Announcements

1. Schedule: http://www.wfu.edu/~natalie/s05phy114/homework/
   - Class this week: electromagnetic waves (Chap. 34)
   - Monday 3/21/05: Review Chap. 33-34
   - Wednesday 3/23/05: Exam #3 (practice exam available)
2. Special lectures this week – Prof. Clifford Will will give physics colloquium and public lecture
3. Today's topic – Chapter 34

   Maxwell's equations and electromagnetic radiation
   - What is the relationship between Maxwell's equations and electromagnetic waves
   - How are electromagnetic waves related to other waves – water waves, sound waves, etc.
   - What is the relationship between the E and B fields in an electromagnetic wave
   - Energy and electromagnetic waves

Maxwell's equations and electromagnetic radiation

- The wave equation – seen last semester for mechanical waves – (we will focus on periodic wave solutions)
- Maxwell's equations
  - Coulomb's and Gauss's law for electric field
  - Gauss's law for magnetic field
  - Faraday's law
  - Biot-Savart and Ampere's law with Maxwell's contributions
- Properties of electromagnetic waves due to Maxwell's equations

The mathematical form of the "wave equation":

$$\boxed{\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}}$$

velocity of wave

Solution form for a periodic solution:

$$y(x,t) = A\sin\left(\frac{2\pi}{\lambda}(x - vt) + \varphi\right) \equiv A\sin\left(\frac{2\pi x}{\lambda} - 2\pi ft + \varphi\right) = A\sin(kx - \omega t + \varphi)$$

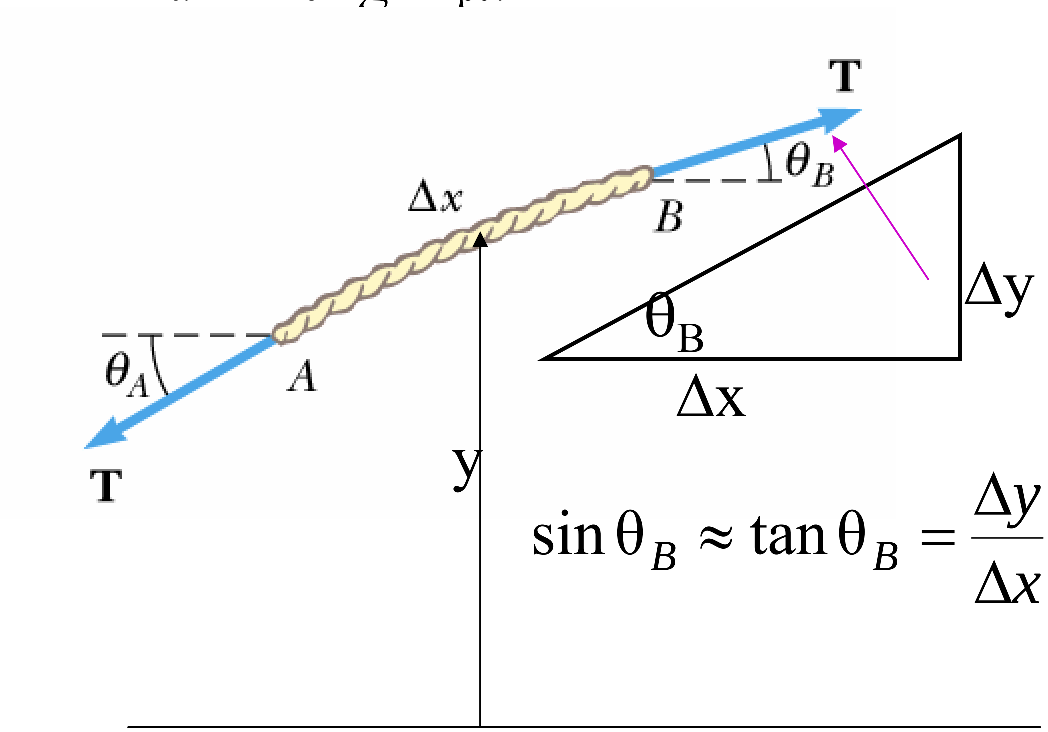
amplitude

phase

$$v = \lambda f = \frac{\omega}{k}$$

Example of mechanical wave motion:

Transverse wave on a string with tension T and mass per unit length μ:



$$\sin\theta_B \approx \tan\theta_B = \left.\frac{\Delta y}{\Delta x}\right|_B$$

$$m\frac{d^2y}{dt^2} = T\sin\theta_B - T\sin\theta_A$$

$$m \approx \mu\Delta x$$

$$\Rightarrow \mu\Delta x\frac{d^2y}{dt^2} \approx T\left(\left.\frac{\Delta y}{\Delta x}\right|_B - \left.\frac{\Delta y}{\Delta x}\right|_A\right)$$

$$\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left(\left.\frac{\Delta y}{\Delta x}\right|_B - \left.\frac{\Delta y}{\Delta x}\right|_A\right) = \frac{\partial^2 y}{\partial x^2}$$

$$\boxed{\frac{\partial^2 y}{\partial t^2} = \frac{T}{\mu}\frac{\partial^2 y}{\partial x^2} \qquad \Rightarrow v = \sqrt{\frac{T}{\mu}}}$$

Comparison of mechanical and electromagnetic waves

Mechanical	Electromagnetic
Satisfy wave equation	Satisfy wave equation
v depends upon propagation material.	v depends upon propagation material (or vacuum).
Can be transverse or longitudinal	Can only be transverse
Can only propagate within materials (solids, liquids, gases, strings, etc.)	Can propagate within a vacuum and within some materials.
Doppler effect: $f' = f\dfrac{1 \pm u_O/v}{1 \mp u_S/v}$	Doppler effect: $f' = f\sqrt{\dfrac{1+u/v}{1-u/v}}$

Maxwell's equations:

Coulomb-Gauss law: $\oiint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\varepsilon_0}$

Gauss's for magnetic field: $\oiint \mathbf{B} \cdot d\mathbf{A} = 0$

Biot-Savart-Ampere-Maxwell law: $\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$

Faraday's law: $\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$

Electric and magnetic fluxes:

$\Phi_E \equiv \iint \mathbf{E} \cdot d\mathbf{A} \qquad \Phi_B \equiv \iint \mathbf{B} \cdot d\mathbf{A}$

Maxwell's equations in absence of sources (*Q=0*, *I=0*):

$$\oint\!\!\oint \mathbf{E}\cdot d\mathbf{A} = 0$$

Coulomb-Gauss law

$$\oint\!\!\oint \mathbf{B}\cdot d\mathbf{A} = 0$$

Gauss's for magnetic field

$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}$$

Ampere-Maxwell law

$$\oint \mathbf{E}\cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

Faraday's law

Peer instruction question

How can we have electric and/or magnetic fields without sources? Which of the following statements is not true?

(A) Charges and/or currents are necessary to create electric and magnetic fields.

(B) Electric and magnetic fields can exist far away from charge and/or current sources.

(C) Statements (A) and (B) are both false.

(D) Statements (A) and (B) are both true.

Solutions to Maxwell's equations

"Plane" waves –

- **Mathematically and physically simplest solution of Maxwell's equations**
- **At each instant of time, E and B fields are uniform in a plane perpendicular to the propagation direction of the wave**

