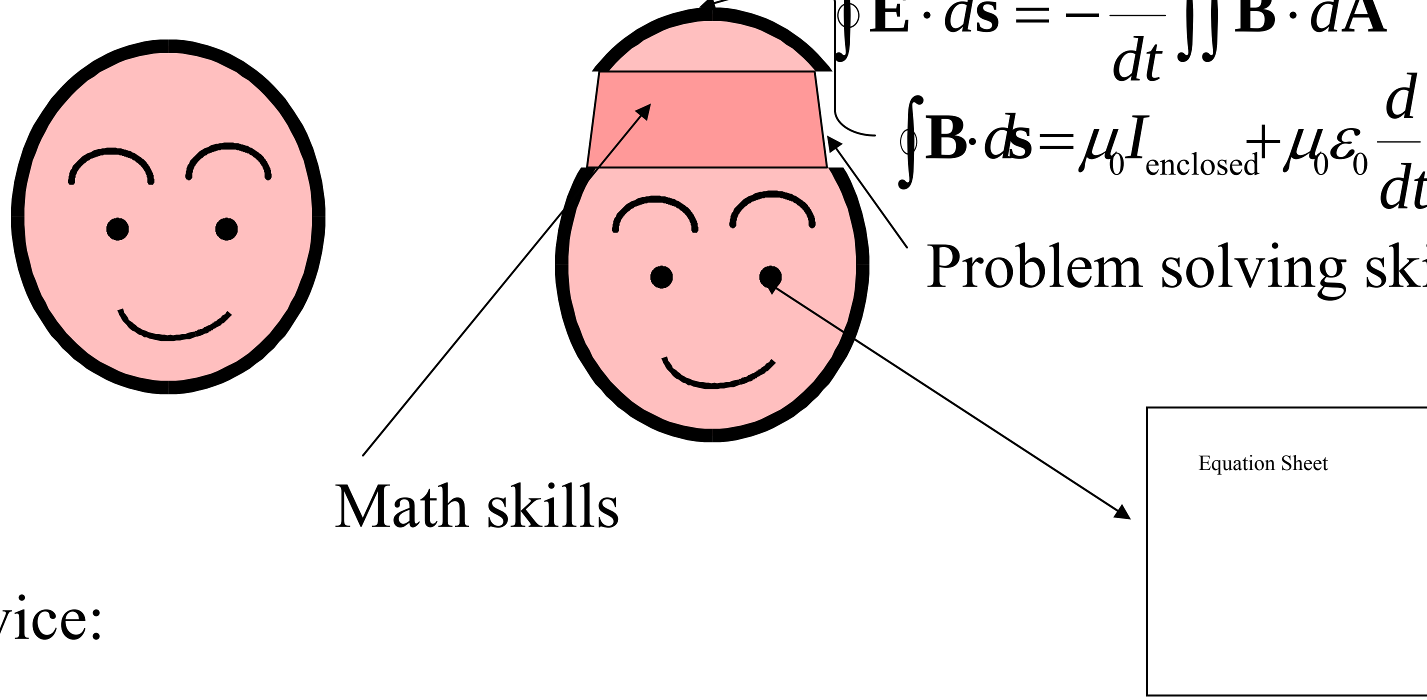
Announcements

1. **Second hour exam – Friday, February 25, 2005 – covering Chapters 28.8-32.**
   - 4 problems – show your work and reasoning for possible partial credit.
   - May bring one 8½" x 11" sheet of paper to the exam (to be turned in with your exam papers).
   - Details
2. **Problem solving session tonight at 6 PM.**
3. Example exam available on website.
4. Today's lecture –
   Advice for studying   Systematic review

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$$

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt} \iint \mathbf{B} \cdot d\mathbf{A}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{enclosed}} + \mu_0 \varepsilon_0 \frac{d}{dt} \iint \mathbf{E} \cdot d\mathbf{A}$$



Advice:

1. Keep basic concepts and equations at the top of your head.
2. Practice problem solving and math skills
3. Develop an equation sheet that you can consult.

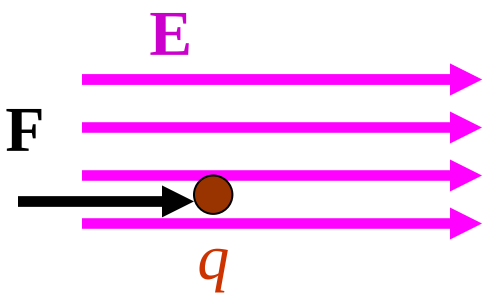
Problem solving steps

1. Visualize problem – labeling variables
2. Determine which basic physical principle(s) apply
3. Write down the appropriate equations using the variables defined in step 1.
4. Check whether you have the correct amount of information to solve the problem (same number of knowns and unknowns).
5. Solve the equations.
6. Check whether your answer makes sense (units, order of magnitude, etc.).

