Announcements

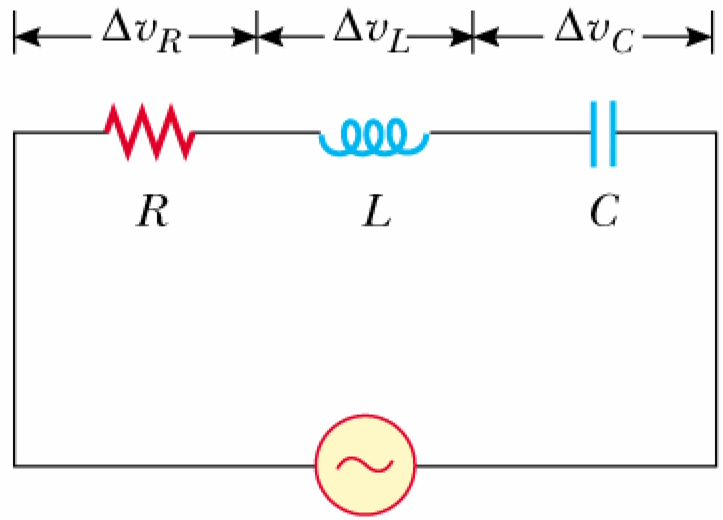
1. Reworked second exam – due today
2. Assignment #18 & 19 due 3/16/05
3. Today's topics

   Continue discussion of AC circuits

   Power associated with AC circuits

   Transformers
4. Have a great spring break!

AC emf with LCR circuit:



$$\text{Differential eq:}\quad -RI - L\frac{dI}{dt} - \frac{q}{C} + \mathcal{E}_{\max}\sin\omega t = 0$$

Steady-state term

Transient term

$$\text{Solution for } I(t):\quad I = I_{\max}\sin(\omega t - \varphi) + Ke^{-t/\tau}\cos(\varpi_0 t + \alpha)$$

$$\text{where:}\qquad I_{\max} = \frac{\mathcal{E}_{\max}}{Z} \qquad Z \equiv \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$

$$\tan\varphi = \frac{\omega L - 1/\omega C}{R}$$

Phase relationships of individual circuit components:

$$\frac{q}{C} = -\frac{I_{max}}{\omega C}\cos(\omega t)$$

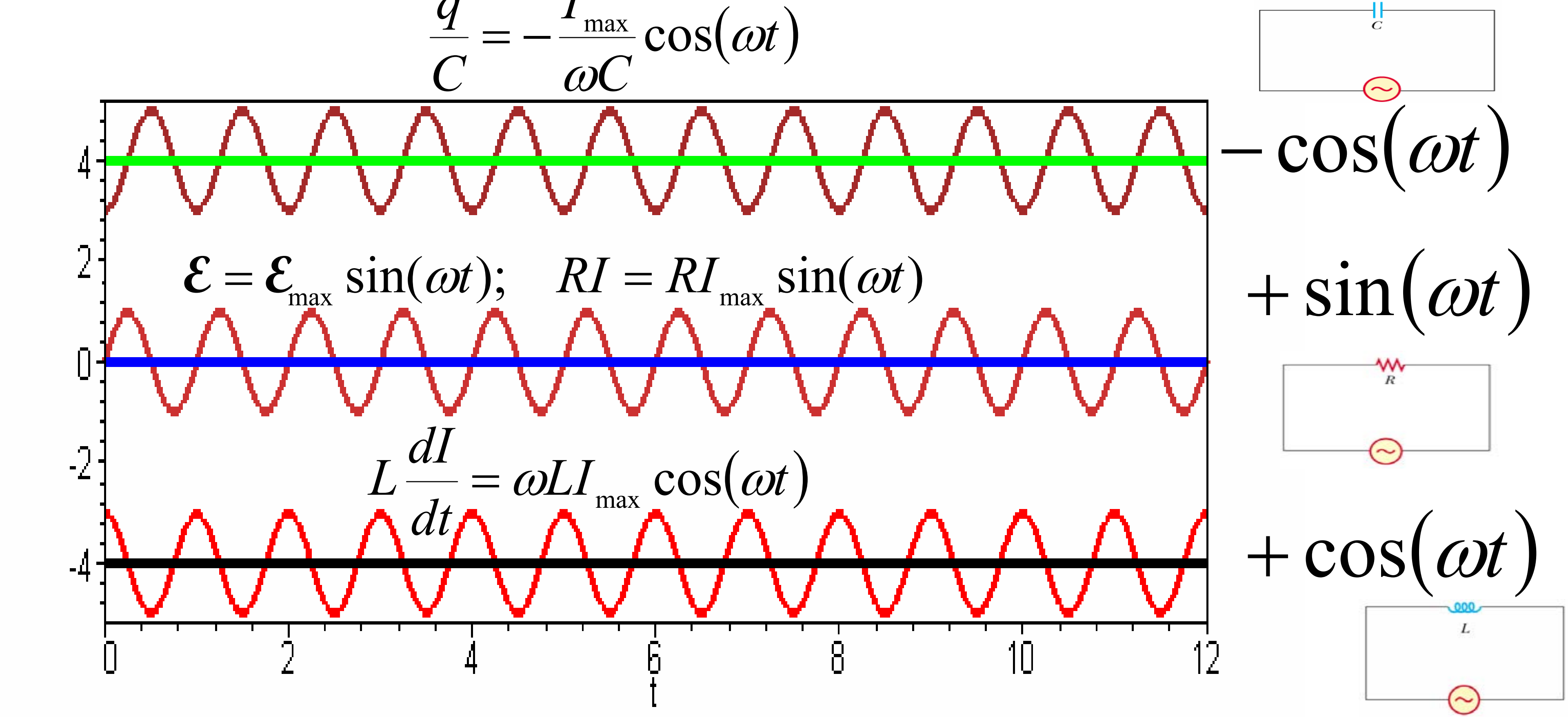
$$-\cos(\omega t)$$

$$\mathcal{E} = \mathcal{E}_{max}\sin(\omega t); \quad RI = RI_{max}\sin(\omega t)$$

$$+\sin(\omega t)$$

$$L\frac{dI}{dt} = \omega L I_{max}\cos(\omega t)$$

$$+\cos(\omega t)$$



Differential equation for circuit components in serial:

$$-RI - L\frac{dI}{dt} - \frac{q}{C} + \mathcal{E}_{\max}\sin\omega t = 0$$

Construct solution of the form:

$$I(t) = I_{\max}\sin(\omega t - \varphi) \;\text{ or }\; I(t) = I_s\sin(\omega t) + I_c\cos(\omega t)$$

Substituting form into differential equation:

$$-RI_{\max}\sin(\omega t - \varphi) - \omega L I_{\max}\cos(\omega t - \varphi) + \frac{1}{\omega C} I_{\max}\cos(\omega t - \varphi) + \mathcal{E}_{\max}\sin\omega t = 0$$

$$\left(-RI_{\max}\cos(\varphi) - \omega L I_{\max}\sin(\varphi) + \frac{1}{\omega C} I_{\max}\sin(\varphi) + \mathcal{E}_{\max} \right)\sin\omega t = 0$$

$$\left(+RI_{\max}\sin(\varphi) - \omega L I_{\max}\cos(\varphi) + \frac{1}{\omega C} I_{\max}\cos(\varphi) \right)\cos\omega t = 0$$

$$-RI - L\frac{dI}{dt} - \frac{q}{C} + \mathcal{E}_{\max}\sin\omega t = 0$$

$$\left(-RI_{\max}\cos(\varphi) - \omega L I_{\max}\sin(\varphi) + \frac{1}{\omega C}I_{\max}\sin(\varphi) + \mathcal{E}_{\max}\right)\sin\omega t = 0$$

