Announcements

1. Schedule:
   - This week – continuing to study electromagnetic waves (Chap. 34)
   - Problem solving session tonight at 6 PM in Olin 101
   - Wednesday 3/23/05: Exam #3 (practice exam available)

2. Special lectures this week – Prof. Clifford Will from Washington University will give physics colloquium and public lecture on Thursday, 3/17/05.

3. Today's topic – Chapter 34
   Maxwell's equations and electromagnetic radiation
   - Polarization properties of EM waves
   - Refraction and reflection of EM waves

Maxwell's equations:

$$\oiint \mathbf{E} \cdot d\mathbf{A} = Q / \varepsilon_0$$
Coulomb-Gauss law

$$\oiint \mathbf{B} \cdot d\mathbf{A} = 0$$
Gauss's for magnetic field

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$
Ampere-Maxwell law

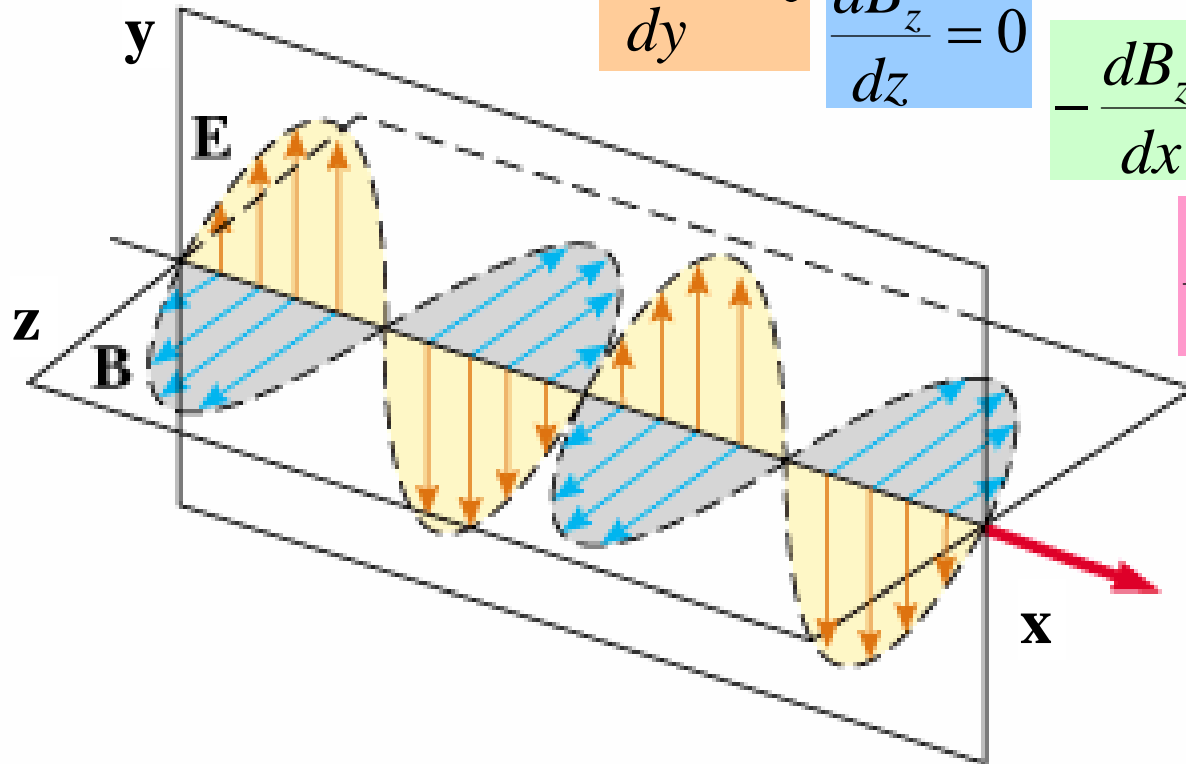$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$
Faraday's law

Maxwell's equations far away from sources ($Q=0$, $I=0$):

$$\oiint \mathbf{E} \cdot d\mathbf{A} = 0 \qquad \oiint \mathbf{B} \cdot d\mathbf{A} = 0 \qquad \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \varepsilon_0 \frac{d\left(\iint \mathbf{E} \cdot d\mathbf{A}\right)}{dt} \qquad \oint \mathbf{E} \cdot d\mathbf{s} = - \frac{d\left(\iint \mathbf{B} \cdot d\mathbf{A}\right)}{dt}$$