Force, torque, and energy due to magnetic fields

1. Magnetic field exerts a force on charged moving particles
2. Magnetic field exerts a torque on current loops and/or magnetic dipoles

Electric field

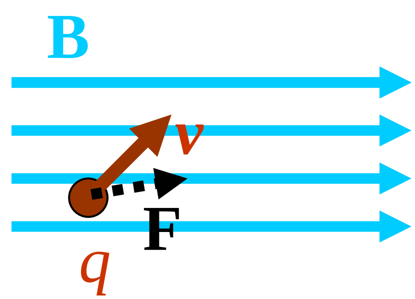


$F = q\mathbf{E}$

(force direction is determined by direction of **E**)

units: N = C · N/C

Magnetic field

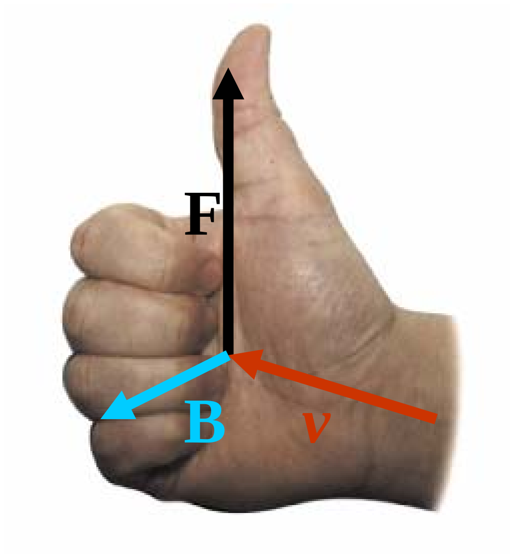


$F = q\,\boldsymbol{v} \times \mathbf{B}$

(force direction is determined by vector cross product of ***v*** and **B**)

units: N = C · m/s · Tesla

Right hand rule:



$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

$$|\mathbf{F}| = |q||\mathbf{v}||\mathbf{B}|\sin\theta$$

$F = q\,v \times \mathbf{B}$

In plane containing v and $\mathbf{B}$:

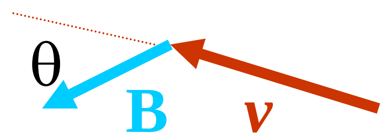
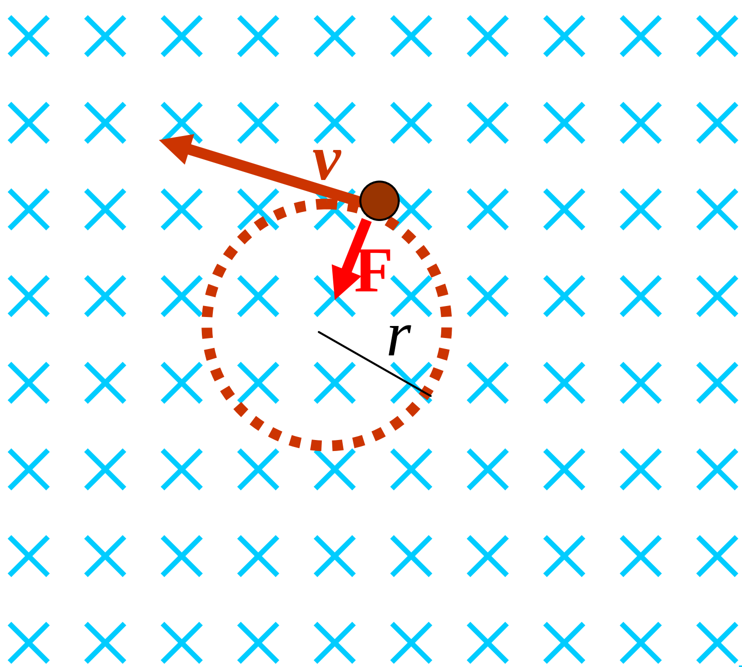


Diagram of positive particle (charge q mass m) moving perpendicular to magnetic field.