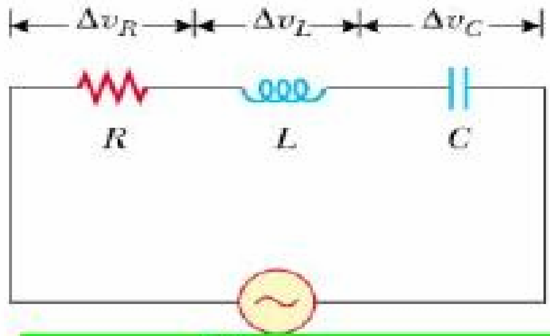
$$\left(+RI_{\max}\sin(\varphi) - \omega L I_{\max}\cos(\varphi) + \frac{1}{\omega C}I_{\max}\cos(\varphi)\right)\cos\omega t = 0$$

$$\Rightarrow \varphi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right) \qquad I_{\max} = \frac{\mathcal{E}_{\max}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \equiv \frac{\mathcal{E}_{\max}}{Z}$$

Complete solution :

$$I(t) = I_{\max}\sin(\omega t - \varphi) + Ke^{-t/\tau}\cos(\varpi_0 t + \alpha)$$

Online Quiz for Lecture 15
AC circuits -- Mar. 02, 2005



$$-RI - L\frac{dI}{dt} - \frac{q}{C} + \Delta V_{max} \sin\omega t = 0$$

$$I = I_{max} \sin(\omega t + \varphi)$$

$$I_{max} = \frac{\Delta V_{max}}{Z} \qquad Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$

$$\tan\varphi = \frac{\omega L - 1/\omega C}{R}$$

Consider the circuit above and the corresponding analysis.

1. At what frequency ω will the current I have the largest value?
   (a) 0 (b) sqrt(LC) (c) 1/sqrt(LC) (d) ∞
2. What is the largest amplitude of the current at that value of the frequency ω ?
   (a) ΔV/R (b) ΔV/(ωL) (c) ΔV(ωC) (d) ∞

1. [HRW6 33.P.045.] An *RLC* circuit such as that of Fig. 33-7 has R = 5.24 Ω, C = 21.1 μF, L = 1.14 H, and $\mathscr{E}_m$ = 33.1 V.

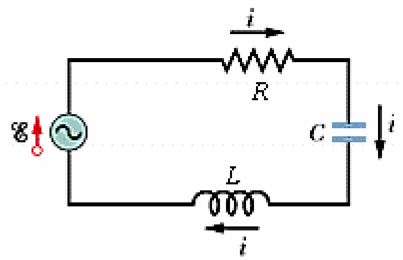


Figure 33-7.

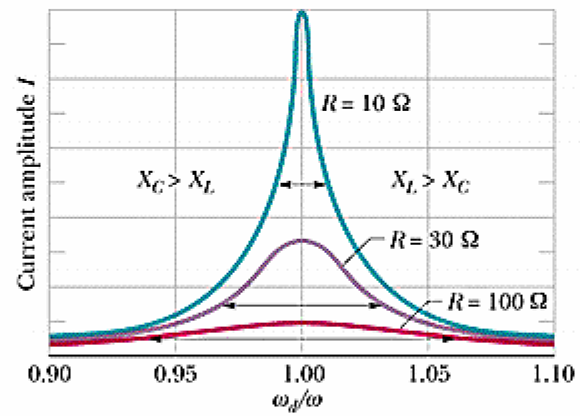


Figure 33-13.

(a) At what angular frequency ω_d will the current amplitude have its maximum value, as in the resonance curves of Fig. 33-13?

[ ] rad/s

(b) What is this maximum value?

[ ] A

(c) At what two angular frequencies ω_{d1} and ω_{d2} will the current amplitude be half this maximum value?

[ ] rad/s (larger of the two)

[ ] rad/s (smaller of the two)

(d) What is the fractional half-width [= $(\omega_{d1} - \omega_{d2})/\omega$] of the resonance curve for this circuit?

