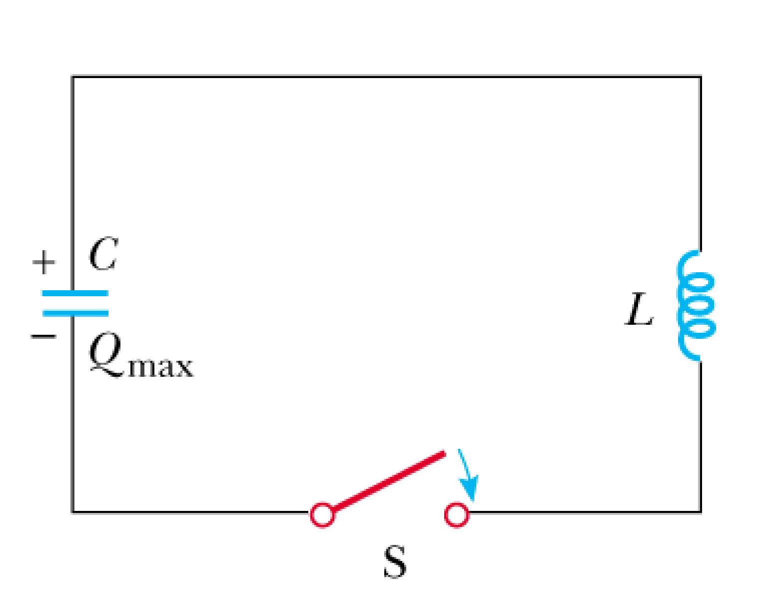
Announcements

1. Second exams will be returned at the end of class. Mid-term grades = ½(Exam#1+Exam#2) (hopefully an underestimate).
2. Assignments 16-21 now posted on WebAssign
   - This week: LCR & AC circuits (Chapter 33)
   - Next week: EM waves (Chapter 34) (after Spring Break)
3. Today's topics
   LC circuits
   Effects of an AC emf

LC – circuits:



$$-\frac{q}{C} - L\frac{dI}{dt} = 0$$

$$\text{or:} \quad -\frac{q}{C} - L\frac{d^2q}{dt^2} = 0$$

Peer instruction question

The LC circuit equation can be written in the form:

$$\frac{d^2q}{dt^2} = -\frac{1}{LC}q$$

What does this remind of?

(A) A bad dream.

(B) A point mass moving in response to a constant force.

(C) A mass on a spring.

(D) A charged mass moving in a magnetic field.

Solutions to the LC circuit equations

Recall that:

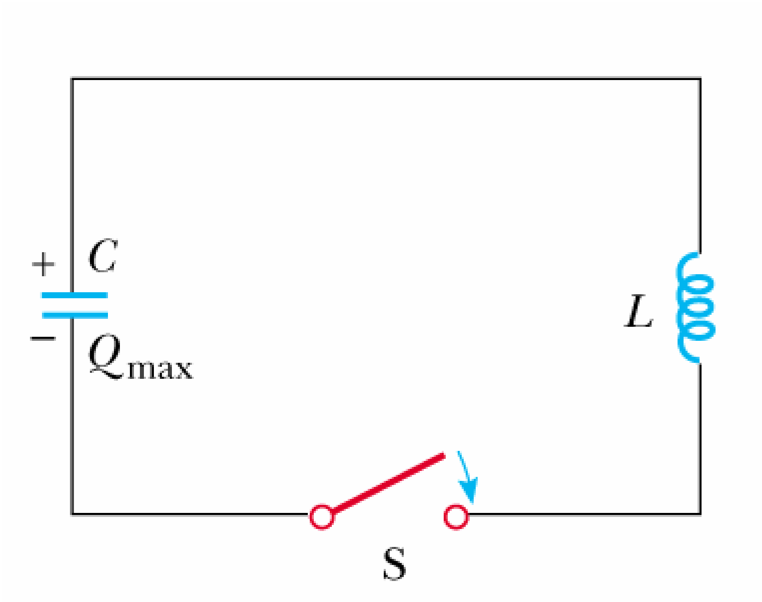
$$\frac{d^2x}{dt^2} = -\frac{k}{m}x \qquad \Rightarrow \; x(t) = X_0 \cos(\omega_0 t + \varphi) \;\text{ with }\; \omega_0 \equiv \sqrt{\frac{k}{m}}$$