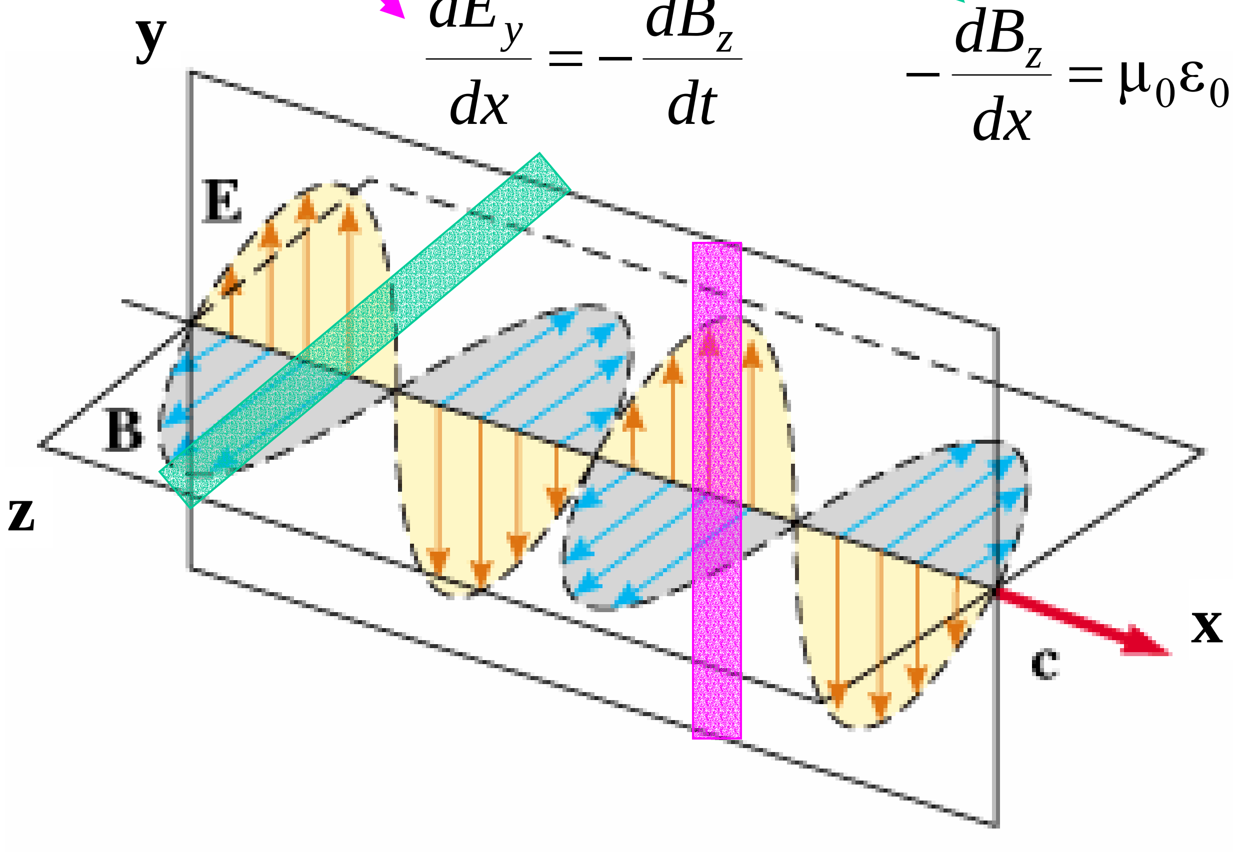
Results – it follows that:

1. **Electromagnetic waves can propagate in a vacuum with a speed $c \cong 3\times10^8$ m/s.**
2. **The E and B fields are perpendicular to each other and to the propagation direction (transverse waves).**
3. **Field magnitudes are related: $|B|=|E|/c$**

“Plane wave” Maxwell’s equations:

$$\oint_{\text{line}} \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt} \qquad \oint_{\text{line}} \mathbf{B} \cdot d\mathbf{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

