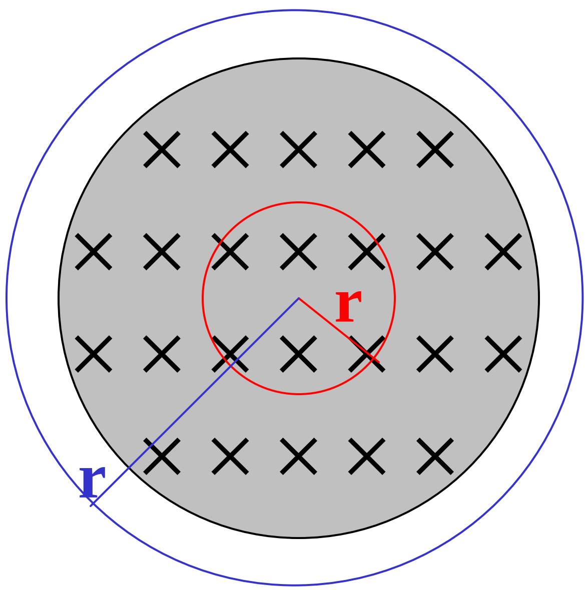


Top view:



View of electric field between two capacitor plates:

(**E** pointing into screen)



charge on plates

$$\mathbf{E}(t) = \frac{\sigma(t)}{\varepsilon_0} = \frac{q(t)/A_P}{\varepsilon_0}$$

area of plates

$$\frac{d\mathbf{E}(t)}{dt} = \frac{1}{A_P\varepsilon_0}\frac{dq(t)}{dt}$$

$$\frac{d\Phi_E(t)}{dt} = \iint \frac{d\mathbf{E}}{dt}\cdot d\mathbf{A}$$

Smaller circle:

$$\frac{d\Phi_E(t)}{dt} = \frac{1}{A_P\varepsilon_0}\frac{dq(t)}{dt}\pi r^2$$

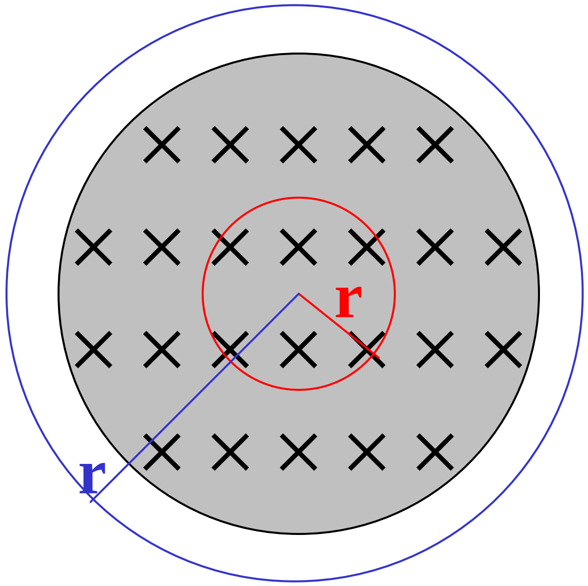
Larger circle:

$$\frac{d\Phi_E(t)}{dt} = \frac{1}{A_P\varepsilon_0}\frac{dq(t)}{dt}A_P = \frac{1}{\varepsilon_0}\frac{dq(t)}{dt}$$

Ampere-Maxwell equation:

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{enclosed}} + \mu_0 \varepsilon_0 \frac{d}{dt} \iint \mathbf{E} \cdot d\mathbf{A}$$

(0 in this case)



Smaller circle:

$$\frac{d\Phi_E(t)}{dt} = \frac{1}{A_P \varepsilon_0} \frac{dq(t)}{dt} \pi r^2$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = |\mathbf{B}| 2\pi r = \frac{\mu_0 \pi r^2}{A_P} \frac{dq(t)}{dt}$$

$$\Rightarrow |\mathbf{B}(t)| = \frac{\mu_0 r}{2A_P} \frac{dq(t)}{dt}$$

Ampere-Maxwell equation:

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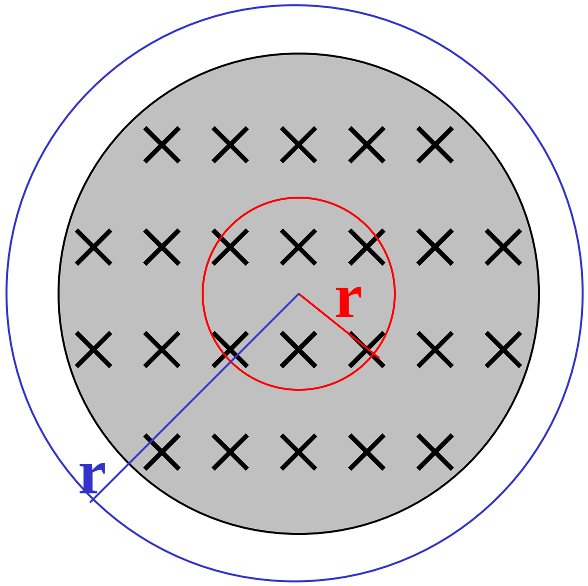


Larger circle:

$$\frac{d\Phi_E(t)}{dt} = \frac{1}{\varepsilon_0} \frac{dq(t)}{dt}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = |\mathbf{B}| 2\pi r = \mu_0 \frac{dq(t)}{dt}$$

$$\Rightarrow |\mathbf{B}(t)| = \frac{\mu_0}{2\pi r} \frac{dq(t)}{dt}$$

1. [HRW6 32.P.026.] A parallel-plate capacitor with circular plates of radius R = 55 mm is being charged as in Fig 32-15*a*. At what radius r is the induced magnetic field equal to 50% of its maximum value?

[ ]	mm (for r < R) or
[ ]	mm (for r > R)

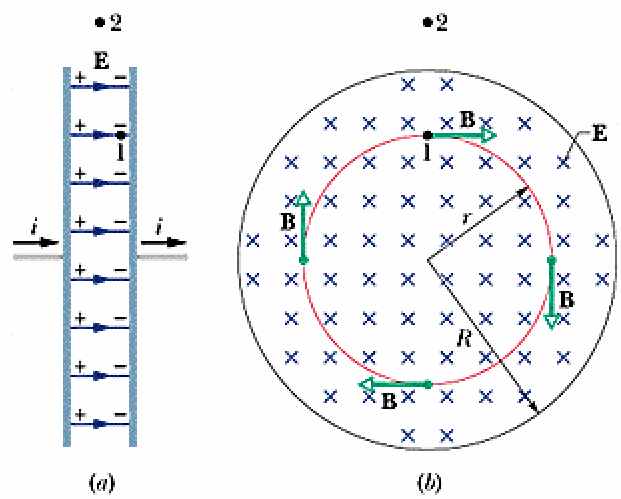


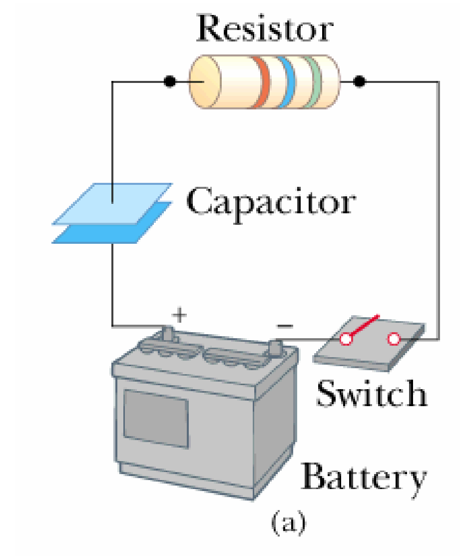
Figure 32-15.

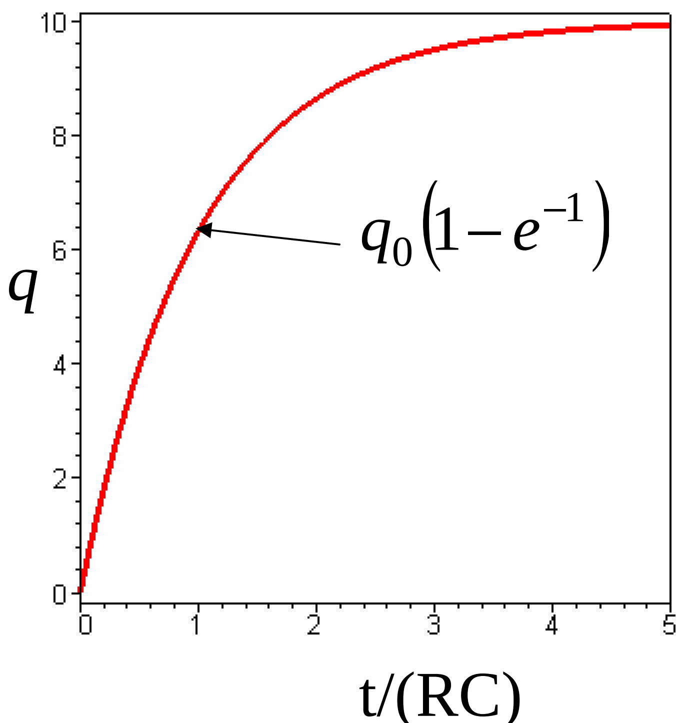
$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \varepsilon_0 \frac{d}{dt} \iint \mathbf{E} \cdot d\mathbf{A}$$