Plane-polarized solution:

$$\frac{dE_y}{dy} = 0 \qquad \frac{dB_z}{dz} = 0$$

$$-\frac{dB_z}{dx} = \mu_0 \varepsilon_0 \frac{dE_y}{dt}$$

$$\frac{dE_y}{dx} = -\frac{dB_z}{dt}$$

Results:

$$-\frac{dB_z}{dx} = \mu_0\varepsilon_0\frac{dE_y}{dt} \qquad \frac{dE_y}{dx} = -\frac{dB_z}{dt}$$

Wave equations for E_y and B_z:

$$\frac{\partial^2 E_y}{\partial t^2} = \frac{1}{\mu_0\varepsilon_0}\frac{\partial^2 E_y}{\partial x^2} \qquad \frac{\partial^2 B_z}{\partial t^2} = \frac{1}{\mu_0\varepsilon_0}\frac{\partial^2 B_z}{\partial x^2}$$

$$\Rightarrow v \equiv c = \sqrt{\frac{1}{\mu_0\varepsilon_0}} = 2.99792458\times10^8\, m/s$$

Periodic solution:

$$E_y(x,t) = E_{max}\sin\left(\frac{2\pi}{\lambda}(x-ct)\right) = E_{max}\sin(kx-\omega t)$$

$$B_z(x,t) = \frac{E_{max}}{c}\sin\left(\frac{2\pi}{\lambda}(x-ct)\right) = \frac{E_{max}}{c}\sin(kx-\omega t)$$

Summary of significant properties of electromagnetic waves:

➡ "Self-sustaining" electric and magnetic fields which can propagate in vacuum at a velocity of $c = 2.99792458 \times 10^{8}$ m/s (or within matter at a velocity of $v = c/n$) .

➡ **E** and **B** fields are perpendicular to each other and perpendicular to propagation direction (transverse waves).
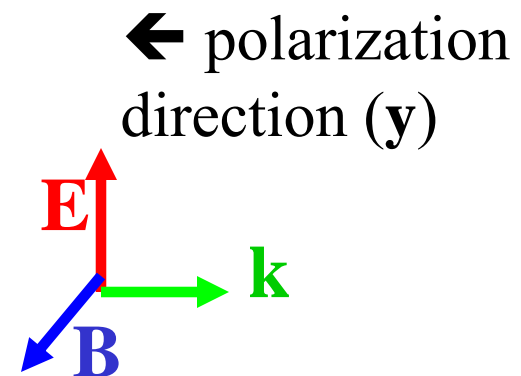
➡ $|\mathbf{E}(x,t)| = \dfrac{|\mathbf{B}(x,t)|}{v}$

➡ Periodic waves have the form:

$$E_y(x,t) = E_{max}\sin(kx - \omega t)$$

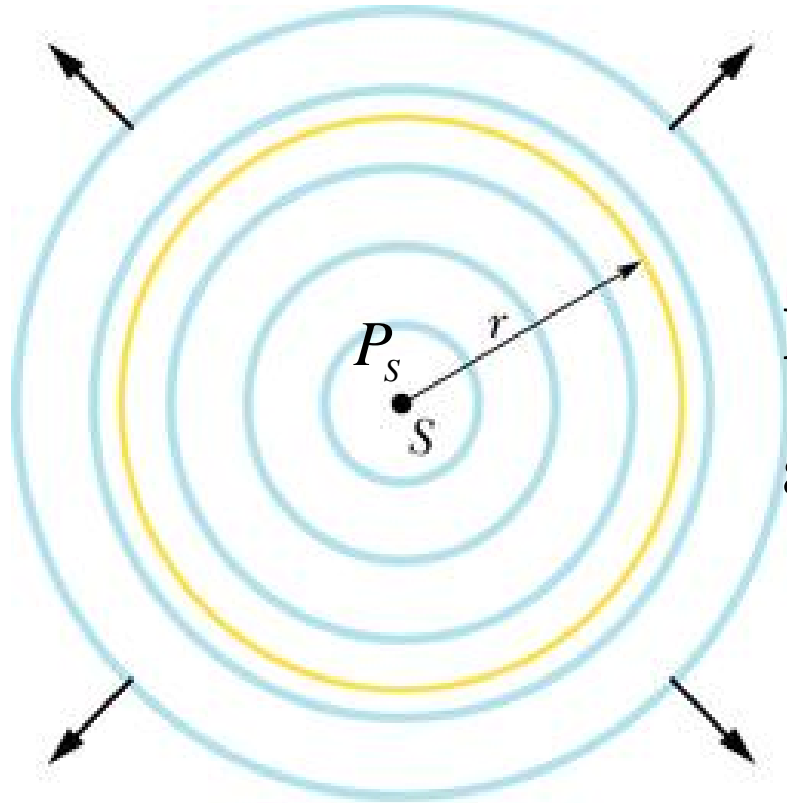$$B_z(x,t) = \frac{E_{max}}{v}\sin(kx - \omega t)$$

$$\frac{\omega}{k} = v$$

← polarization direction (**y**)