If there is no component of the velocity vector $\mathbf{v}$ of the particle in the direction of the magnetic field, then the particle moves in a circular orbit:



B (into screen)

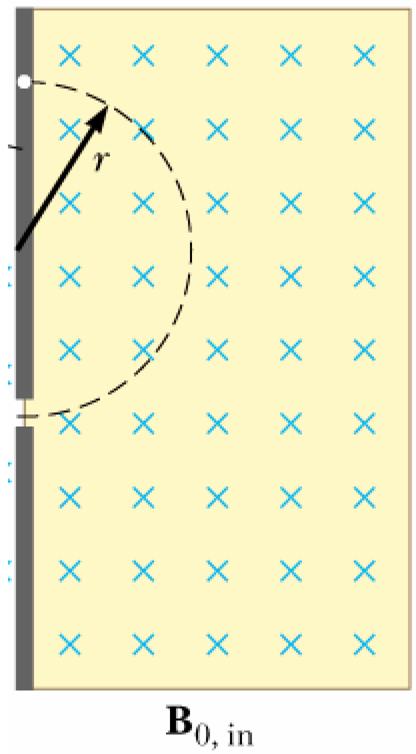
$$F = qvB = ma_r = m\frac{v^2}{r}$$

$$r = \frac{mv}{qB}$$

centripetal acceleration

Principles of the mass spectrometer:

radius r is measured



$\boldsymbol{v}$

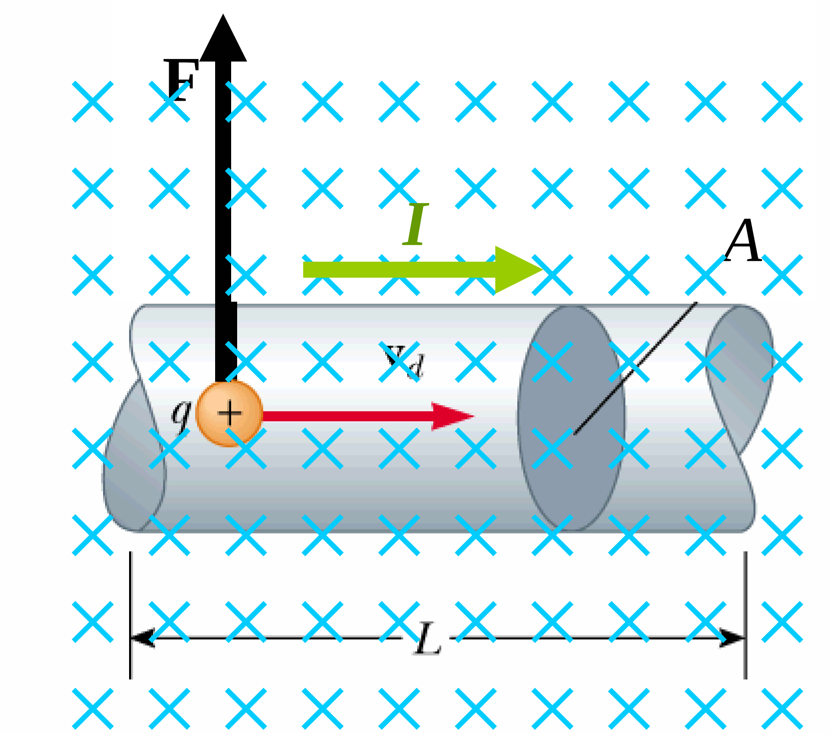
q,m

velocity $\boldsymbol{v}$ is known

field B is known

$$r = \frac{mv}{qB} \quad \Rightarrow \frac{q}{m} = \frac{v}{Br}$$

Magnetic forces acting on moving charges in a wire



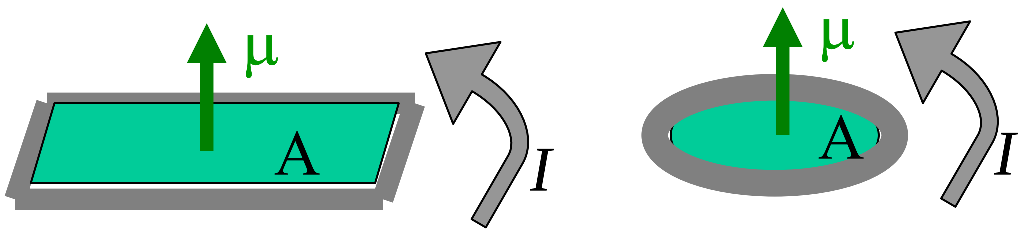
$$I = nqv_d A \qquad n = \frac{N}{AL}$$

$$I = \frac{Nqv_d}{L} \qquad \Rightarrow Nqv_d = IL$$

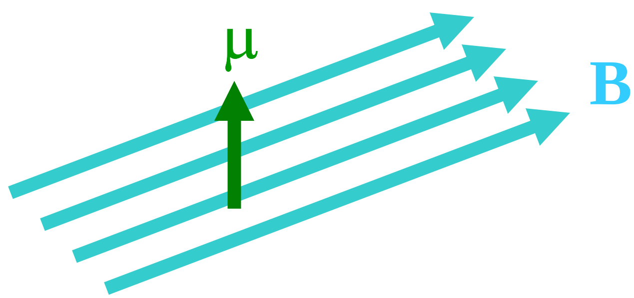
$$\mathbf{F} = L\mathbf{I} \times \mathbf{B}$$