[ ]

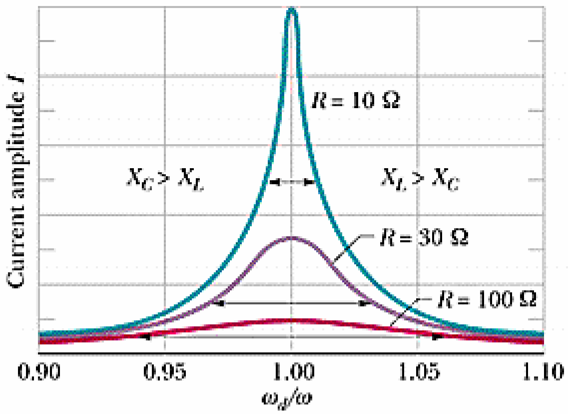


Figure 33-13.

$$I_{\max}(\omega) = \frac{\mathcal{E}_{\max}}{Z(\omega)} \qquad Z(\omega) \equiv \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$

(a) At what angular frequency ω_d will the current amplitude have its maximum value, as in the resonance curves of Fig. 33-13?

[ ] rad/s

(b) What is this maximum value?

[ ] A

$$I_{\max}(\omega)\big|_{\max} = \frac{\mathcal{E}_{\max}}{R} \qquad \text{for } (\omega L - 1/\omega C) = 0$$

(c) At what two angular frequencies ω_{d1} and ω_{d2} will the current amplitude be half this maximum value?

[ ] rad/s (larger of the two)

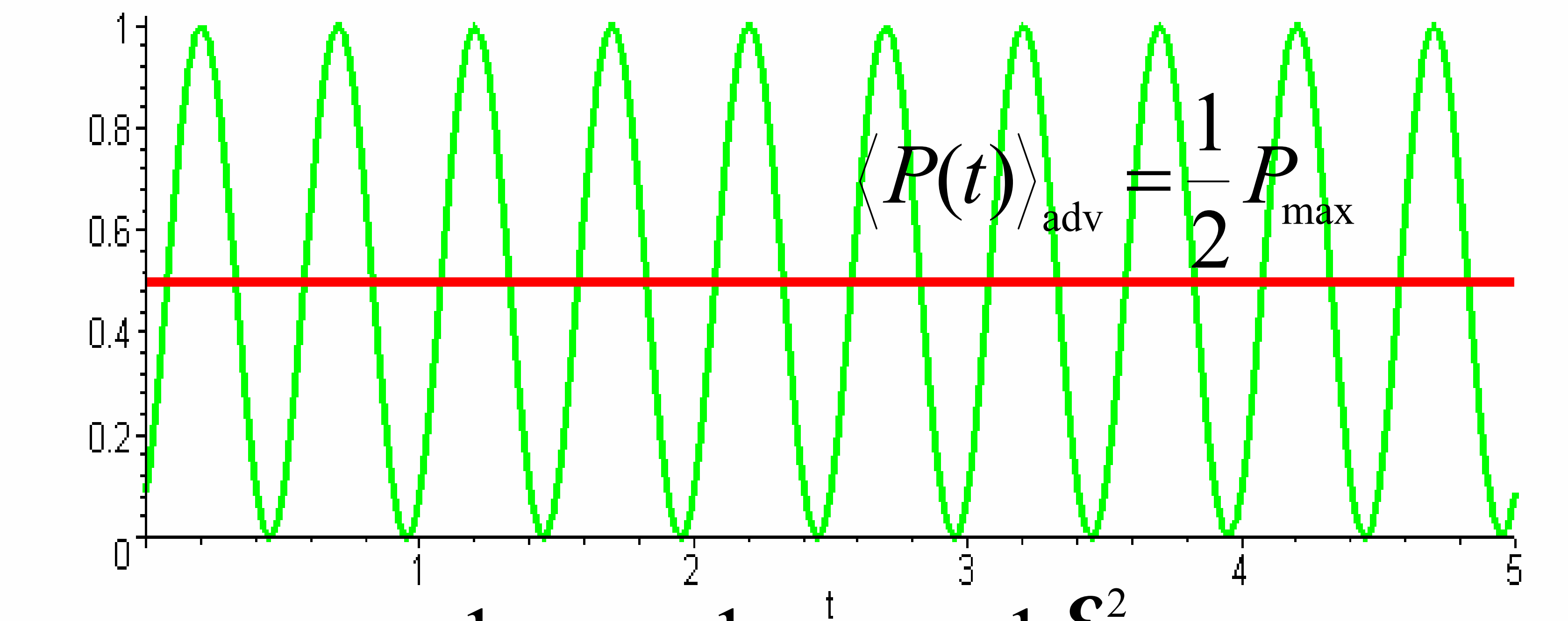
[ ] rad/s (smaller of the two)