E

k

B

Radiation power

Poyntingvector: $\mathbf{S} = \dfrac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$ **units: Watts/area**

pointing in propagation direction

Intensity : $I \equiv \left\langle \left| \mathbf{S} \right| \right\rangle_{avg}$

Intensity for plane wave : $I = \dfrac{E_{max}^2}{2c\mu_0}$
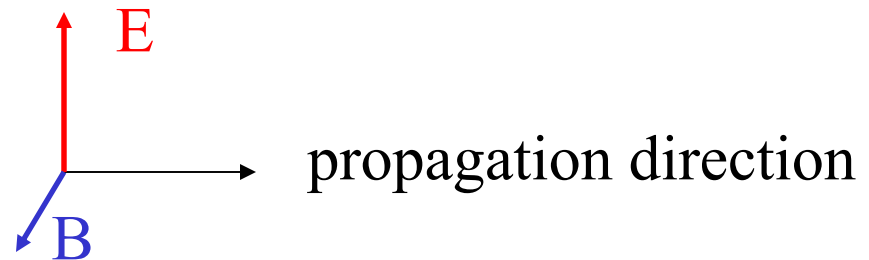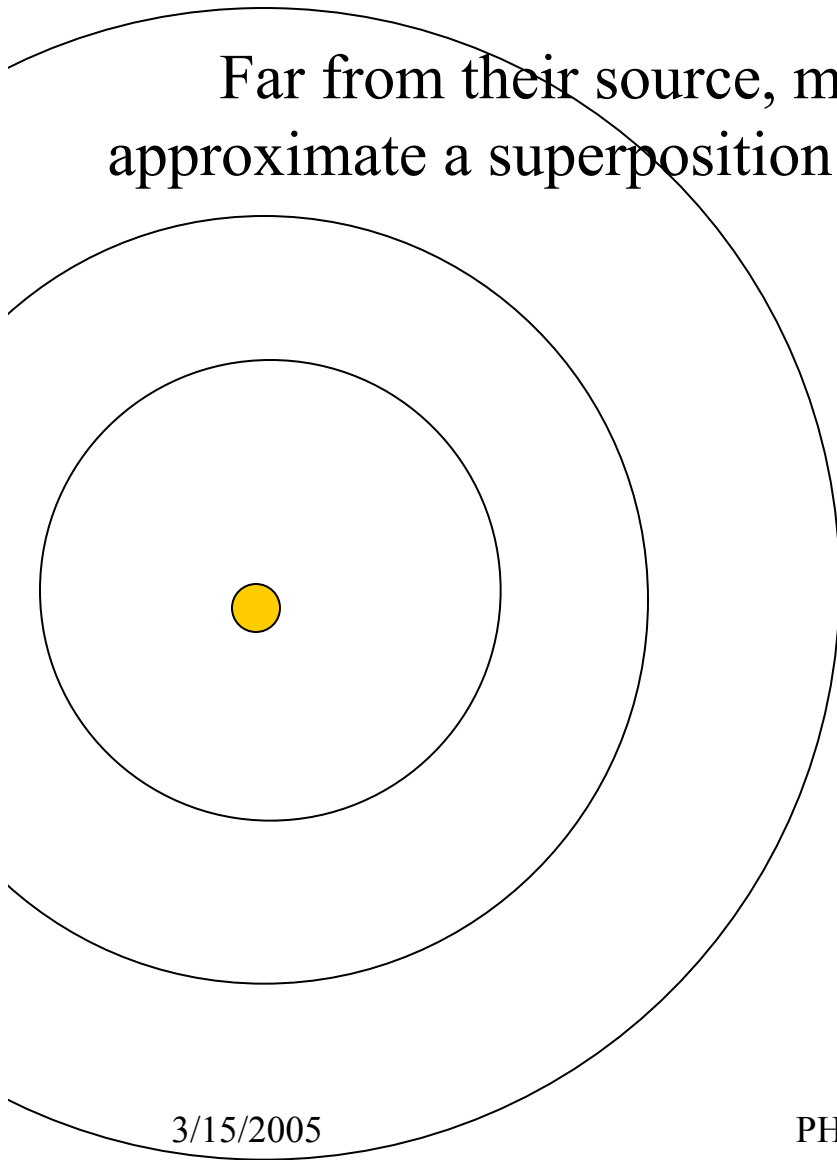
due to averaging