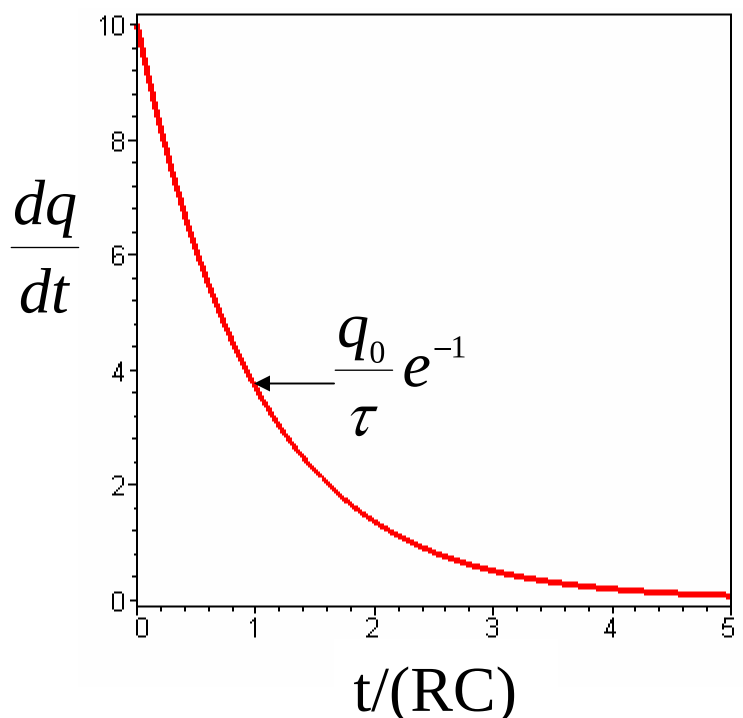
Charging capacitor plates :

$$q(t) = \mathcal{E}C(\,1\text{-}e^{-t/RC}\,)$$

$$\frac{dq}{dt} = \frac{\mathcal{E}}{R}e^{-t/RC}$$







1. [HRW6 32.P.026.] A parallel-plate capacitor with circular plates of radius R = 55 mm is being charged as in Fig 32-15*a*. At what radius r is the induced magnetic field equal to 50% of its maximum value?

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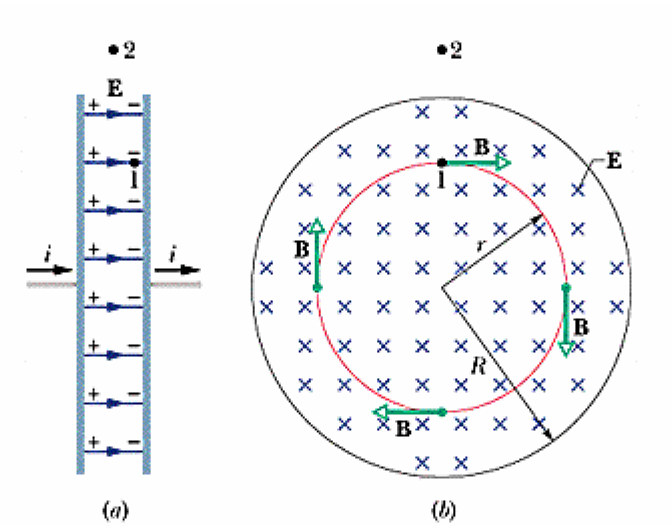
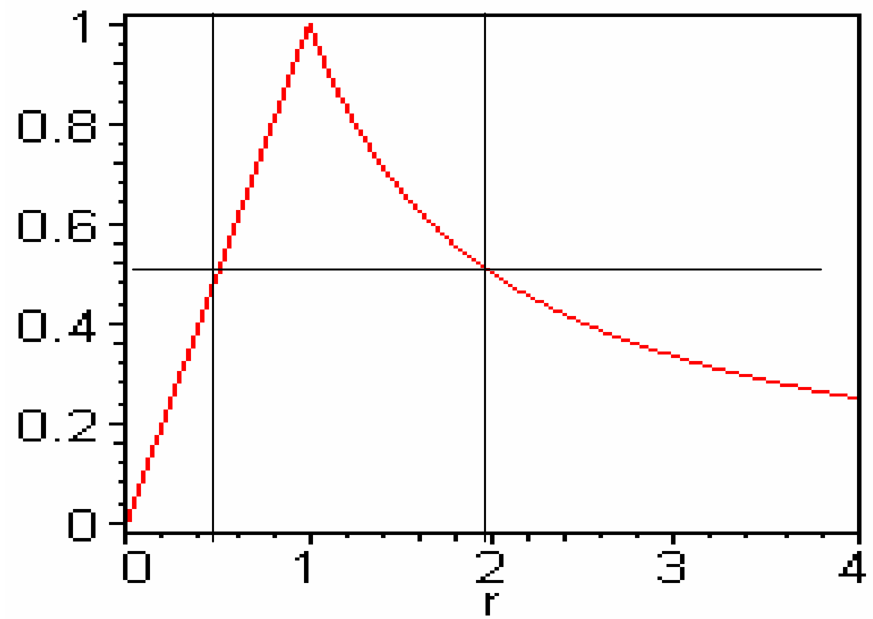