Therefore:

$$\frac{d^2q}{dt^2} = -\frac{1}{LC}q \qquad \Rightarrow \; q(t) = Q_0 \cos(\omega_0 t + \varphi) \;\text{ with }\; \omega_0 \equiv \sqrt{\frac{1}{LC}}$$

$$I(t) = \frac{dq}{dt} = -\omega Q_0 \sin(\omega_0 t + \varphi)$$

LC – circuit:



Mathematical solution :

$$q(t) = Q_0 \cos(\omega_0 t + \varphi) \quad \text{with} \quad \omega_0 \equiv \sqrt{\frac{1}{LC}}$$

Physical constants

Differential equation :

$$-\frac{q}{C} - L\frac{dI}{dt} = 0$$

$$\text{or :} \quad -\frac{q}{C} - L\frac{d^2 q}{dt^2} = 0$$

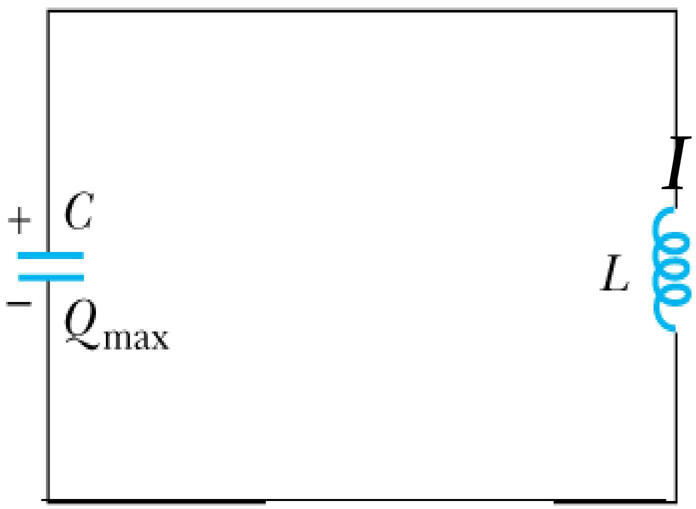
Current :

$$I(t) = \frac{dq}{dt} = -\omega Q_0 \sin(\omega_0 t + \varphi)$$

1. [HRW6 33.P.002.] In an oscillating *LC* circuit *L* = 1.00 mH and *C* = 4.00 μF. The maximum charge on the capacitor is 3.00 μC. Find the maximum current.

[ ] A

$$q(t) = Q_{\max}\cos(\omega_0 t + \varphi) \;\; \text{with} \;\; \omega_0 \equiv \sqrt{\frac{1}{LC}}$$



$$I(t) = \frac{dq}{dt} = -\omega_0 Q_{\max}\sin(\omega_0 t + \varphi)$$

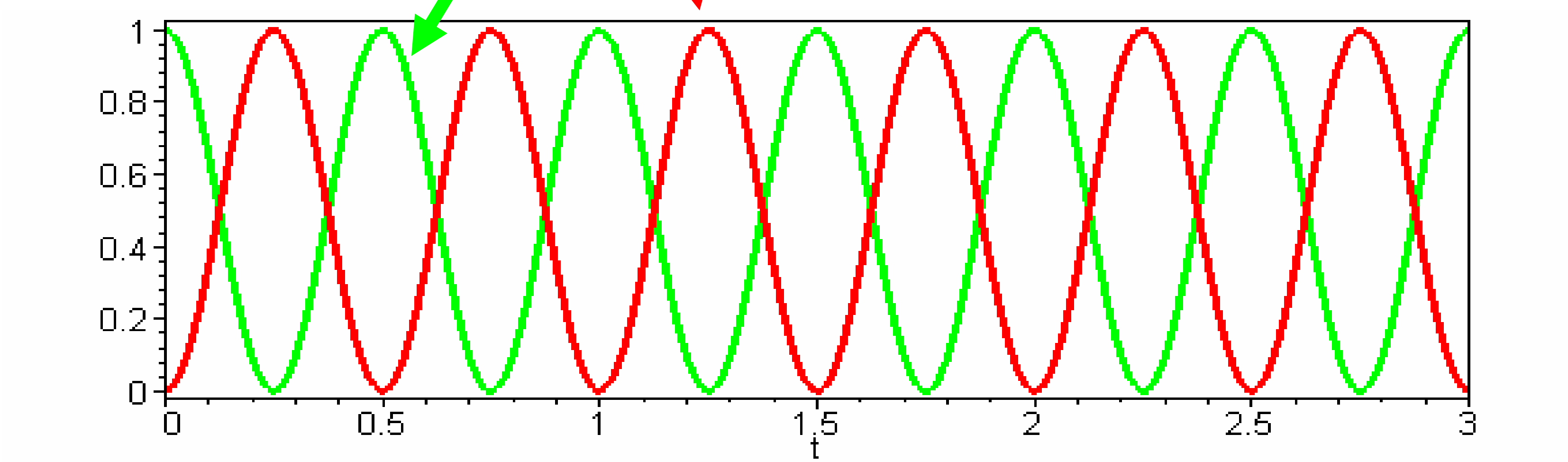
$$\text{Note: } I_{\max} = \omega_0 \, Q_{\max}$$