Radiation power from point source

Intensity from point source
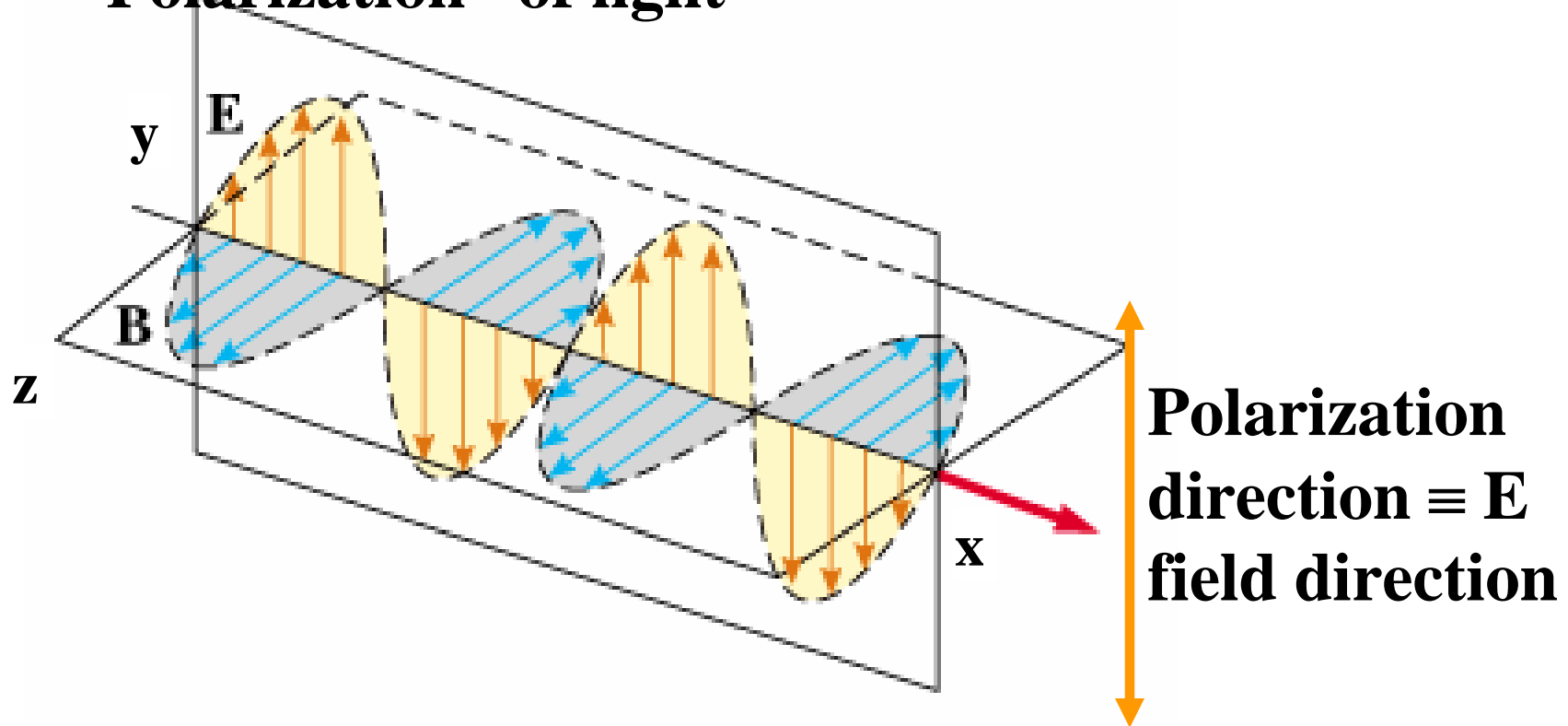
at a distance *r:*

$$I = \frac{P_s}{4\pi r^2}$$

Examples of plane wave electromagnetic radiation:

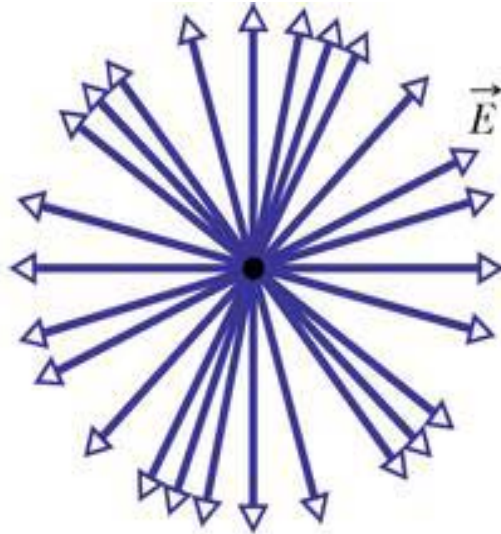Far from their source, most electromagnetic waves approximate a superposition of plane waves.

E

propagation direction

B

Laser?

