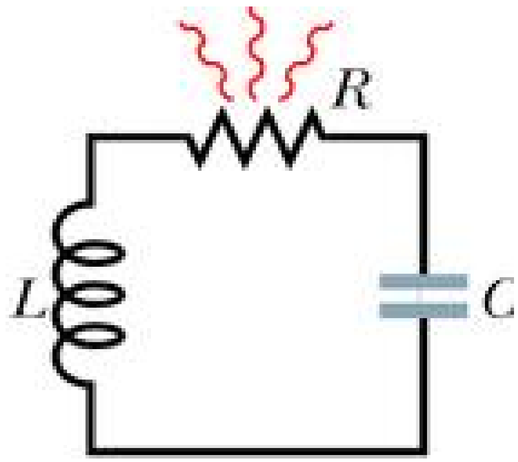
Announcements

1. Extra credit for re-doing test – due Friday 3/4/05
2. Problem session this evening – 6 PM Olin 101 ??
3. Physics seminar tomorrow – Bruce Sherwood and Ruth Chabay from NCSU – experts in physics instruction
4. Today's topics

   AC circuits

LCR circuit



$$-\frac{q}{C} - L\frac{di}{dt} - Ri = 0$$

$$-\frac{1}{C}q - L\frac{d^2q}{dt^2} - R\frac{dq}{dt} = 0$$

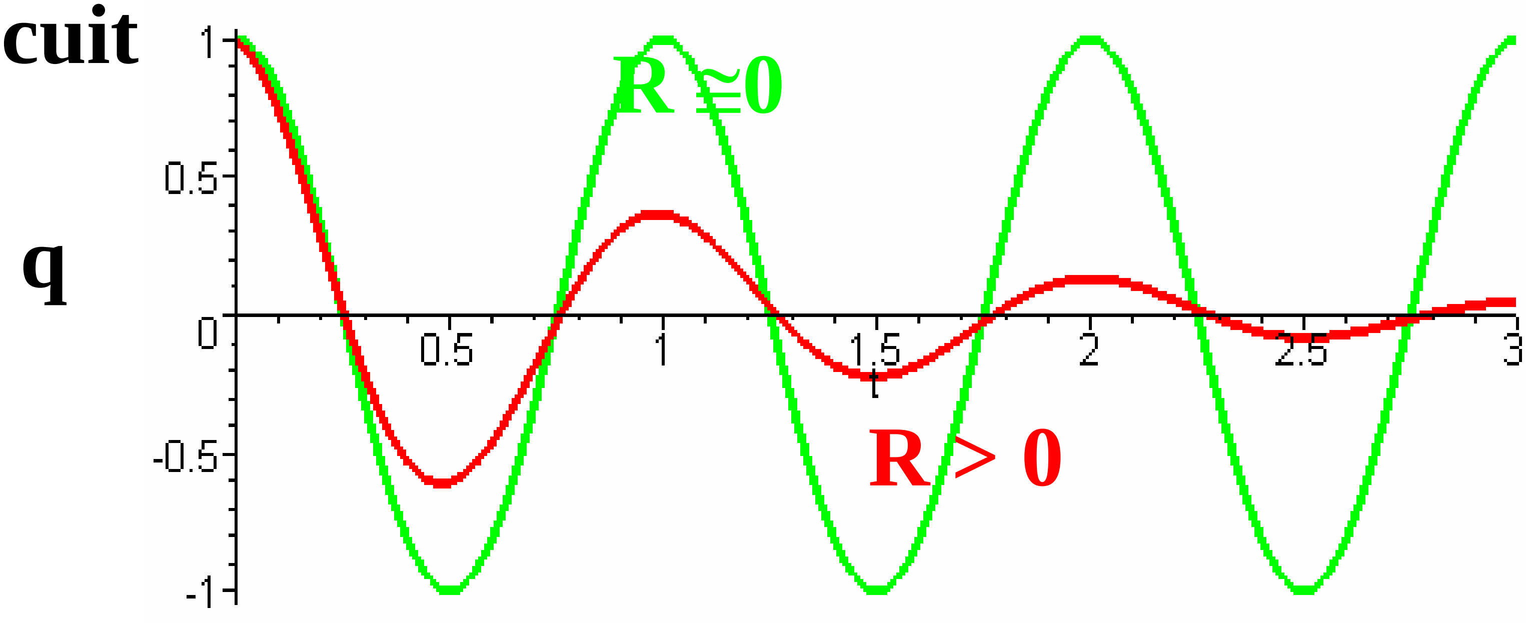
Solution :