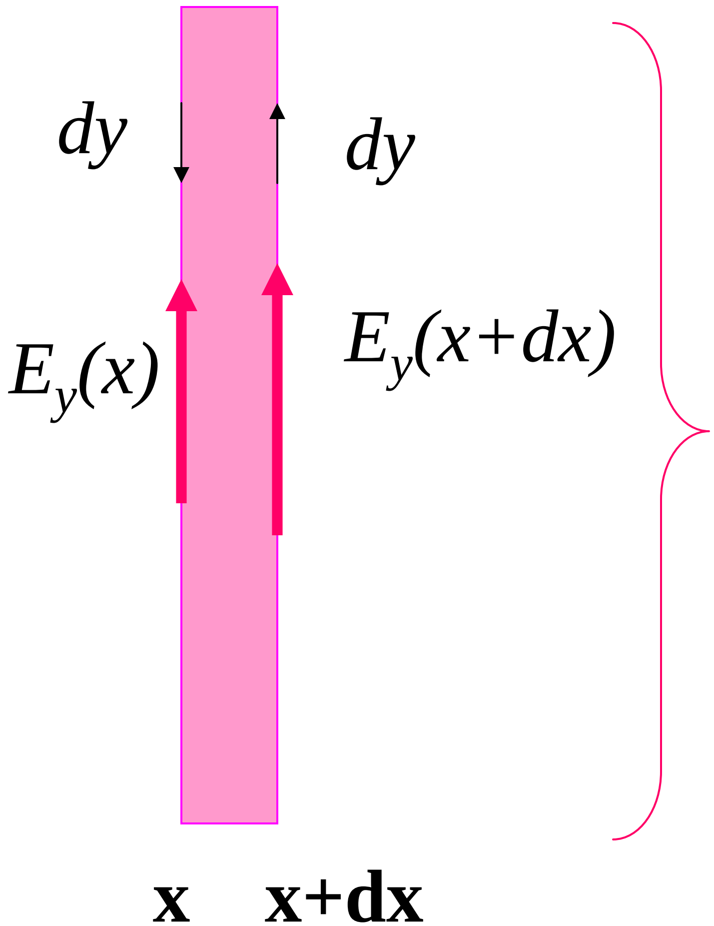
$$\frac{dE_y}{dx} = -\frac{dB_z}{dt} \qquad -\frac{dB_z}{dx} = \mu_0 \varepsilon_0 \frac{dE_y}{dt}$$



Some details:

$$\oint_{\text{line}} \mathbf{E}\cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

$$\left[E_y(x+dx) - E_y(x)\right]L = -\frac{\partial B_z}{\partial t} L\,dx$$



$$\textbf{L} \Rightarrow \frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$

More details –

Differential form of Faraday's and Maxwell-Ampere's laws:

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \qquad -\frac{\partial B_z}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t}$$

Solving for E_y or B_z:

$$\frac{\partial^2 E_y}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \frac{\partial^2 E_y}{\partial x^2} \qquad \frac{\partial^2 B_z}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \frac{\partial^2 B_z}{\partial x^2}$$

$$v \equiv c = \sqrt{\frac{1}{\mu_0 \varepsilon_0}} = 2.99792458 \times 10^8 \, m/s$$

- **Both E_y and B_z satisfy a wave equation with the same wave velocity c**
- **The E and B fields are perpendicular to each other**

Wave equations:

$$\frac{\partial^2 E_y}{\partial t^2} = c^2 \frac{\partial^2 E_y}{\partial x^2} \qquad \frac{\partial^2 B_z}{\partial t^2} = c^2 \frac{\partial^2 B_z}{\partial x^2}$$

$$v \equiv c = \sqrt{\frac{1}{\mu_0 \varepsilon_0}} = 2.99792458 \times 10^8 \, m/s$$

Periodic wave solutions:

$$E_y(x,t) = E_{\max} \sin\left(\frac{2\pi}{\lambda}(x - ct) + \varphi\right) = E_{\max} \sin(kx - \omega t + \varphi)$$