B (into screen)

Magnetic moment associated with current loop:



$\mu = NI\,\mathbf{A}$



Torque:

$\tau = \mu \times \mathbf{B}$

Potential energy:

$U = -\,\mu \cdot \mathbf{B}$

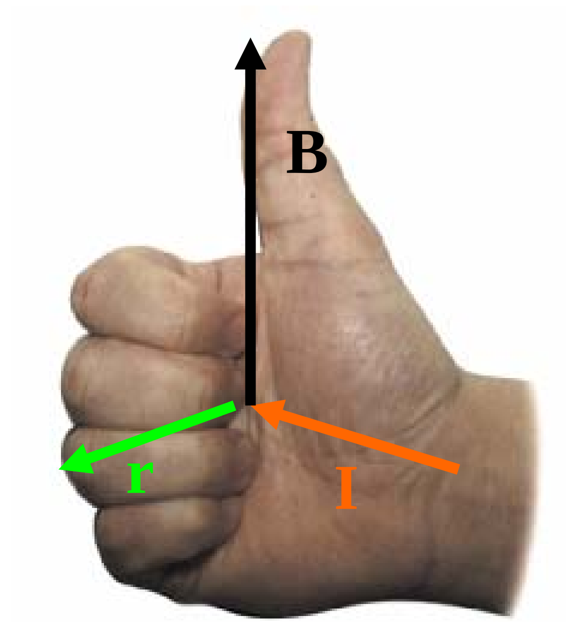
Sources of magnetic field

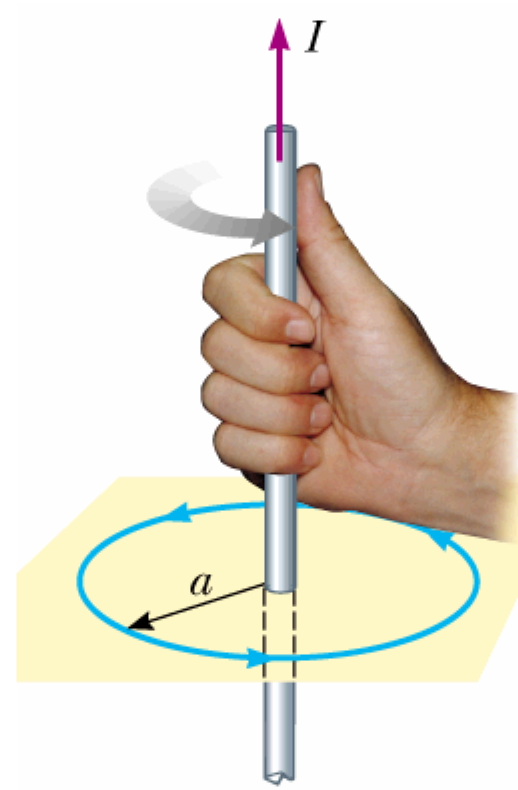
1. **Currents**
2. **Electric flux which varies in time**

$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 I_{\text{enclosed}} + \mu_0 \varepsilon_0 \frac{d}{dt}\iint \mathbf{E}\cdot d\mathbf{A}$$

Digression on the right-hand rule:

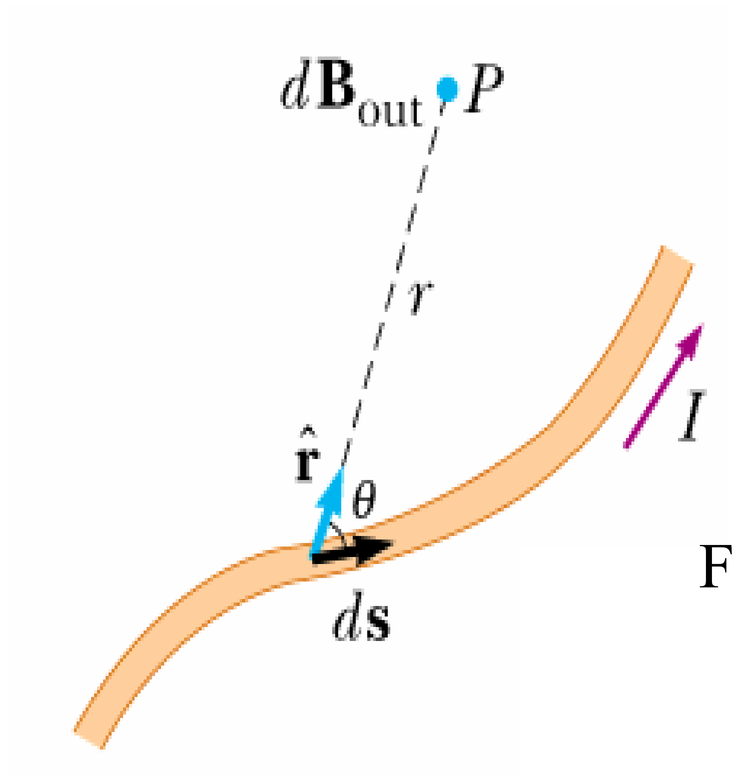
$\mathbf{B}$	$\rightarrow$ $\mathbf{I}$	$\times$ $\mathbf{r}$
thumb	palm	fingers
palm	fingers	thumb
fingers	thumb	palm