$$q(t) = Q_{\max} e^{-t/\tau}\cos(\varpi_0 t + \varphi)$$

constants

$$\tau = \frac{2L}{R} \qquad \varpi_0 = \sqrt{\omega_0^2 - 1/\tau^2}$$

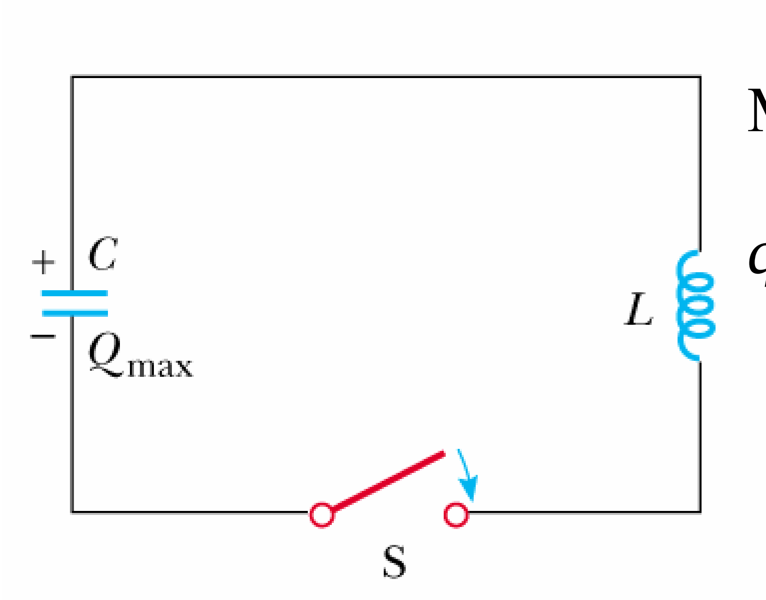
LCR circuit



$$q(t) = Q_{\max} e^{-t/\tau} \cos(\varpi_0 t + \varphi)$$

$$\tau = \frac{2L}{R} \qquad \varpi_0 = \sqrt{\omega_0^2 - 1/\tau^2}$$

For $R \cong 0$ -- LC – circuit:



Mathematical solution :

$$q(t) = Q_{max} \cos(\omega_0 t + \varphi) \quad \text{with} \quad \omega_0 \equiv \sqrt{\frac{1}{LC}}$$

Physical constants

Differential equation :

$$-\frac{q}{C} - L\frac{dI}{dt} = 0$$

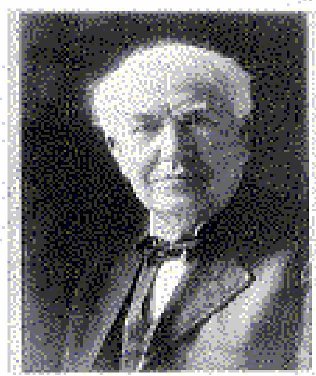
$$\text{or :} \quad -\frac{q}{C} - L\frac{d^2q}{dt^2} = 0$$

Current :

$$I(t) = \frac{dq}{dt} = -\omega Q_{max} \sin(\omega_0 t + \varphi)$$

Emf -- DC vs AC

Power Struggles



One thing physics is good for is deciding who is right, even when large sums of money, titanic egos, and political influence are all involved.

A good example is the conflict between the gentleman on the left, Thomas Edison (1847-1931), and the gentleman on the right, George Westinghouse (1846-1914). A little over 100 years ago, these two men squared off in a technological battle that makes and

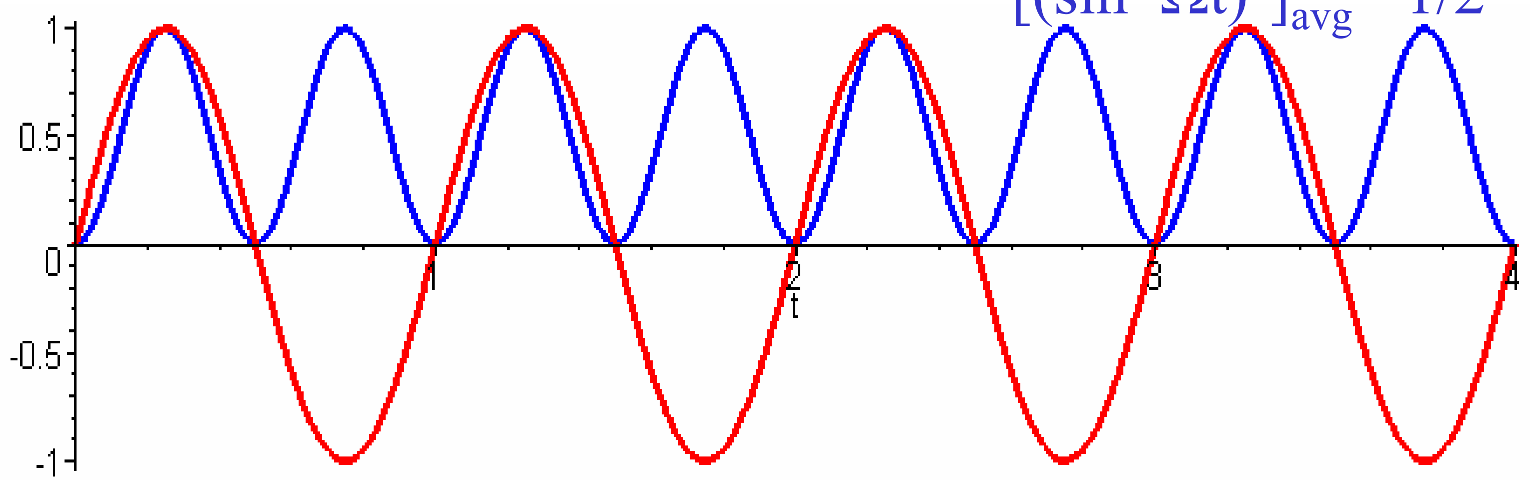
http://www.usafa.af.mil/dfp/physics/webphysics/jittworkshop/251Sp98GFMar23.htm