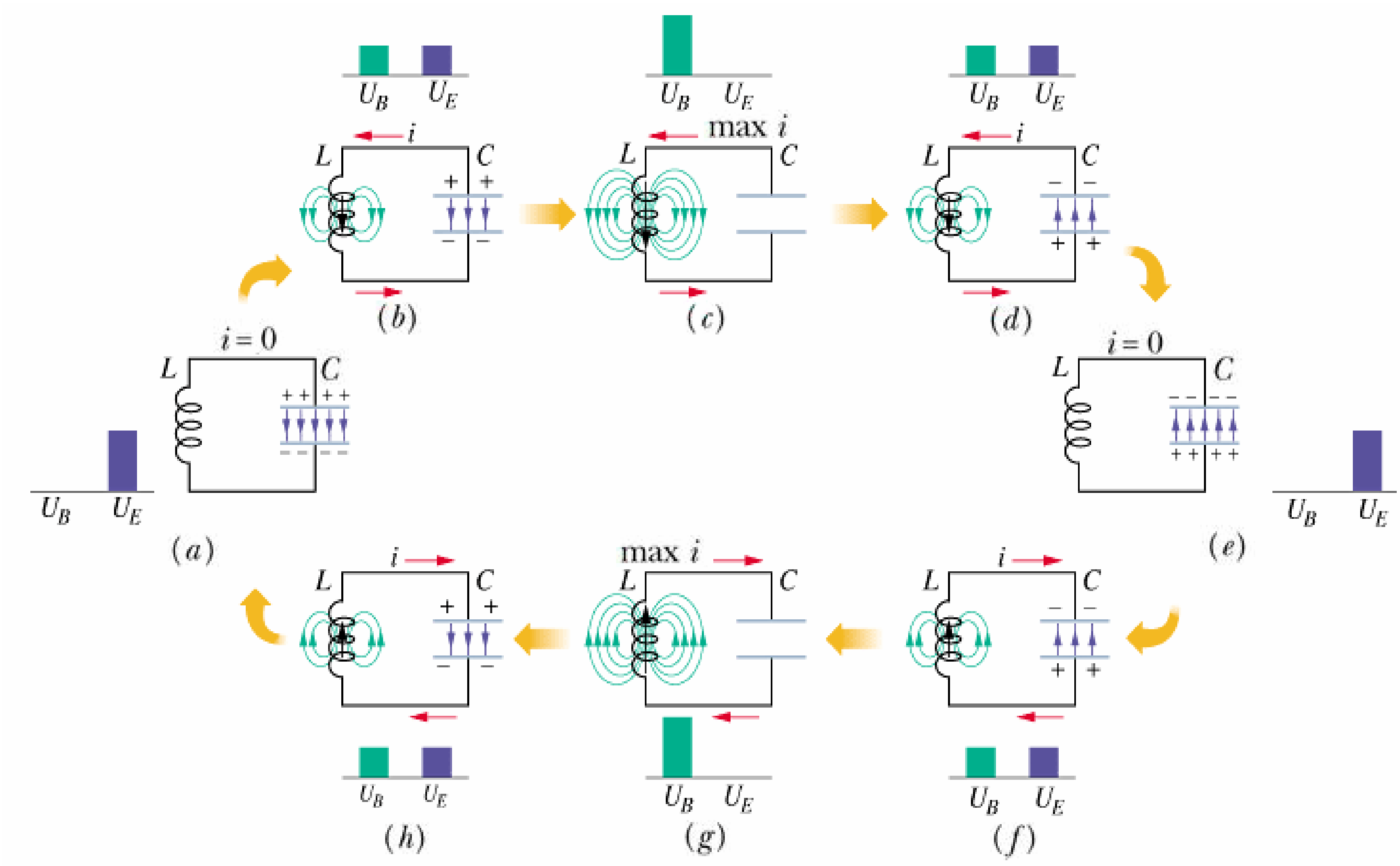
$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{1\times10^{-3}\cdot 4\times10^{-6}}}\,\text{rad/s}$$