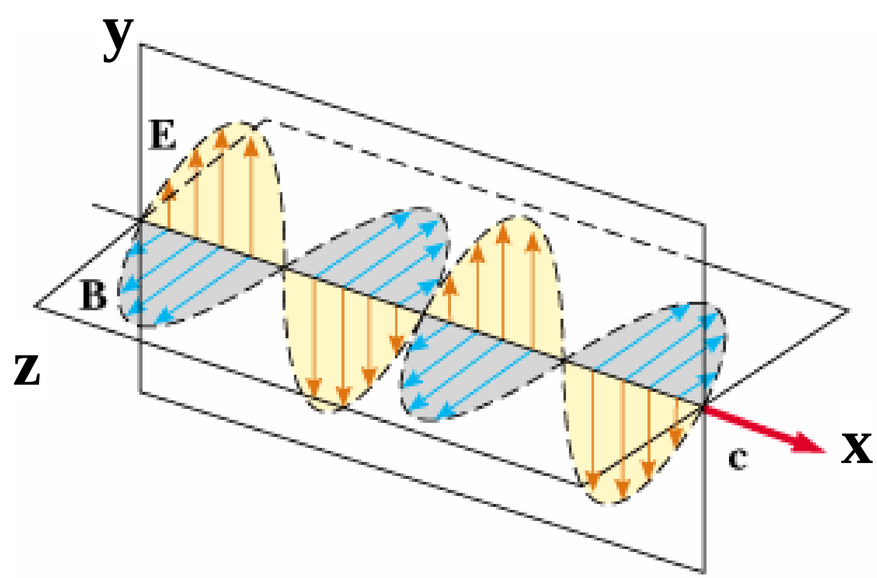
$$B_z(x,t) = \frac{E_{\max}}{c} \sin\left(\frac{2\pi}{\lambda}(x - ct) + \varphi\right) = \frac{E_{\max}}{c} \sin(kx - \omega t + \varphi)$$

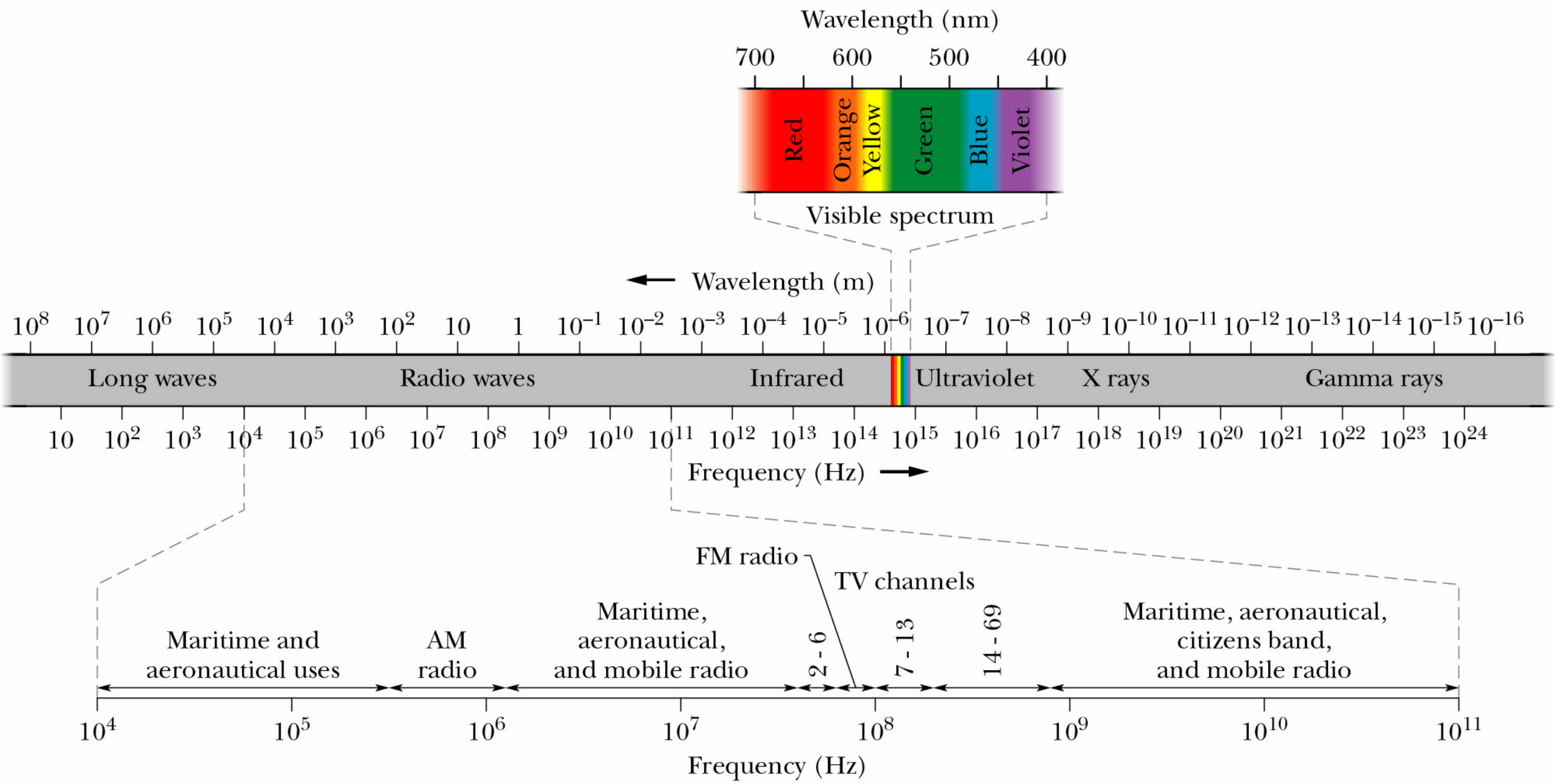
Summary of results for linearly polarized electromagnetic plane waves: (Setting phase factor φ = 0)