Sources of magnetic field – currents

Biot-Savart law



$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I d\mathbf{s}\times\hat{\mathbf{r}}}{r^2}$$

Field from a single moving charge:

$$\mathbf{B} \approx \frac{\mu_0}{4\pi}\frac{q\mathbf{v}\times\hat{\mathbf{r}}}{r^2}$$

μ_0 = Permeability constant

$= 4\pi \times 10^{-7}$ Tm^2/A (or H/m)

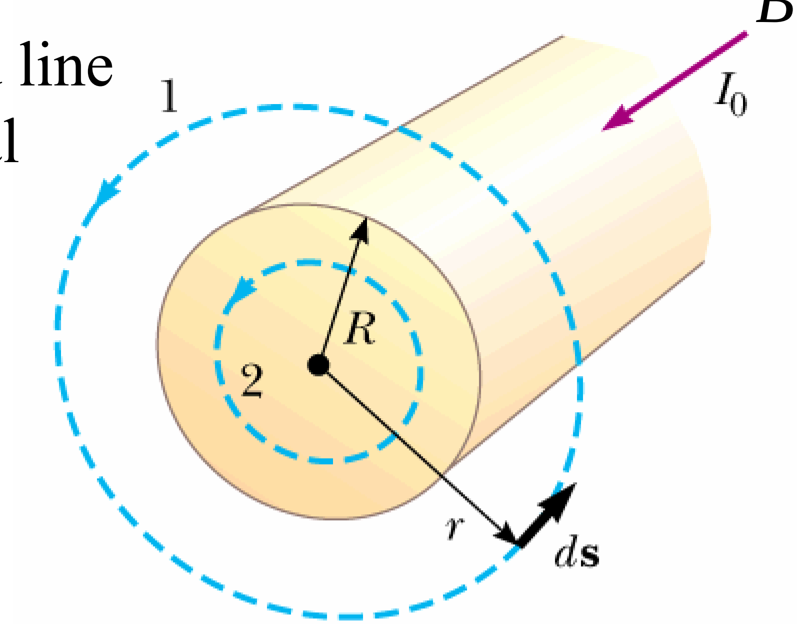
Ampere's law (Gauss's law for currents $\frac{dE}{dt} = 0$)

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{in}$$

Closed line integral

1. For $r > R$:

$$B(2\pi r) = \mu_0 I_0 \quad \Rightarrow \quad B = \frac{\mu_0 I_0}{2\pi r}$$



2. For $r < R$:

$$B(2\pi r) = \mu_0 I_0 \frac{r^2}{R^2} \quad \Rightarrow \quad B = \frac{\mu_0 I_0 r}{2\pi R^2}$$

Summary of magnetic fields from currents: $\left(\frac{dE}{dt}=0\right)$

Ampere's law: $\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 I_{in}$

Closed line integral

Field from a single long wire having current *I*:

$$\boxed{\mathbf{B}=\frac{\mu_0 I}{2\pi r}}$$

➔ **at distance r from wire**

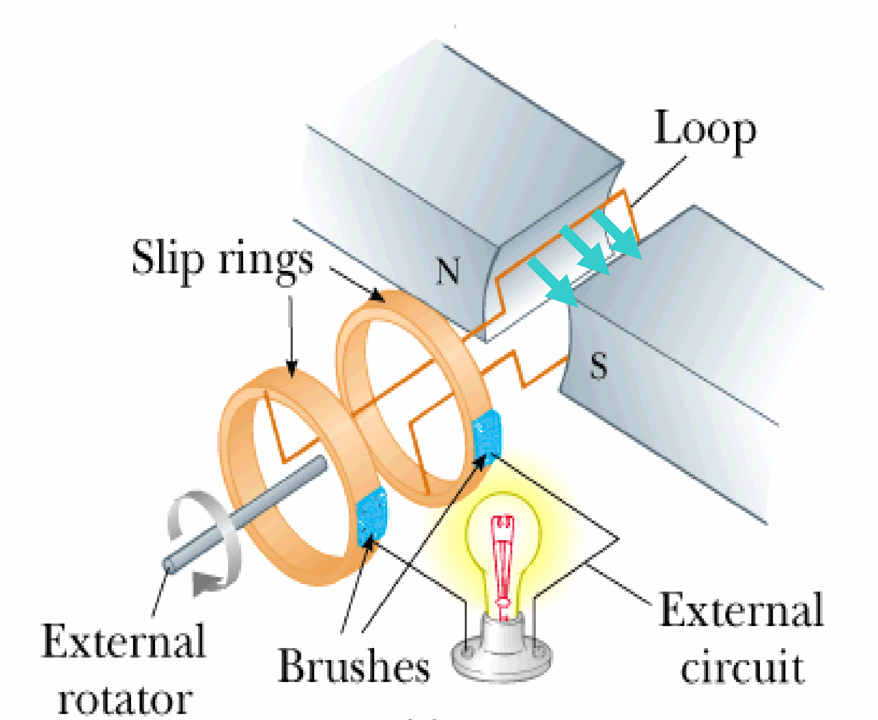
direction tangent to circle

Field inside a long solenoid having current *I* and loops/length *n*: $\boxed{\mathbf{B}=\mu_0 n I}$

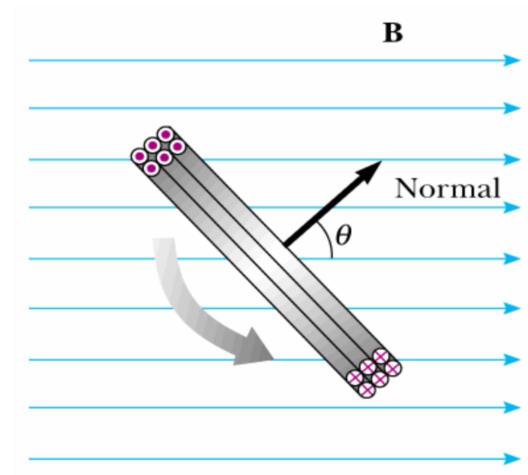
Faraday's law ➔ an electric field can be produced a magnetic field which changes as a function of time in a wire and also in a free space.

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt}\int \mathbf{B} \cdot d\mathbf{A}$$

Example: AC generator



(a)

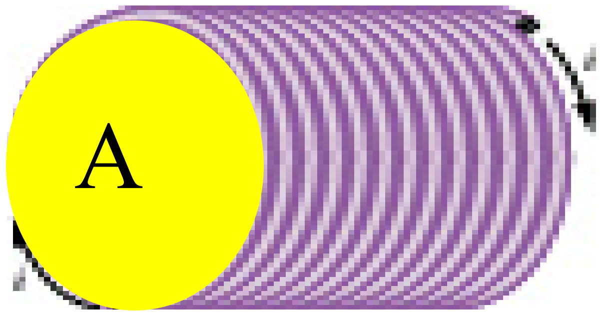


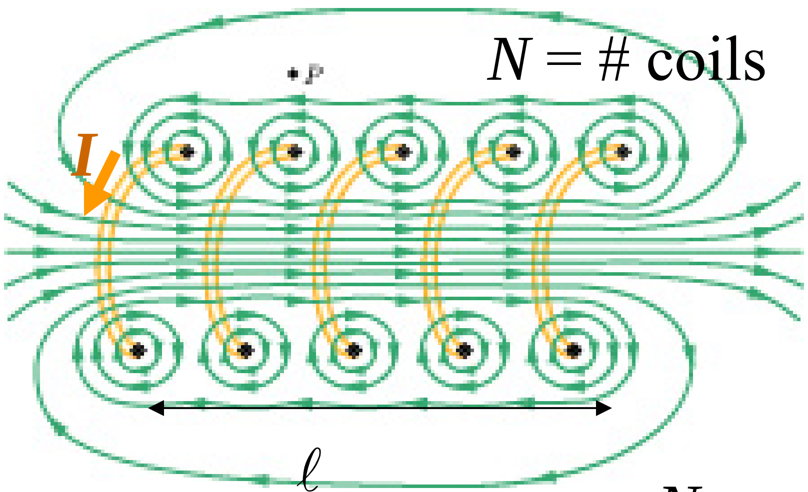
$$\frac{d}{dt}\int \mathbf{B}\cdot d\mathbf{A} = \frac{d}{dt}\left(BA\cos\omega t\right)$$

$$= -\omega BA\sin\omega t$$

$$\Rightarrow \mathcal{E} = \underbrace{\omega BA}_{\mathcal{E}_{\max}}\sin\omega t$$

Faraday's law in a wire in the solenoid geometry





$$B = \mu_0 \frac{N}{\ell} I$$

$$\int \mathbf{B} \cdot d\mathbf{A} = \mu_0 \frac{N}{\ell} I(NA)$$

$$\mathcal{E} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{A} = -\mu_0 \frac{N^2 A}{\ell} \frac{dI}{dt}$$

➔ In this geometry, a coil can induce an emf in itself!