$$I_{\max}(\omega') = \frac{\mathcal{E}_{\max}}{2R} \qquad \Rightarrow \sqrt{R^2 + (\omega' L - 1/\omega' C)^2} = 2R$$

(d) What is the fractional half-width [= $(\omega_{d1} - \omega_{d2})/\omega$] of the resonance curve for this circuit?

[ ]

Power in AC circuit: $P = I^2 R = \left(I_{\max} \sin(\omega t - \varphi)\right)^2 R$



$$\langle P(t) \rangle_{\mathrm{adv}} = \frac{1}{2} P_{\max} = \frac{1}{2} I_{\max}^2 R = \frac{1}{2} \frac{\mathcal{E}_{\max}^2}{Z^2} R$$

$$I_{rms} = \sqrt{\frac{1}{2} I_{\max}^2} = \sqrt{\frac{1}{2}} I_{\max}$$

Peer instruction question

If the wall voltage is 120V (rms) what is the peak voltage?

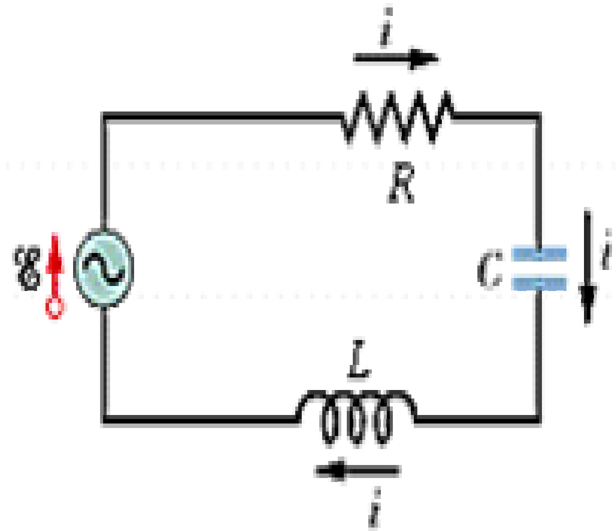
A. 120 V

B. 240 V

C. 60 V

D. 170 V

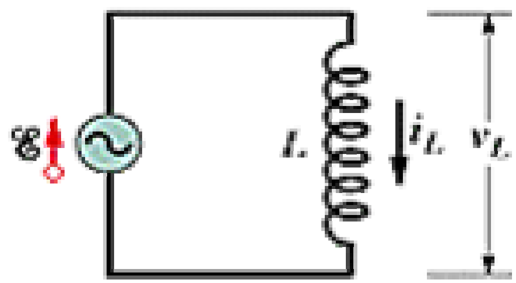
Example:



$$\langle P(t)\rangle_{\text{adv}} = \frac{1}{2}\frac{\mathcal{E}_{\max}^2}{Z^2}R = \frac{1}{2}\frac{\mathcal{E}_{\max}^2}{R^2+\left(\omega L - 1/\omega C\right)^2}R$$

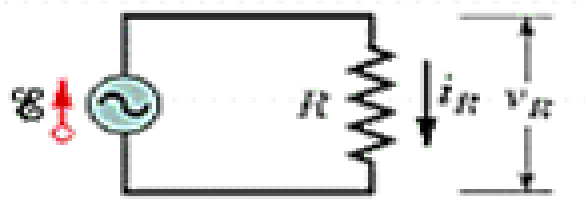
Can be calculated from a knowledge of $\mathcal{E}_{\max}$, R,L,C, and $\omega=2\pi f$