$$E_y(x,t) = E_{\max}\sin\left(\frac{2\pi}{\lambda}(x-ct)\right) = E_{\max}\sin\left(\frac{2\pi x}{\lambda} - 2\pi f t\right) = E_{\max}\sin(kx-\omega t)$$

$$B_z(x,t) = \frac{E_{\max}}{c}\sin\left(\frac{2\pi}{\lambda}(x-ct)\right) = \frac{E_{\max}}{c}\sin\left(\frac{2\pi x}{\lambda} - 2\pi f t\right) = \frac{E_{\max}}{c}\sin(kx-\omega t)$$

$$\lambda f = \frac{\omega}{k} = c = \sqrt{\frac{1}{\mu_0\varepsilon_0}} = 2.99792458\times 10^8\, m/s$$





Wavelength (nm)
700 600 500 400
Red Orange Yellow Green Blue Violet
Visible spectrum
Wavelength (m)
10^8 10^7 10^6 10^5 10^4 10^3 10^2 10 1 10^-1 10^-2 10^-3 10^-4 10^-5 10^-6 10^-7 10^-8 10^-9 10^-10 10^-11 10^-12 10^-13 10^-14 10^-15 10^-16
Long waves
Radio waves
Infrared
Ultraviolet
X rays
Gamma rays
10 10^2 10^3 10^4 10^5 10^6 10^7 10^8 10^9 10^10 10^11 10^12 10^13 10^14 10^15 10^16 10^17 10^18 10^19 10^20 10^21 10^22 10^23 10^24
Frequency (Hz)
FM radio
TV channels
Maritime and aeronautical uses
AM radio
Maritime, aeronautical, and mobile radio
2 - 6
7 - 13
14 - 69
Maritime, aeronautical, citizens band, and mobile radio
10^4 10^5 10^6 10^7 10^8 10^9 10^10 10^11
Frequency (Hz)

Energy and forces associated with electromagnetic waves:

Energy density of electromagnetic wave

$$u \equiv u_E + u_B = \tfrac{1}{2}\varepsilon_0 E^2 + \tfrac{1}{2} B^2/\mu_0$$

$$= \tfrac{1}{2}\left(\varepsilon_0 E_{\max}^2 + B_{\max}^2/\mu_0\right)\sin^2\left(\frac{2\pi x}{\lambda} - 2\pi f t + \varphi\right)$$

Time averaged energy density:

(noting that $B_{\max} = E_{\max}/c = \sqrt{\varepsilon_0\mu_0}E_{\max}$)

$$\langle u \rangle_{avg} = \tfrac{1}{2}\varepsilon_0 E_{\max}^2 = \tfrac{1}{2} B_{\max}^2/\mu_0$$

units: joules/m^3

Peer instruction question

Suppose an electromagnetic field has an electric field amplitude of E_{max} = 40 N/C, what is the average energy density associated with this radiation (in units of Joules/m^3)?