DC: $\mathcal{E}$=(constant)   AC: $\mathcal{E}=\mathcal{E}_{max}\sin(\Omega t)$

Properties of AC circuits

$$\mathcal{E}=\mathcal{E}_{max} \sin \Omega t \quad \text{or} \quad \mathcal{E}_{max} \cos \Omega t$$

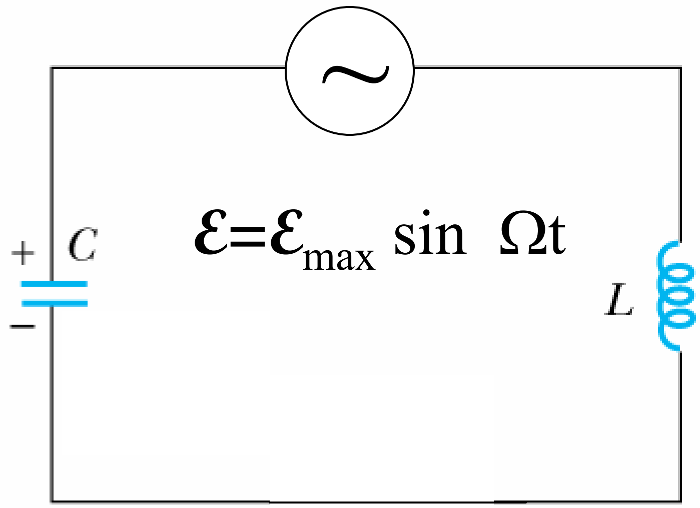
$[(\sin \Omega t)^2]_{avg} = 1/2$



$[\sin \Omega t]_{avg} = 0$

$$\mathcal{E}_{rms} = \sqrt{1/2}\ \mathcal{E}_{max} \qquad \text{similarly,} \qquad I_{rms} = \sqrt{1/2}\ I_{max}$$

LC circuit with AC emf:



$$\mathcal{E}=\mathcal{E}_{max} \sin \Omega t$$

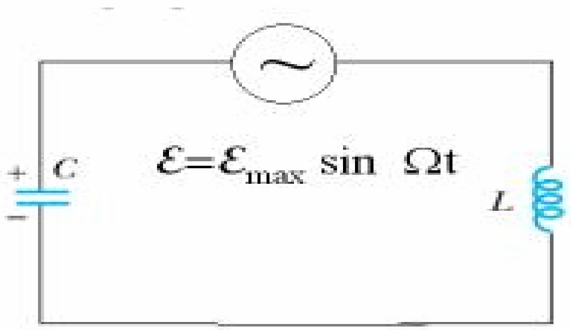
$$-\frac{q}{C}-L\frac{d^2q}{dt^2}+\mathcal{E}_{max}\sin\Omega t=0$$

$$q(t)=\frac{\mathcal{E}_{max}/L}{\frac{1}{LC}-\Omega^2}\sin\Omega t$$

large response when

$$\Omega=\frac{1}{\sqrt{LC}}$$

Online Quiz for Lecture 16
RLC circuits -- Feb. 28, 2005



LC circuit with AC emf

➔ "driven" oscillator

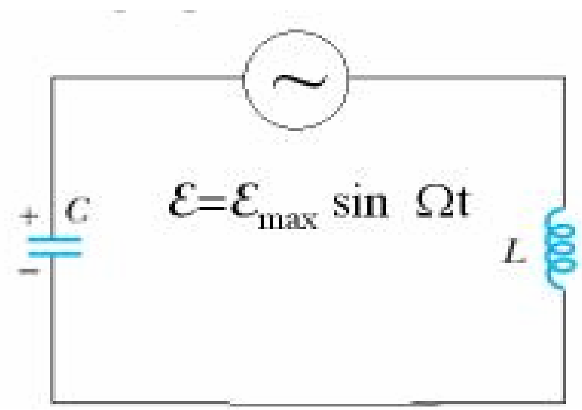
In this circuit, ω=50 π rad/s, L=0.03 H, C=0.0003 F, and $\mathcal{E}_{max}$=120V.

1. What is the largest charge on the capacitor (in Coulombs)?
   (A) 0.036 (B) 0.046 (C) 120 (D) 4000
2. What is the period of time (in seconds) between maximum charge values on the capacitor?
   (A) 0.0188 (B) 0.02 (C) 0.04 (D) 50

$$q(t)=\frac{\mathcal{E}_{max}/L}{\frac{1}{LC}-\Omega^2}\sin\Omega t$$

$$Q_{max}=\frac{\mathcal{E}_{max}/L}{\frac{1}{LC}-\Omega^2}=\frac{120/0.03}{\frac{1}{0.03\cdot 0.0003}-(50\pi)^2}=0.046\,C$$