"Polarization" of light

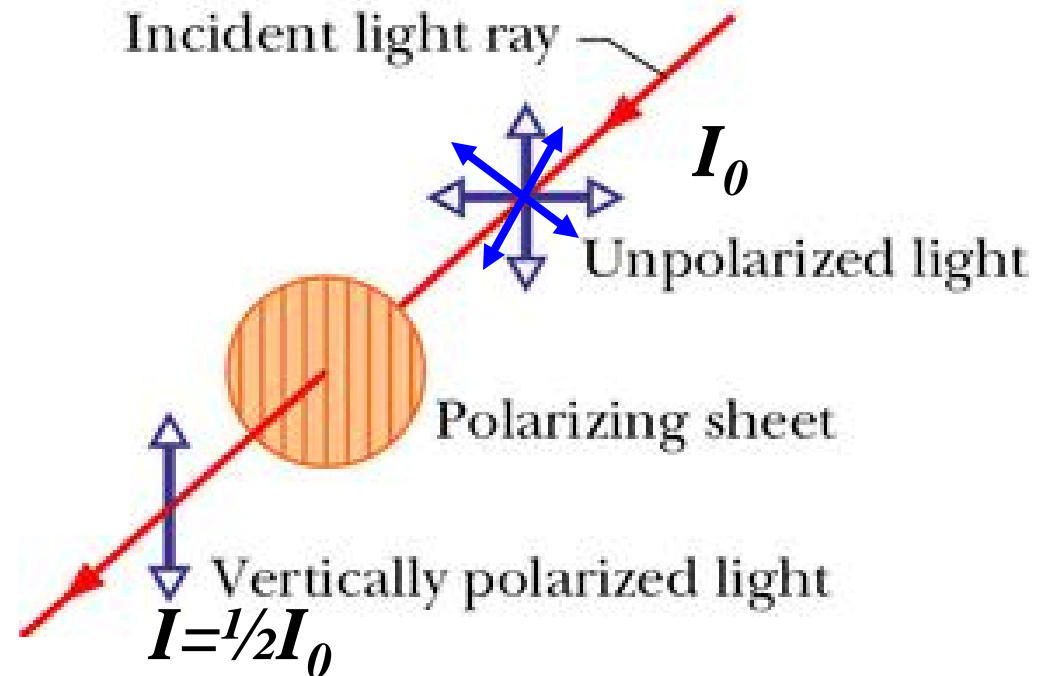Polarization direction $\equiv$ E field direction

Some radiation sources produce polarized light –

For example: lasers, antennas

Many sources produced *un*polarized light

Creating polarized light by filtering

Incident light ray

I_0

Unpolarized light

Polarizing sheet

Vertically polarized light

$I=\frac{1}{2}I_0$

Filtration of polarized light as a function of angle

Polarizing direction

θ

P_1

$I_1 = \frac{1}{2} I_0$

P_2

$I_2 = I_1 \cos^2\theta = \frac{1}{2} I_0 \cos^2\theta$
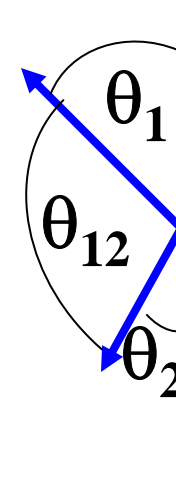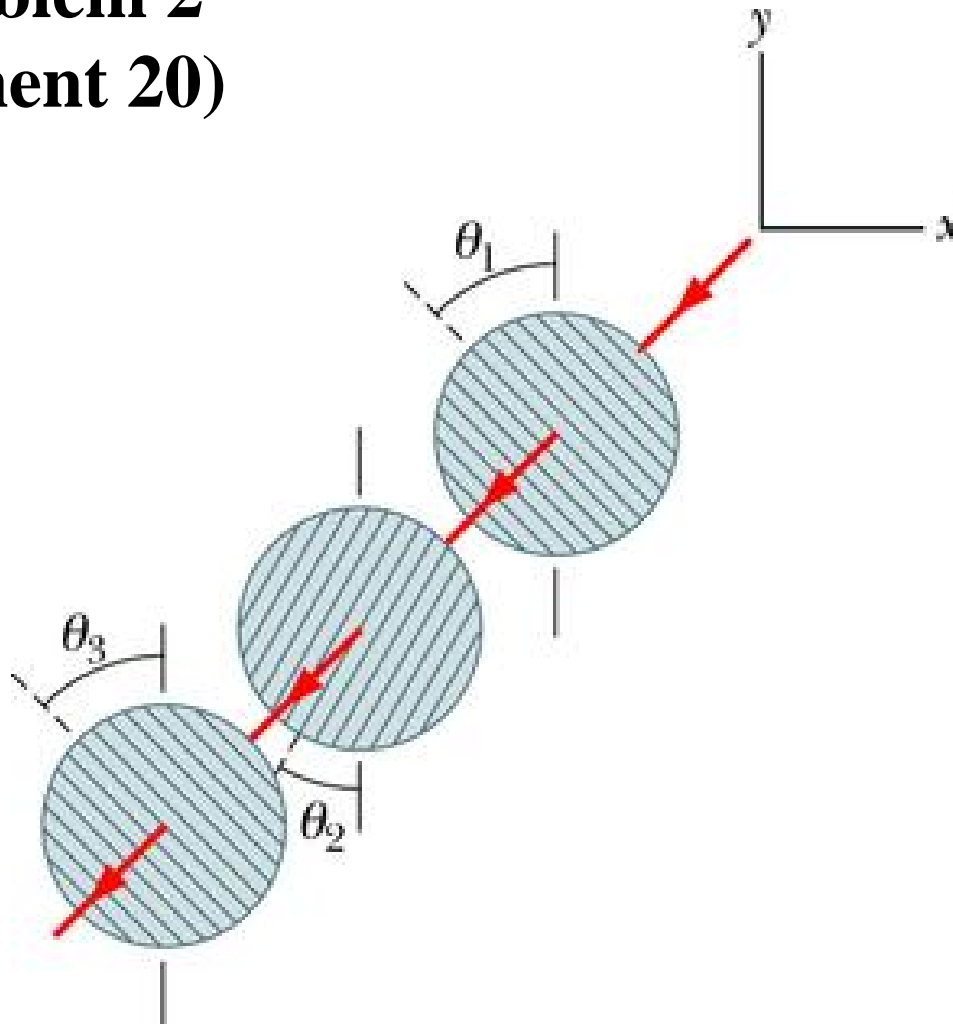
E_1