Faraday's law in a solenoid:

$$\mathcal{E} = -\underbrace{\mu_0 \frac{N^2 A}{\ell}}_{\equiv L \text{ "inductance"}} \frac{dI}{dt}$$

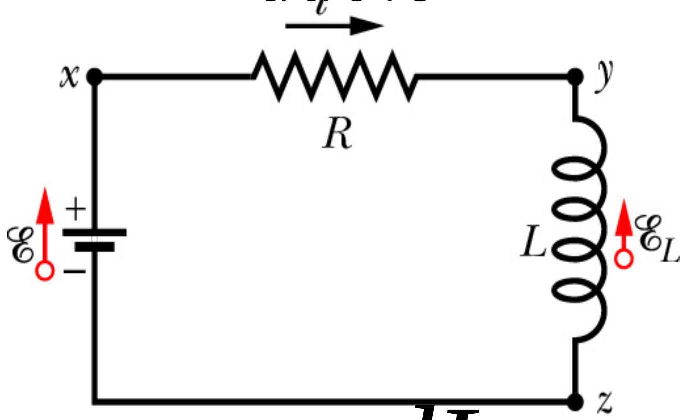
1 henry ≡ Volt · s/A = T · m^2/A

Time-varying circuits

- So far, we have studied RC and LR circuits
- Later, we will study LC and RLC circuits.

Solution of differential equations:

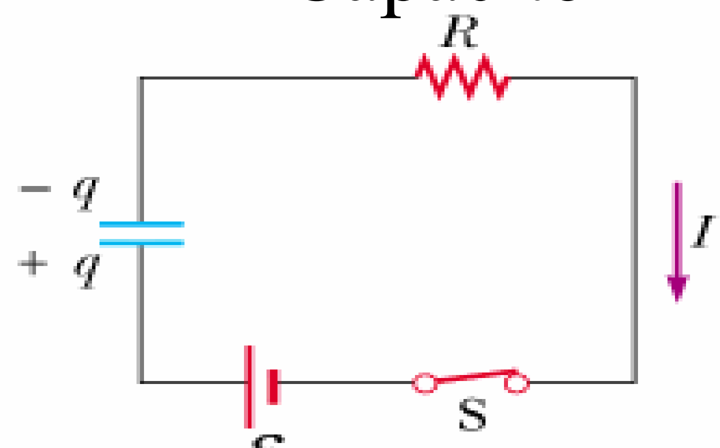
Inductor



$$\mathcal{E} - IR - \frac{dI}{dt}L = 0$$

$$I(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/(L/R)}\right) + Ke^{-t/(L/R)}$$

Capacitor



$$\mathcal{E} - \frac{q}{C} - \frac{dq}{dt}R = 0$$

$$q(t) = C\mathcal{E}\left(1 - e^{-t/(RC)}\right) + Ke^{-t/(RC)}$$

Some differentials and integrals

$$\frac{de^{ax}}{dx} = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}$$

$$\frac{d\sin(ax)}{dx} = a\cos(ax)$$

$$\int \sin(ax) dx = -\frac{1}{a}\cos(ax)$$

Complete Maxwell's Equations

$$\oiint \mathbf{E} \cdot d\mathbf{A} = q_{\text{enclosed}} / \varepsilon_0$$

$$\oiint \mathbf{B} \cdot d\mathbf{A} = 0$$

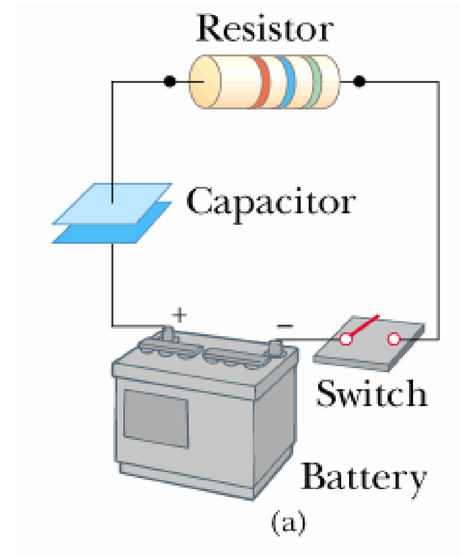
$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{enclosed}} + \mu_0 \varepsilon_0 \frac{d}{dt} \iint \mathbf{E} \cdot d\mathbf{A}$$