More details:



$$-\frac{q}{C}-L\frac{d^2q}{dt^2}+\mathcal{E}_{\max}\sin\Omega t=0$$

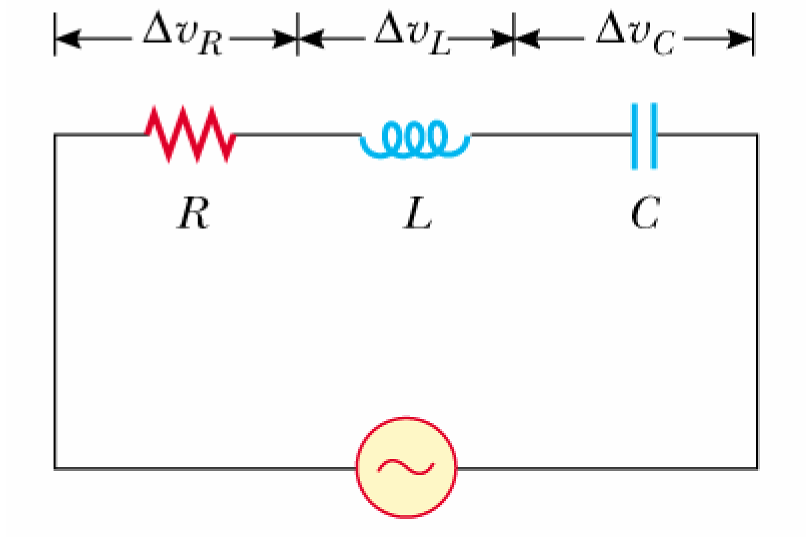
$$q(t)=\frac{\mathcal{E}_{\max}/L}{\frac{1}{LC}-\Omega^2}\sin\Omega t+K\cos(\omega_0 t+\varphi) \qquad \omega_0=\sqrt{\frac{1}{LC}}$$

$$\text{or:}\quad q(t)=\frac{\mathcal{E}_{\max}/L}{\omega_0^{\ 2}-\Omega^2}\sin\Omega t+K\cos(\omega_0 t+\varphi)$$

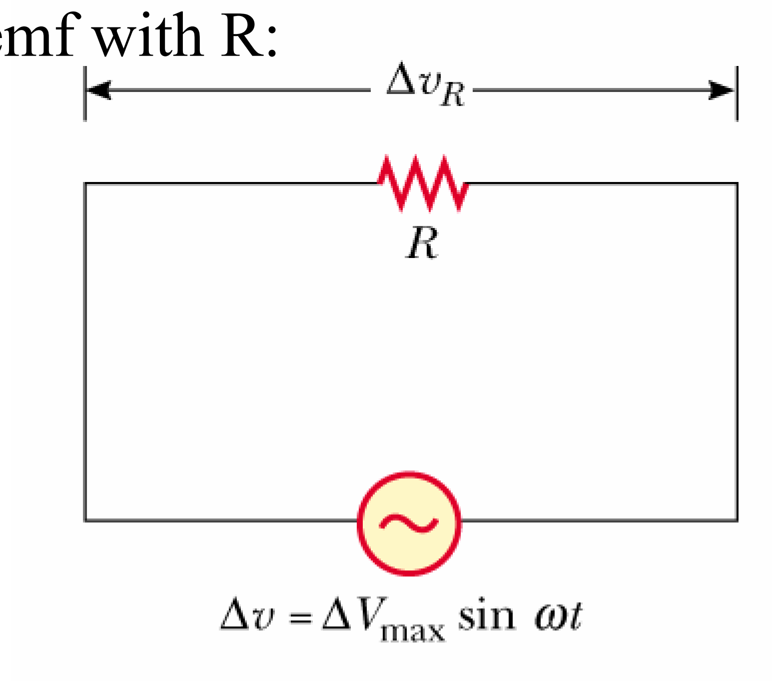
"driven" term

"homogeneous" term

More general treatment of LCR circuits with AC emf:



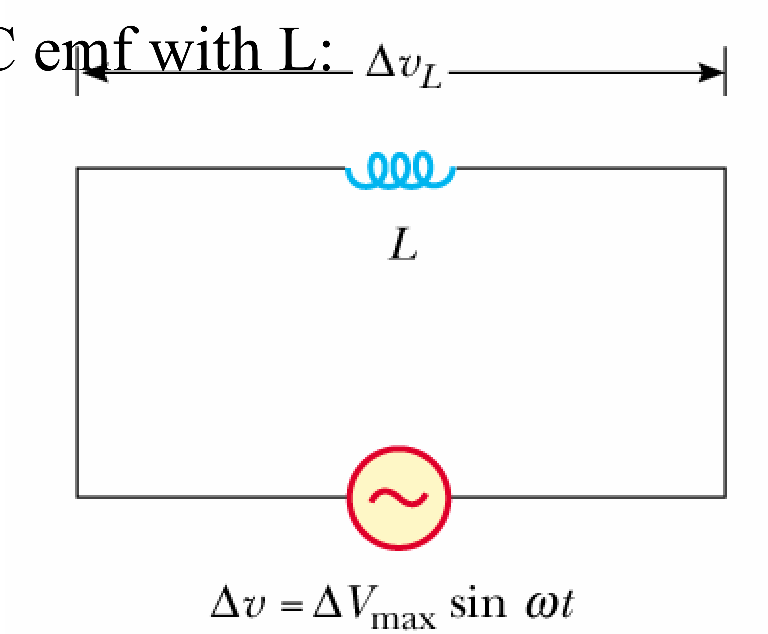
AC emf with R:



$$-IR + \Delta V_{max} \sin \omega t = 0$$

$$I = \frac{\Delta V_{max}}{R} \sin \omega t$$

AC emf with L:

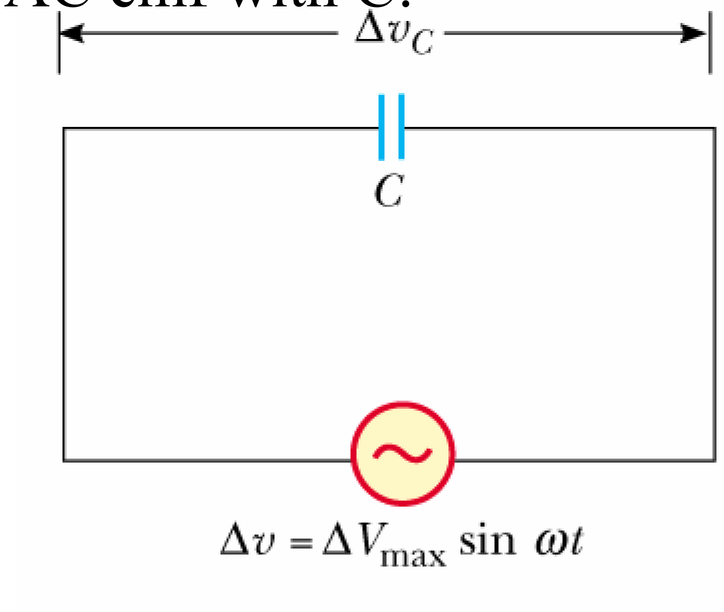


$$-L\frac{dI}{dt} + \Delta V_{\max} \sin \omega t = 0$$

$$\frac{dI}{dt} = \frac{\Delta V_{\max}}{L} \sin \omega t$$

$$I = -\frac{\Delta V_{\max}}{\omega L} \cos \omega t$$

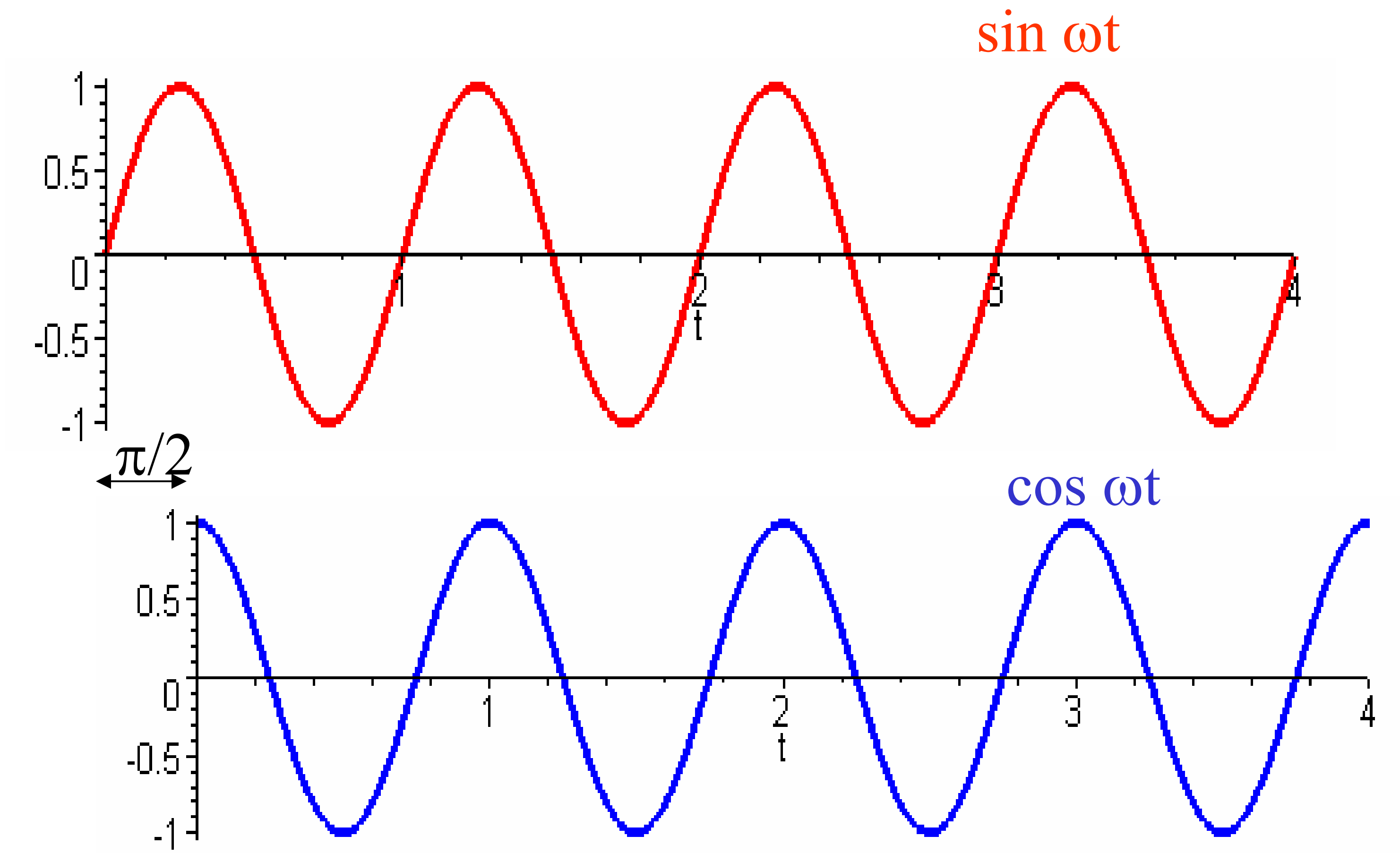
AC emf with C:



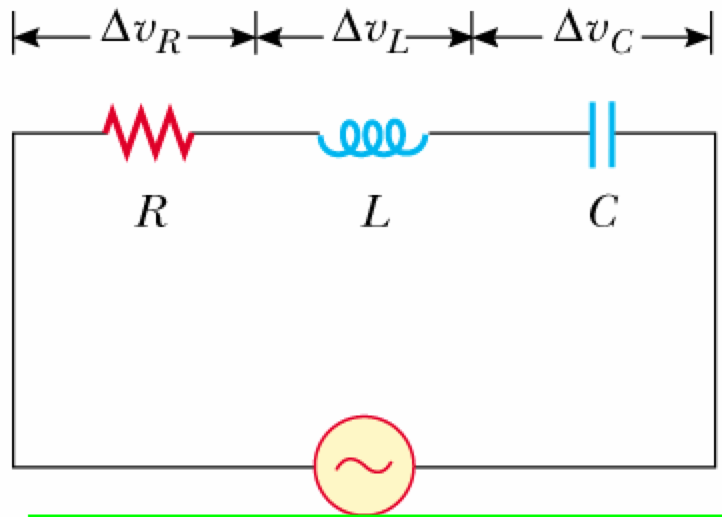
$$-\frac{q}{C} + \Delta V_{\max} \sin \omega t = 0$$

$$q = C\Delta V_{\max} \sin \omega t$$

$$I = \omega C\Delta V_{\max} \cos \omega t$$


sin ωt
1
0.5
0
-0.5
-1
1
2
t
3
4
π/2
cos ωt
1
0.5
0
-0.5
-1
1
2
t
3
4

AC emf with LCR:



$$-RI - L\frac{dI}{dt} - \frac{q}{C} + \Delta V_{\max}\sin\omega t = 0$$

$$I = I_{\max}\sin(\omega t - \varphi)$$

$$I_{\max} = \frac{\Delta V_{\max}}{Z} \qquad Z \equiv \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$

$$\tan\varphi = \frac{\omega L - 1/\omega C}{R}$$

Differential equation:

$$-RI - L\frac{dI}{dt} - \frac{Q}{C} + \Delta V_{\max}\sin\omega t = 0$$

Solution form: $I(t) = I_{\max}\sin(\omega t - \varphi)$

Strategy: substitute form into differential equation and demand the equality must be true at all times.

$$-RI_{\max}\sin(\omega t-\varphi) - \omega L I_{\max}\cos(\omega t-\varphi) + \frac{1}{\omega C} I_{\max}\cos(\omega t-\varphi)$$
$$+\Delta V_{\max}\sin\omega t = 0$$

$$-RI_{\max}\sin(\omega t-\varphi)-\omega LI_{\max}\cos(\omega t-\varphi)+\frac{1}{\omega C}I_{\max}\cos(\omega t-\varphi)$$
$$+\Delta V_{\max}\sin\omega t=0$$

Strategy: group terms as factors of sin ωt and cos ωt, noting that:

$$\sin(\omega t-\varphi)=\left(\sin(\omega t)\cos\varphi-\cos(\omega t)\sin\varphi\right)$$
$$\cos(\omega t-\varphi)=\left(\cos(\omega t)\cos\varphi+\sin(\omega t)\sin\varphi\right)$$

$$\left(-RI_{\max}\cos(\varphi)-\omega LI_{\max}\sin(\varphi)+\frac{1}{\omega C}I_{\max}\sin(\varphi)+\Delta V_{\max}\right)\sin\omega t=0$$
$$\left(+RI_{\max}\sin(\varphi)-\omega LI_{\max}\cos(\varphi)+\frac{1}{\omega C}I_{\max}\cos(\varphi)\right)\cos\omega t=0$$

$$\Rightarrow\varphi=\tan^{-1}\left(\frac{\omega L-\frac{1}{\omega C}}{R}\right)\qquad I_{\max}=\frac{\Delta V_{\max}}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}}$$

Summary:

$$-RI - L\frac{dI}{dt} - \frac{q}{C} + \Delta V_{\max}\sin\omega t = 0$$

$$I(t) = I_{\max}\sin(\omega t - \varphi)$$

$$I_{\max} = \frac{\Delta V_{\max}}{Z} \qquad Z \equiv \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$

$$\tan\varphi = \frac{\omega L - 1/\omega C}{R}$$

Note -- Complete solution includes homogeneous terms :