Energy in LC circuits

Energy in capacitor : $U_E = \dfrac{q^2}{2C} = \dfrac{Q_{\max}^2 \cos^2(\omega t + \varphi)}{2C}$

Energy in inductor : $U_B = \dfrac{Li^2}{2} = \dfrac{\omega^2 L Q_{\max}^2 \sin^2(\omega t + \varphi)}{2}$





U_B U_E
i
L C
(b)
max i
(c)
(d)
$i = 0$
(a)
(e)
(h)
(g)
(f)

Total energy in LC circuit:

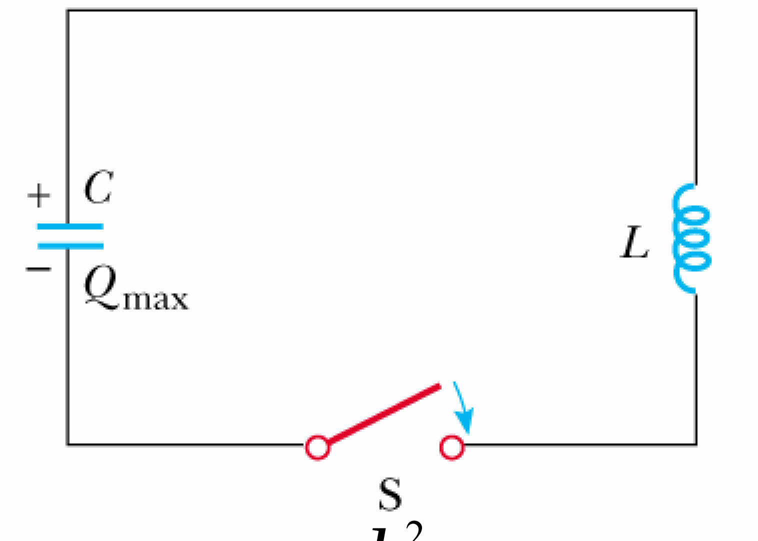
$$U_E + U_B = \frac{q^2}{2C} + \frac{Li^2}{2}$$

$$= \frac{Q_{\max}^2}{2C}\left(\cos^2(\omega_0 t + \varphi) + \sin^2(\omega_0 t + \varphi)\right)$$

$$= \frac{Q_{\max}^2}{2C} = \frac{LI_{\max}^2}{2}$$

$$\text{Note:} \quad \omega_0{}^2 L = \frac{1}{LC} L = \frac{1}{C}$$

Summary – Oscillator circuit



Resonant frequency:

$$\omega_0 = \sqrt{\frac{1}{LC}} = 2\pi f_0 = \frac{2\pi}{T_0}$$

$$-\frac{q}{C} - L\frac{d^2q}{dt^2} = 0$$

$$q(t) = Q_{max}\cos(\omega_0 t + \varphi)$$

$$i(t) = \frac{dq}{dt} = -\omega_0 Q_{max}\sin(\omega_0 t + \varphi)$$

Example: Suppose that you have an inductor L = 0.001 H, what capacitance do you need to tune your oscillator to the following frequencies:

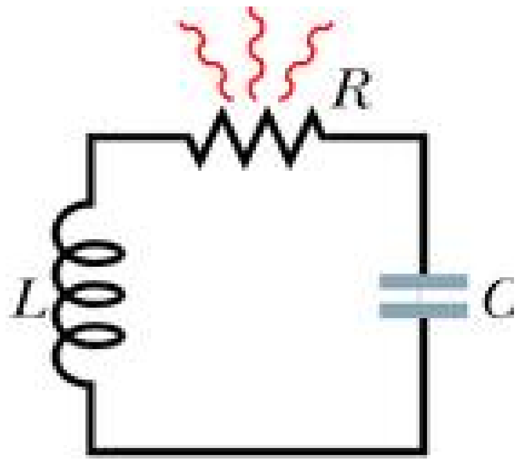
$\boldsymbol{f_0}$ **= 88.9 MHz**

$$C = \frac{1}{L(2\pi f)^2} = \frac{1}{0.001 \cdot \left(2\pi \cdot 88.9 \times 10^6\right)^2} = 3.2 \times 10^{-14} F$$