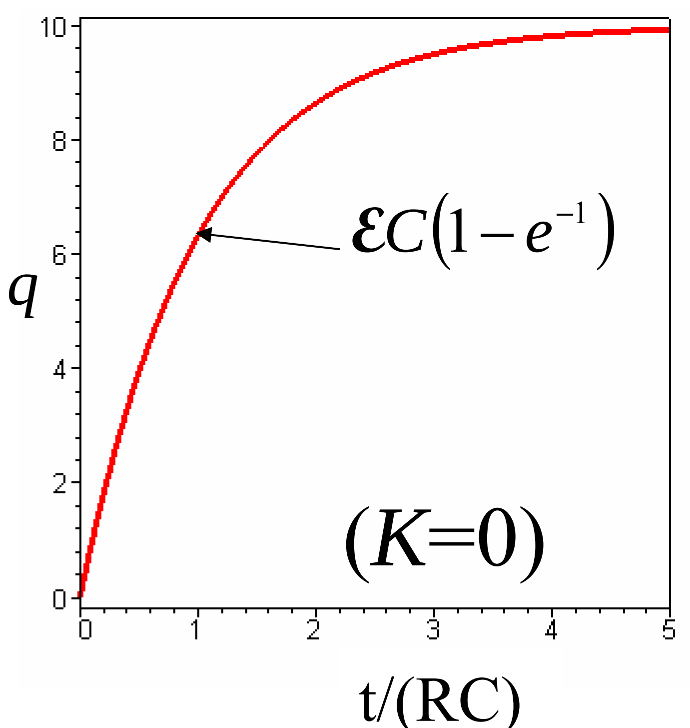
$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt} \iint \mathbf{B} \cdot d\mathbf{A}$$

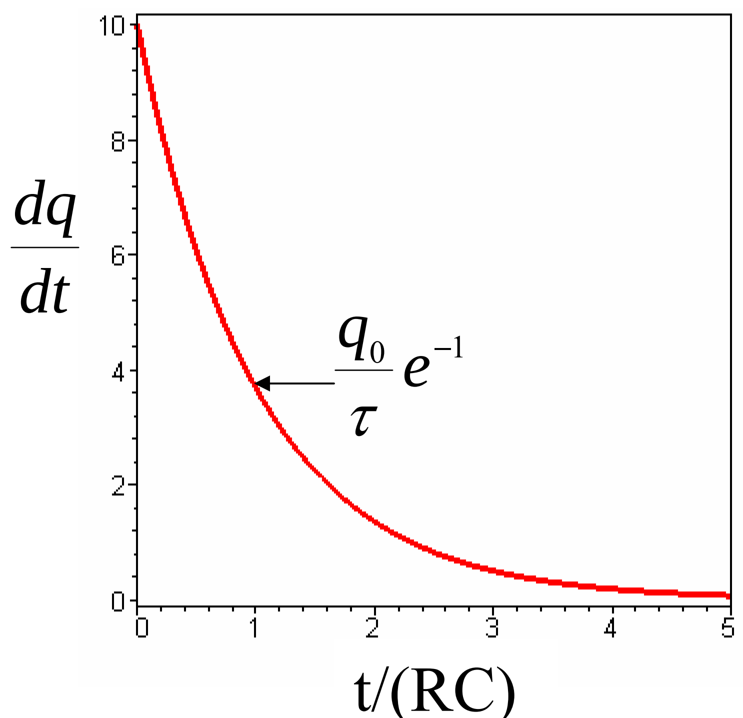
Example:
Charging capacitor plates :

$$q(t) = \mathcal{E}C(1-e^{-t/RC}) + Ke^{-t/RC}$$

$$\frac{dq}{dt} = \left(\frac{\mathcal{E}}{R} - \frac{K}{RC}\right)e^{-t/RC}$$



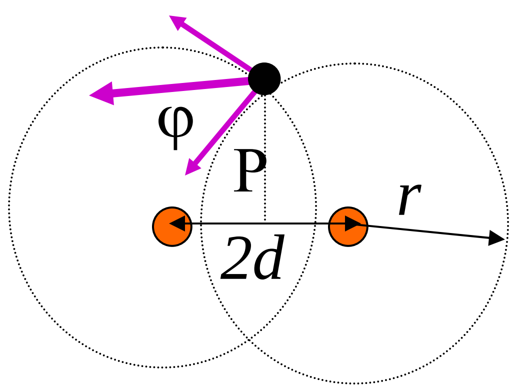




Example of Ampere's law for two wires:

Suppose that there are two wires perpendicular to the screen both with currents *I* flowing out of the screen. What is the magnitude and direction of the magnetic field at the point P?

$$B = \frac{\mu_0 I}{2\pi r} 2\cos\varphi = \frac{2\mu_0 I}{2\pi\sqrt{d^2+P^2}} \frac{d}{\sqrt{d^2+P^2}} = \frac{\mu_0 I d}{\pi\left(d^2+P^2\right)}$$



Note: The two wires exert a force on eachother. In this case, the force is attractive and, in terms of the length *l* of the wires, has the magnitude:

$$\frac{F_B}{l} = \frac{\mu_0 I^2}{4\pi d}$$