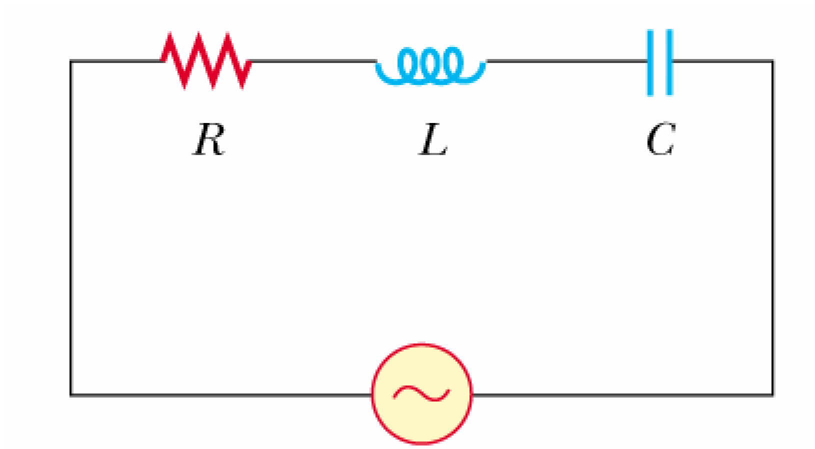
$$I(t) = I_{\max}\sin(\omega t - \varphi) + Ke^{-t/\tau}\cos(\varpi_0 t + \alpha)$$

Important facts to remember in analyzing AC circuits

- $\Delta V_{max} = \sqrt{2}\ \Delta V_{rms}$
- Steady-state currents and voltages will have sinusoidal time dependence such as sin ωt or cos ωt or a linear combination of the two.
- Inductors and capacitors change the "phase" of the current relative to that of the emf.
- Solution of the circuit equations for *I(t)* → equations must be satisfied for all times *t*.

DC circuits	AC circuits
$\mathcal{E}$ = constant in time	$\mathcal{E} = \mathcal{E}_{max} \sin \omega t$ or $\mathcal{E}_{max} \cos \omega t$
$I = \begin{cases} \text{constant in time} \\ \text{constant} + I_0 e^{-t/\tau} \\ \text{damped oscillations} \end{cases}$	I= transients + $I_0 \sin(\omega t - \phi)$
Kirchhoff's rules apply	Kirchhoff's rules apply

Example



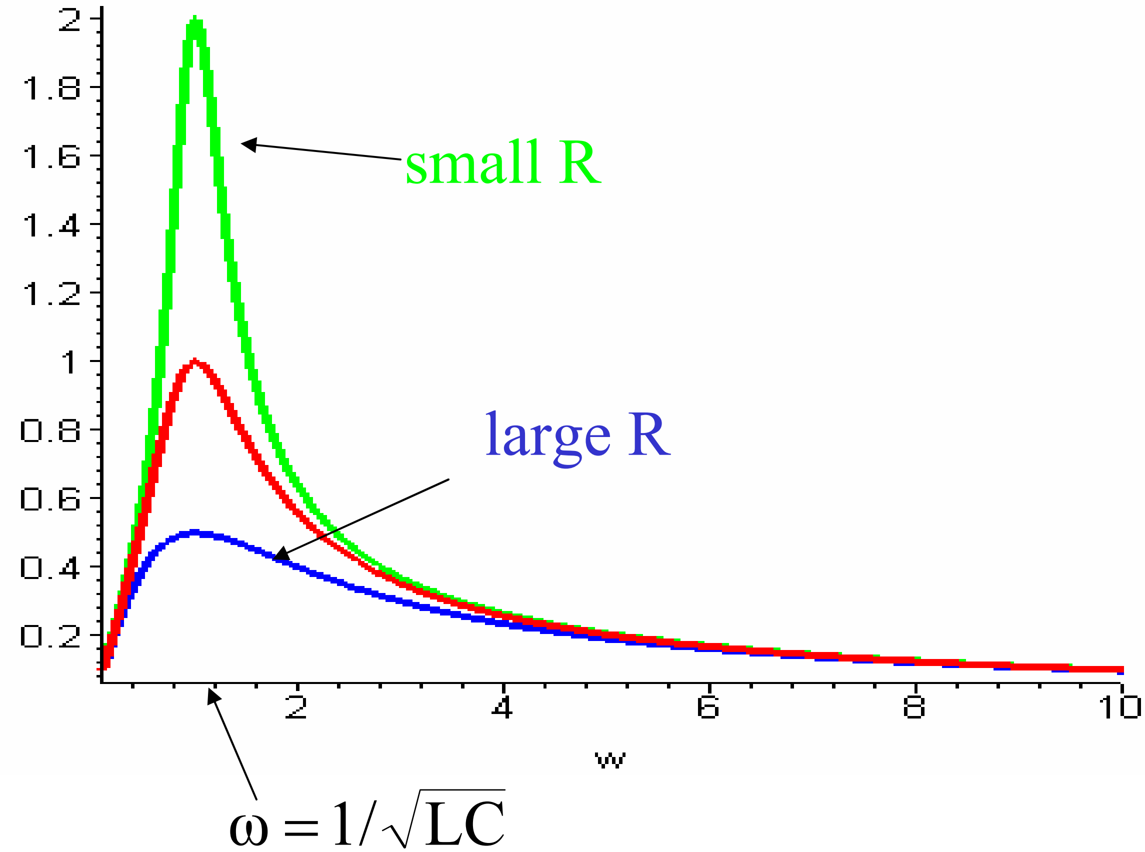
could also be cos ωt

Kirchhoff's rule:

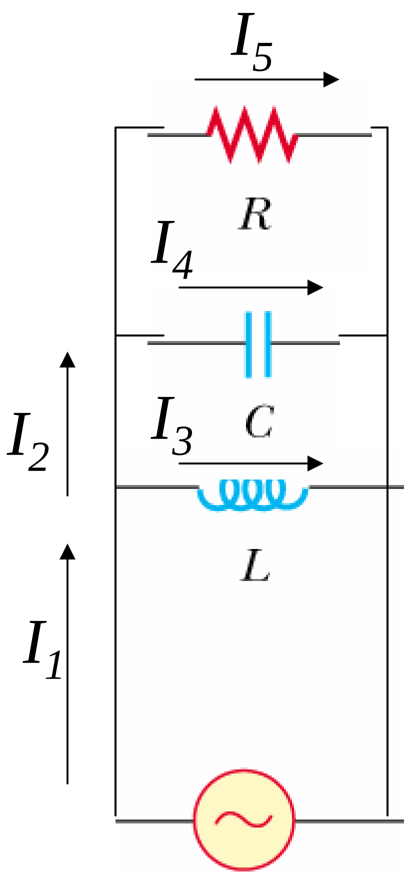
$$-RI - L\frac{dI}{dt} - \frac{Q}{C} + \Delta V_{\max}\sin\omega t = 0$$

Behavior of I_{max}

$$I_{\max} = \frac{\Delta V_{\max}}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$



More complicated circuits can be analyzed in a similar way



$$-L\frac{dI_3}{dt}+\Delta V_{\max}\sin\omega t=0$$

$$-L\frac{dI_3}{dt}+\frac{Q_4}{C}=0$$

$$\frac{Q_4}{C}-RI_5=0$$

$$I_1=I_2+I_3$$

$$I_2=I_4+I_5$$

PHY 230 -- Electronics

Power delivered by generator in a circuit:

$\mathscr{P}(t) = I(t)\,\Delta V(t) = I_{max}\sin(\omega t - \phi)\,\Delta V_{max}\sin(\omega t)$

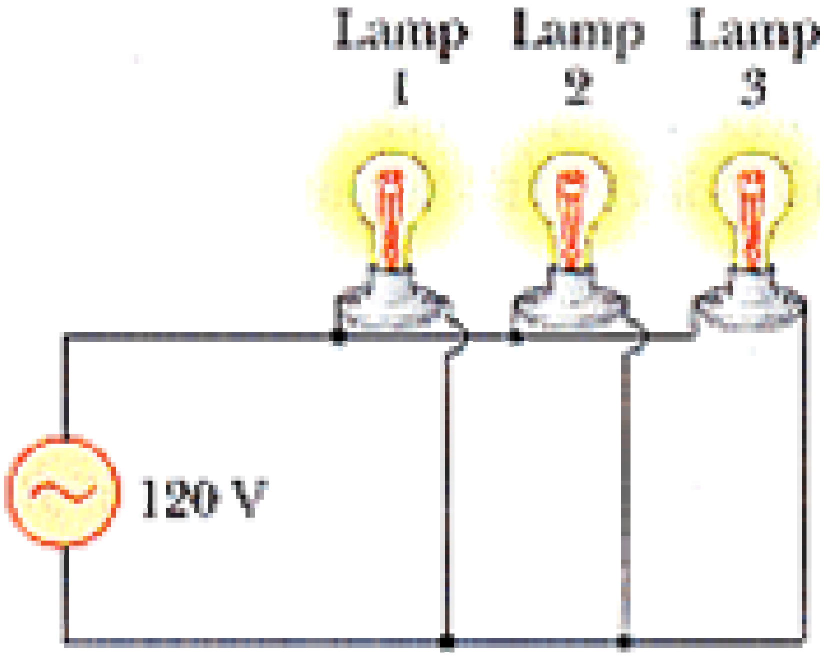
$\langle \mathscr{P} \rangle_{avg} = I_{max}\,\Delta V_{max}\ \tfrac{1}{2}\ \cos(\phi)$

Note that:

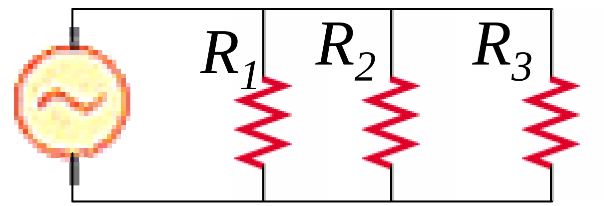
$$\cos\varphi = \frac{R}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

➔ $\langle \mathscr{P} \rangle_{avg} = \tfrac{1}{2}\ R\,I_{max}^2 = R\,I_{rms}^2$

Example – AC circuits



➔



$$I_{rms} = \frac{\Delta V_{rms}}{Z}$$

$$Z = R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$