Figure 32-15.



2. [HRW6 32.P.035.] A uniform electric field collapses to zero from an initial strength of 6.0×10^5 N/C in a time of 15 μs in the manner shown in Fig. 32-34. Calculate the magnitude of the displacement current, through a 2.3 m^2 region perpendicular to the field, during each of the time intervals *a*, *b*, and *c* shown on the graph. (Ignore the behavior at the ends of the intervals.)

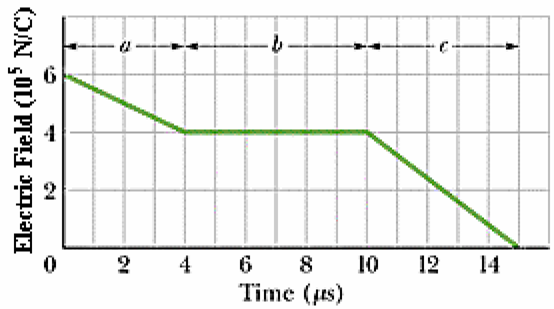


Figure 32-34.

Interval *a*: A

Interval *b*: A

Interval *c*: A

$$i_D \equiv \varepsilon_0 \frac{d\Phi_E}{dt}$$

3. [HRW6 32.P.041.] The capacitor in Fig. 32-36 with circular plates of radius R = 18.9 cm is connected to a source of emf $\mathcal{E} = \mathcal{E}_m \sin \omega t$, where $\mathcal{E}_m$ = 200 V and ω = 130 rad/s. The maximum value of the displacement current is i_d = 7.65 μA. Neglect fringing of the electric field at the edges of the plates.

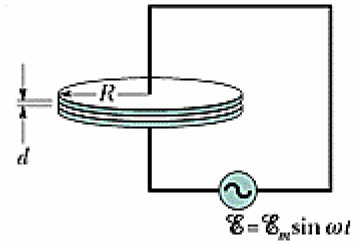


Figure 32-36

(a) What is the maximum value of the current i?

[ ] μA

(b) What is the maximum value of $d\Phi_E/dt$, where Φ_E is the electric flux through the region between the plates?

[ ] V · m/s

(c) What is the separation d between the plates?

[ ] m

(d) Find the maximum value of the magnitude of **B** between the plates at a distance r = 10.4 cm from the center.

[ ] T

4. [HRW6 32.PN.11.] The circuit in Fig. 32N-5 consists of switch S, a 12.0 V ideal battery, a 21.0 MΩ resistor, and an air-filled capacitor. The capacitor has parallel circular plates of radius 5.10 cm, separated by 3.00 mm. At time $t = 0$, switch S is closed to begin charging the capacitor. The electric field between the plates is uniform. At $t = 250$ μs, what is the magnitude of the magnetic field within the capacitor, at radial distance 3.00 cm?

[ ] T

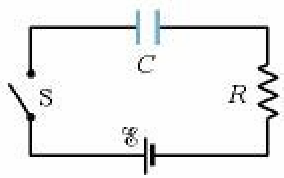


Fig. 32N-5