Example:



$$\langle P(t) \rangle_{\text{adv}} = \frac{1}{2} \frac{\mathcal{E}_{\max}^2}{Z^2} R = \frac{1}{2} \frac{\mathcal{E}_{\max}^2}{R^2 + (\omega L - 1/\omega C)^2} R$$

Example:

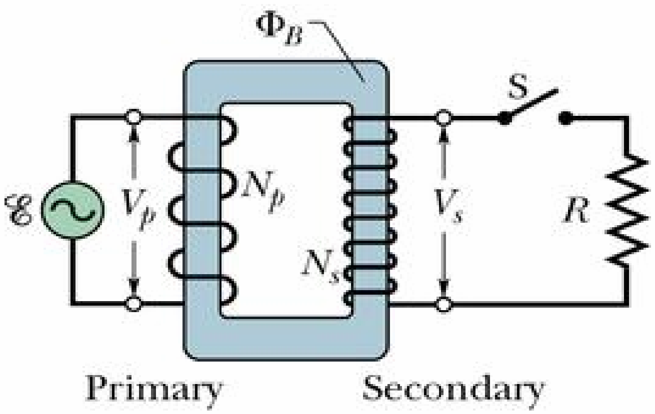


(a)

$$\left\langle P(t)\right\rangle_{\text{adv}} = \frac{1}{2}\frac{\mathcal{E}_{\max}^{2}}{Z^{2}}R = \frac{1}{2}\frac{\mathcal{E}_{\max}^{2}}{R^{2}+\left(\omega L - 1/\omega C\right)^{2}}R$$