$\boldsymbol{f_0}$ **= 800 KHz**

$$C = \frac{1}{L(2\pi f)^2} = \frac{1}{0.001 \cdot \left(2\pi \cdot 800 \times 10^3\right)^2} = 3.96 \times 10^{-11} F$$

Effects of resistance in the circuit



$$-\frac{q}{C}-L\frac{di}{dt}-Ri=0$$

$$-\frac{1}{C}q-L\frac{d^2q}{dt^2}-R\frac{dq}{dt}=0$$

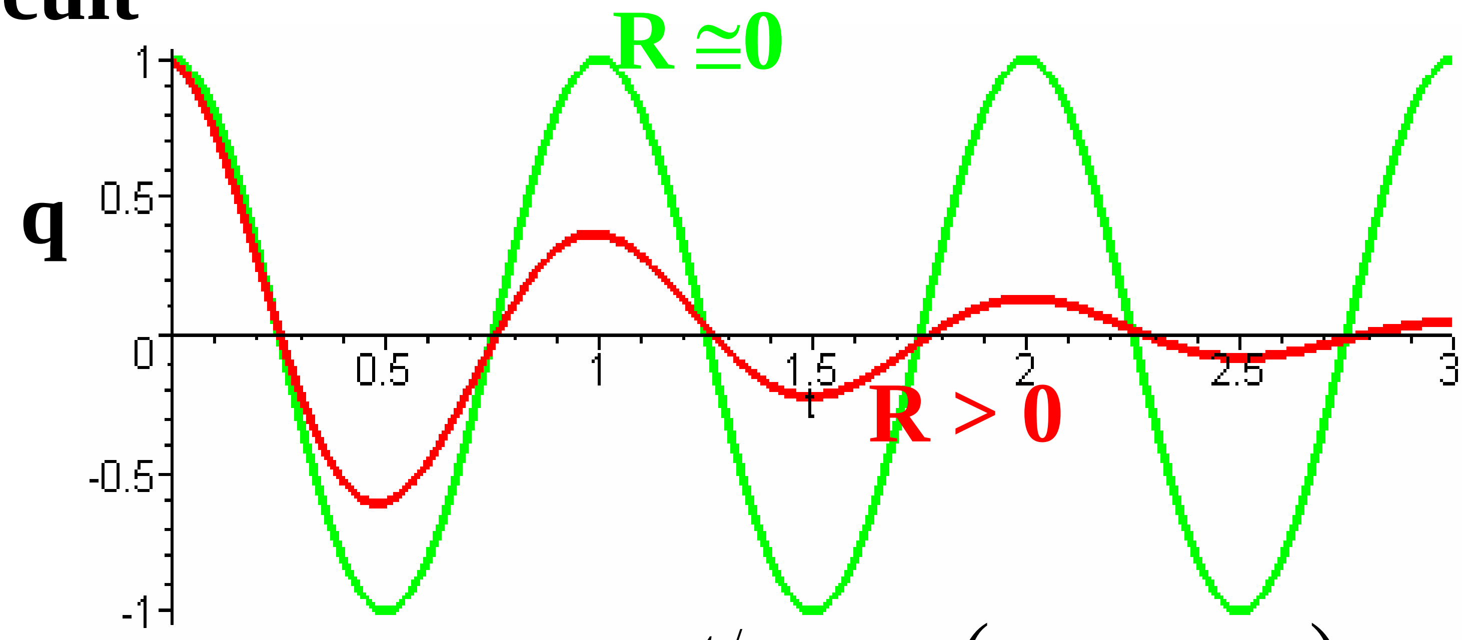
Solution :

$$q(t)=Q_{\max}e^{-t/\tau}\cos\left(\varpi_0 t+\varphi\right)$$

constants

$$\tau=\frac{2L}{R}\qquad \varpi_0=\sqrt{\omega_0^2-1/\tau^2}$$

LCR circuit



$$q(t) = Q_{\max} e^{-t/\tau} \cos(\varpi_0 t + \varphi)$$

$$\tau = \frac{2L}{R} \qquad \varpi_0 = \sqrt{\omega_0^2 - \frac{1}{\tau^2}}$$

4. [HRW6 33.P.026.] A single-loop circuit consists of a 7.13 Ω resistor, a 13.8 H inductor, and a 3.63 μF capacitor. Initially the capacitor has a charge of 6.20 μC and the current is zero. Calculate the charge on the capacitor *N* complete cycles later for *N* = 5, 10, and 100.