θ

$E_2 = E_1 \cos\theta$

(a)

(b)

The figure shows a photo of polarized sunglasses from your text. The next time you have to get new sunglasses, will you get polorized sunglasses or regular sunglasses? Put you answer and a brief comment in the box.

HW problem 2 (assignment 20)

$$\theta_{12}=180-\theta_1-\theta_2$$

Electromagnetic waves in materials

$$-\frac{dB_z}{dx} = \mu\varepsilon\frac{dE_y}{dt}$$

$$\frac{dE_y}{dx} = -\frac{dB_z}{dt}$$

$$\frac{\partial^2 E_y}{\partial t^2} = \frac{1}{\mu\varepsilon}\frac{\partial^2 E_y}{\partial x^2}$$

$$\frac{\partial^2 B_z}{\partial t^2} = \frac{1}{\mu\varepsilon}\frac{\partial^2 B_z}{\partial x^2}$$

$$\Rightarrow v = \sqrt{\frac{1}{\mu\varepsilon}} = \sqrt{\frac{\mu_0\varepsilon_0}{\mu\varepsilon}}\sqrt{\frac{1}{\mu_0\varepsilon_0}} \equiv \frac{c}{n}$$
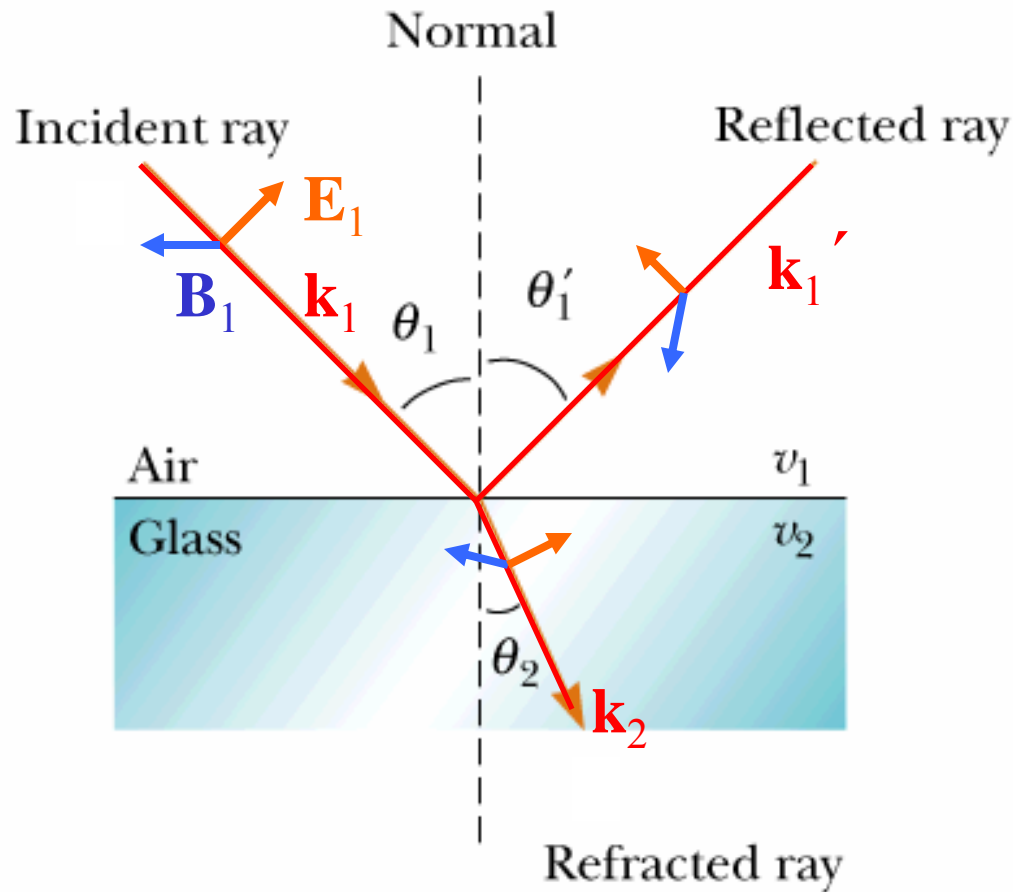
refractive index