Electrical transformer's

Online Quiz for Lecture 18
AC circuits -- Mar. 04, 2005

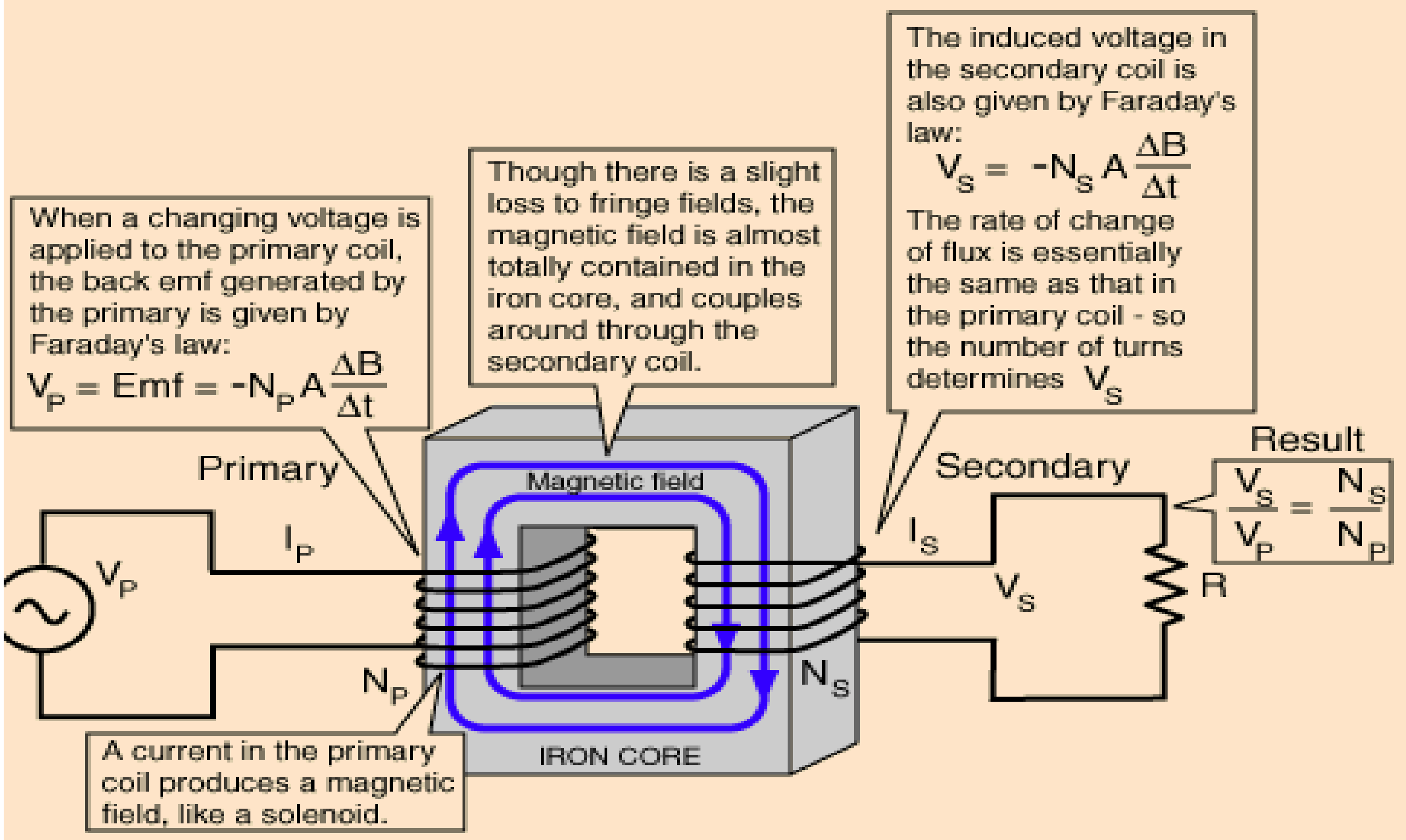


This figure from your text shows a transformer circuit in which two AC circuits wound on an iron form. When the switch is closed, current runs through the "secondary" circuit even though it is not physically connected to the first. What is the basic principle that explains this phenomenon?

A. Gauss's law
B. Coulomb's law
C. Ampere's law
D. Maxwell's law
E. Faraday's law

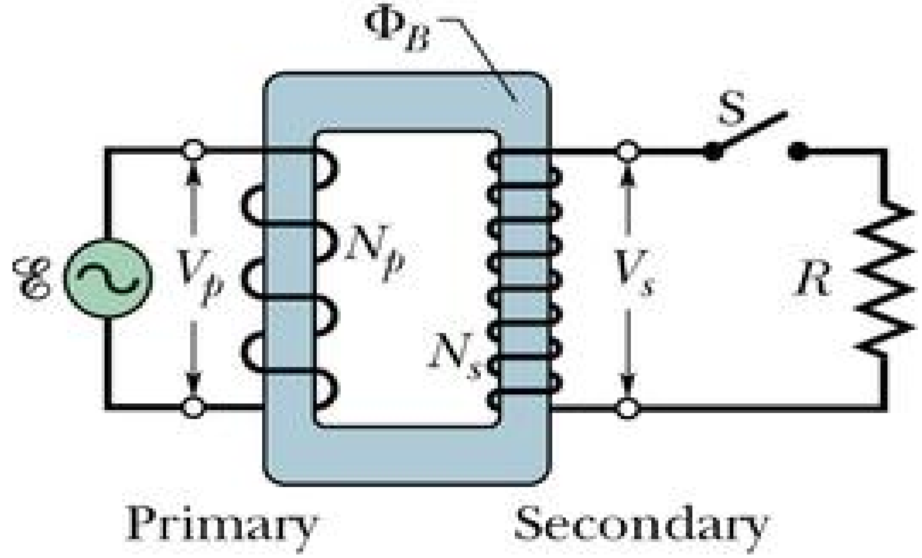
$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

Transformer and Faraday's Law



http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/transf.html#c2

AC transformer



$$\text{Faraday's law for single wire loop:} \quad \mathcal{E} = -\frac{d\Phi_B}{dt}$$

For transformer:

$$\mathcal{E}_P = -N_P \frac{d\Phi_B}{dt} \qquad \mathcal{E}_S = -N_S \frac{d\Phi_B}{dt}$$

$$-\frac{d\Phi_B}{dt} = \frac{\mathcal{E}_P}{N_P} = \frac{\mathcal{E}_S}{N_S} \qquad \Rightarrow \mathcal{E}_S = \frac{N_S}{N_P}\mathcal{E}_P$$

Peer instruction question

Will the transformer relation $\mathcal{E}_S = \frac{N_S}{N_P}\mathcal{E}_P$ be valid for a DC circuit and for an AC circuit?