N = 5

[ ] μC

N = 10

[ ] μC

N = 100

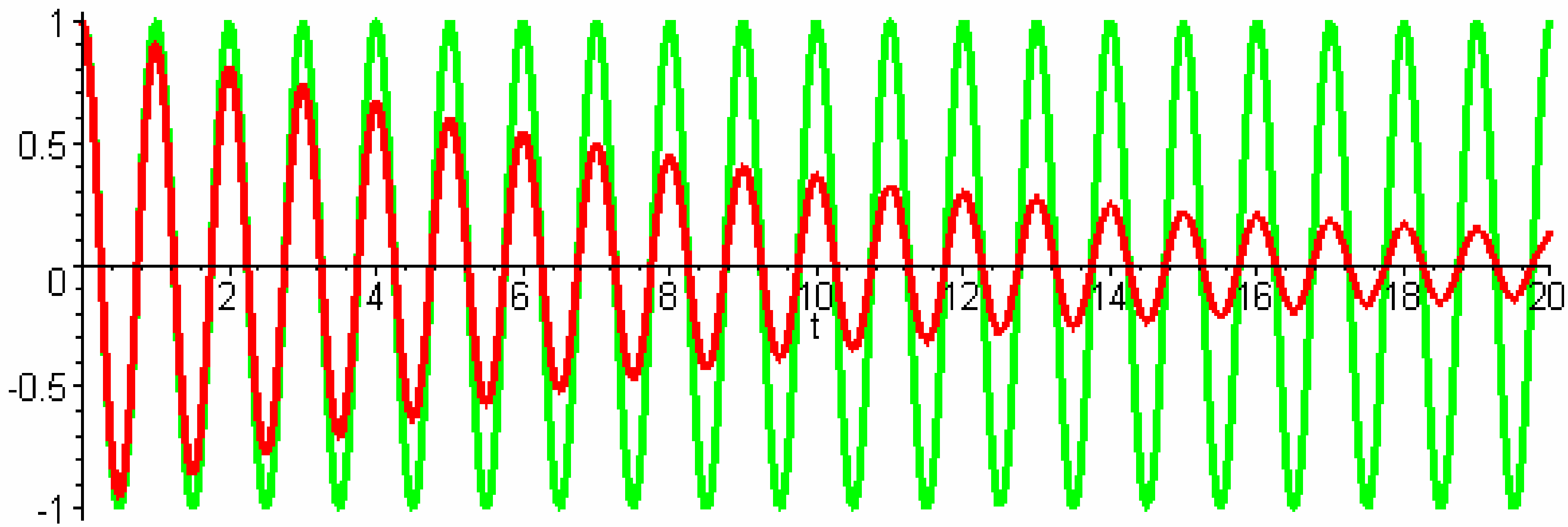
[ ] μC

$$\tau = \frac{2L}{R} \qquad \varpi_0 = \sqrt{\omega_0^2 - 1/\tau^2}$$

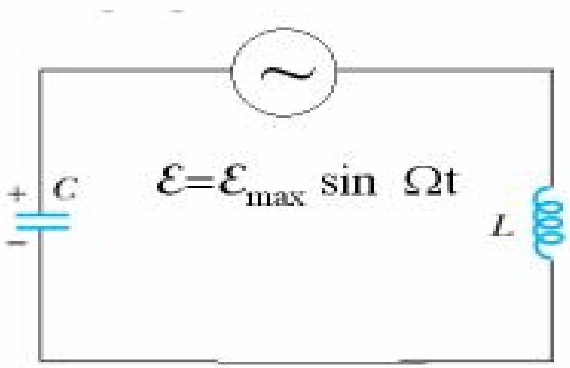
Note that :

$$\frac{1}{\tau} << \omega_0 \quad \Rightarrow \varpi_0 \approx \omega_0 = \frac{2\pi}{T_0}$$

LCR circuit with small R



Online Quiz for Lecture 16
RLC circuits -- Feb. 28, 2005



LC circuit with AC emf

➔ "driven" oscillator

In this circuit, $\omega=50\pi$ rad/s, L=0.03 H, C=0.0003 F, and $\mathcal{E}_{max}=120$V.

1. What is the largest charge on the capacitor (in Coulombs)?
   (A) 0.036 (B) 0.046 (C) 120 (D) 4000
2. What is the period of time (in seconds) between maximum charge values on the capacitor?
   (A) 0.0188 (B) 0.02 (C) 0.04 (D) 50