TABLE 34-1   Some Indexes of Refraction[a]

Medium	Index	Medium	Index
Vacuum	Exactly 1	Typical crown glass	1.52
Air (STP)[b]	1.00029	Sodium chloride	1.54
Water (20°C)	1.33	Polystyrene	1.55
Acetone	1.36	Carbon disulfide	1.63
Ethyl alcohol	1.36	Heavy flint glass	1.65
Sugar solution (30%)	1.38	Sapphire	1.77
Fused quartz	1.46	Heaviest flint glass	1.89
Sugar solution (80%)	1.49	Diamond	2.42

[a]For a wavelength of 589 nm (yellow sodium light).

[b]STP means "standard temperature (0°C) and pressure (1 atm)."

Consider the behavior of a plane-polarized electromagnetic wave near the surface of two materials:
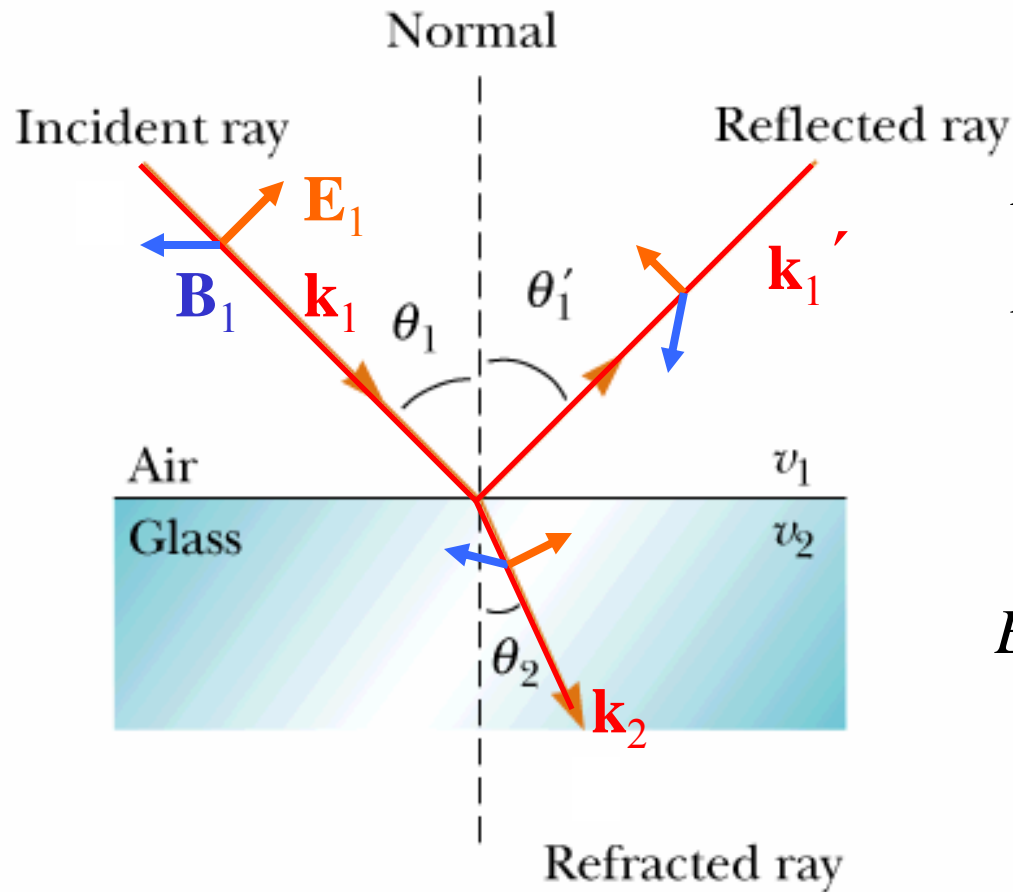
Wave equations