A. DC only

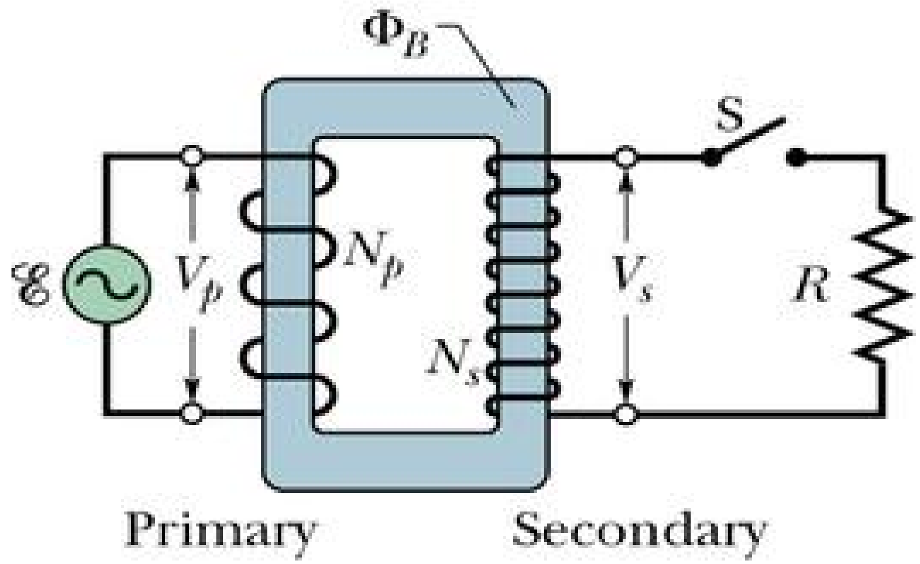
B. AC only

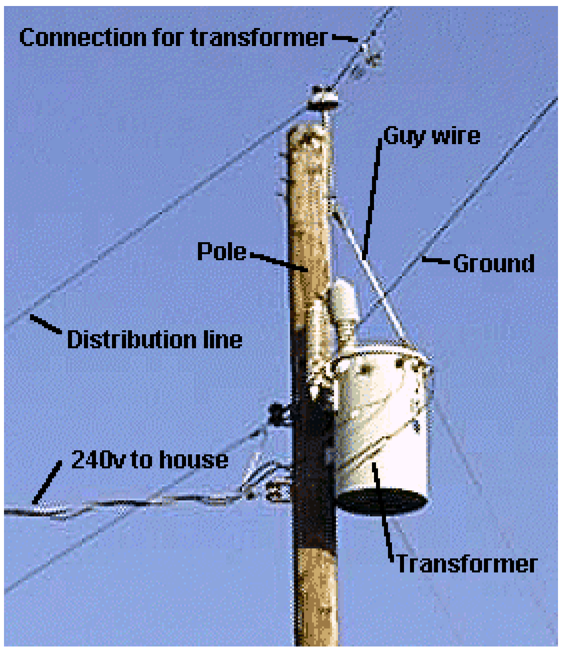
C. DC and AC

D. Not valid

Practical use of transformer:

$$\mathcal{E}_S = \frac{N_S}{N_P}\mathcal{E}_P$$





Example:

$$\mathcal{E}_S = \frac{N_S}{N_P}\mathcal{E}_P = \frac{100}{6000}7200V = 120V$$

http://science.howstuffworks.com/power9.htm