(A) 8×10^{-26} (B) 7×10^{-9} (C) 800 (D) None of these

Energy and forces associated with electromagnetic waves:

"Flow" of energy -- Poynting vector

$$\mathbf{S} \equiv \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$$

$$\mathbf{S} = \frac{\hat{\mathbf{x}}}{\mu_0} E_{\max} B_{\max} \sin^2\left(\frac{2\pi x}{\lambda} - 2\pi f t + \varphi \right)$$

$$\langle \mathbf{S} \rangle_{avg} = \frac{\hat{\mathbf{x}}}{2\mu_0 c} E_{\max}^2 = \frac{\hat{\mathbf{x}} c}{2\mu_0} B_{\max}^2$$

units: (Joules/s)/m^2

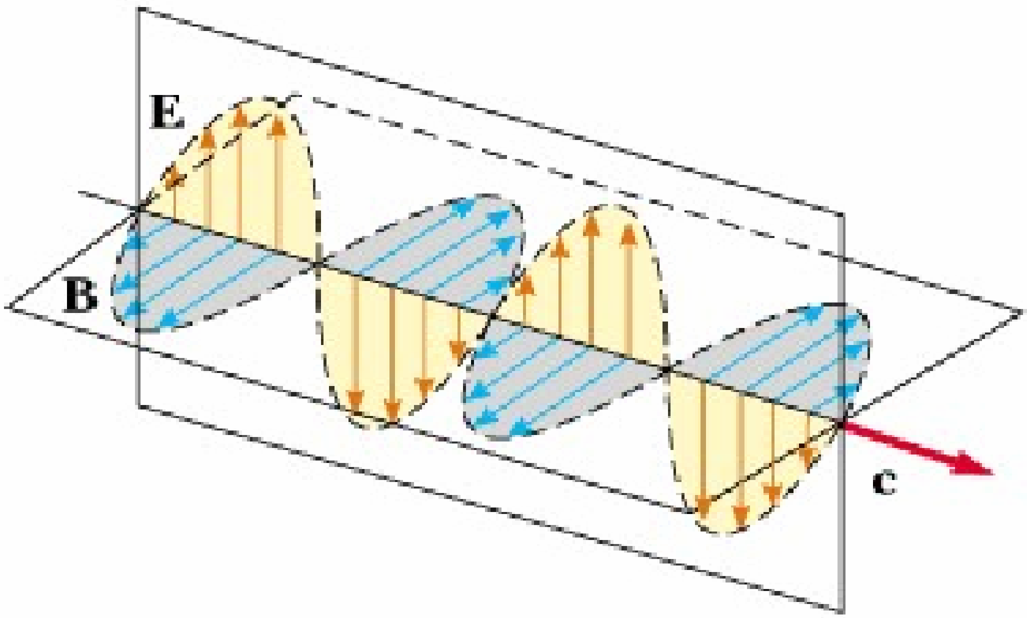
=Watts/m^2

Recalling that :

$$\langle u \rangle_{avg} = \tfrac{1}{2} \varepsilon_0 E_{\max}^2 = \tfrac{1}{2} B_{\max}^2 / \mu_0$$

$$\Rightarrow \langle u \rangle_{avg} = \langle |\mathbf{S}| \rangle_{avg} / c$$

Online Quiz for Lecture 19
Electromagnetic waves -- Mar. 14, 2005



The figure shows a plane-polarized electromagnetic wave with the maximum electric field strength of E_{max}=45 N/C.

1. What is the direction of the Poynting vector for this wave?
   (A) in **E** direction (B) in **B** direction (C) in direction labeled "c" (D) None of these.
2. What is the maximum magnetic field strength?
   (A) 1.5×10^{-7} T (B) 1.35×10^{10} (C) None of these.
3. What is the magnitude of the Poynting vector in units of Watts/m^2 averaged over several complete cycles?
   (A) 1.35×10^{-5} (B) 2.7 (C) 1013 (D)None of these.