$$\frac{\partial^2 E_y}{\partial t^2} = v_1^2 \frac{\partial^2 E_y}{\partial x^2}$$

$$\frac{\partial^2 E_y}{\partial t^2} = v_2^2 \frac{\partial^2 E_y}{\partial x^2}$$

Consider the behavior of a plane-polarized electromagnetic wave near the surface of two materials -- continued:

Periodic waves:

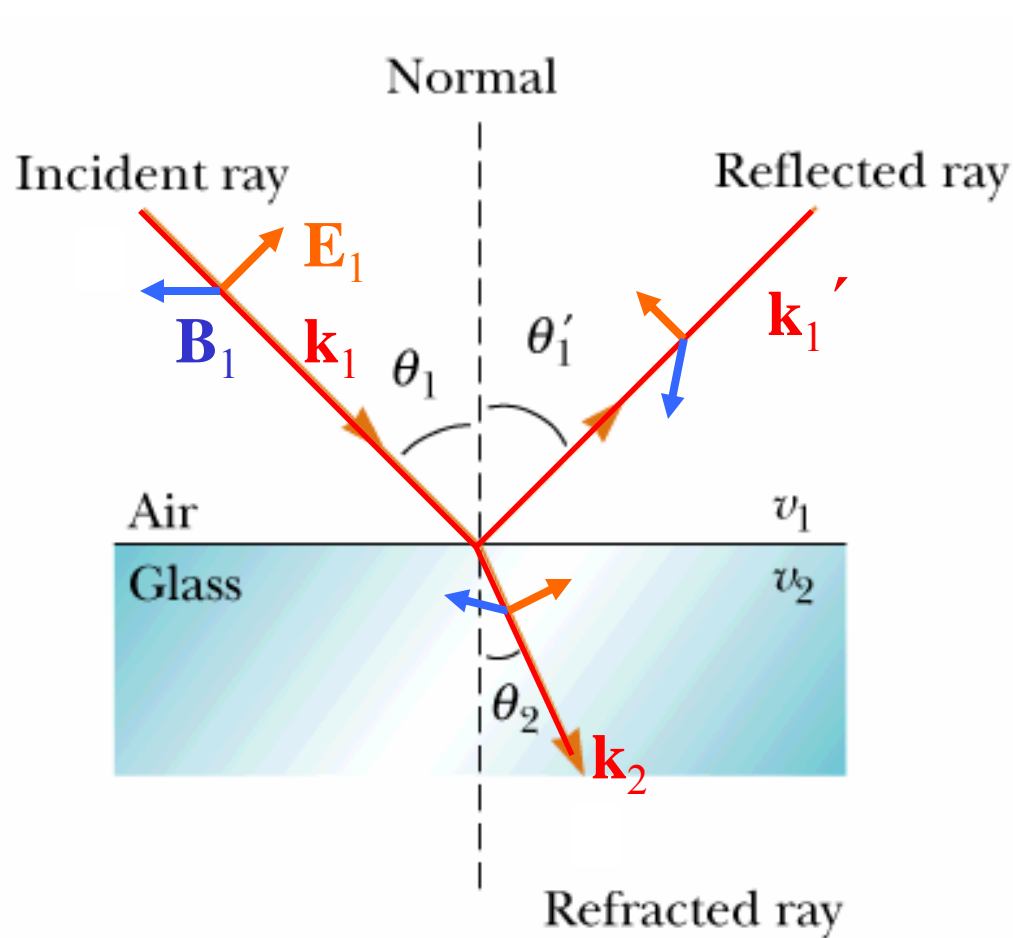$$E_1 = E_{\max_1} \sin(\mathbf{k}_1 \cdot \mathbf{r} - \omega_1 t)$$

$$E_1' = E_{\max_1}' \sin(\mathbf{k}_1' \cdot \mathbf{r} - \omega_1' t)$$

$$\frac{\omega_1}{k_1} = \frac{\omega_1'}{k_1'} = v_1$$

$$E_2 = E_{\max_2} \sin(\mathbf{k}_2 \cdot \mathbf{r} - \omega_2 t)$$

$$\frac{\omega_2}{k_2} = v_2$$

Consider the behavior of a plane-polarized electromagnetic wave near the surface of two materials -- continued:

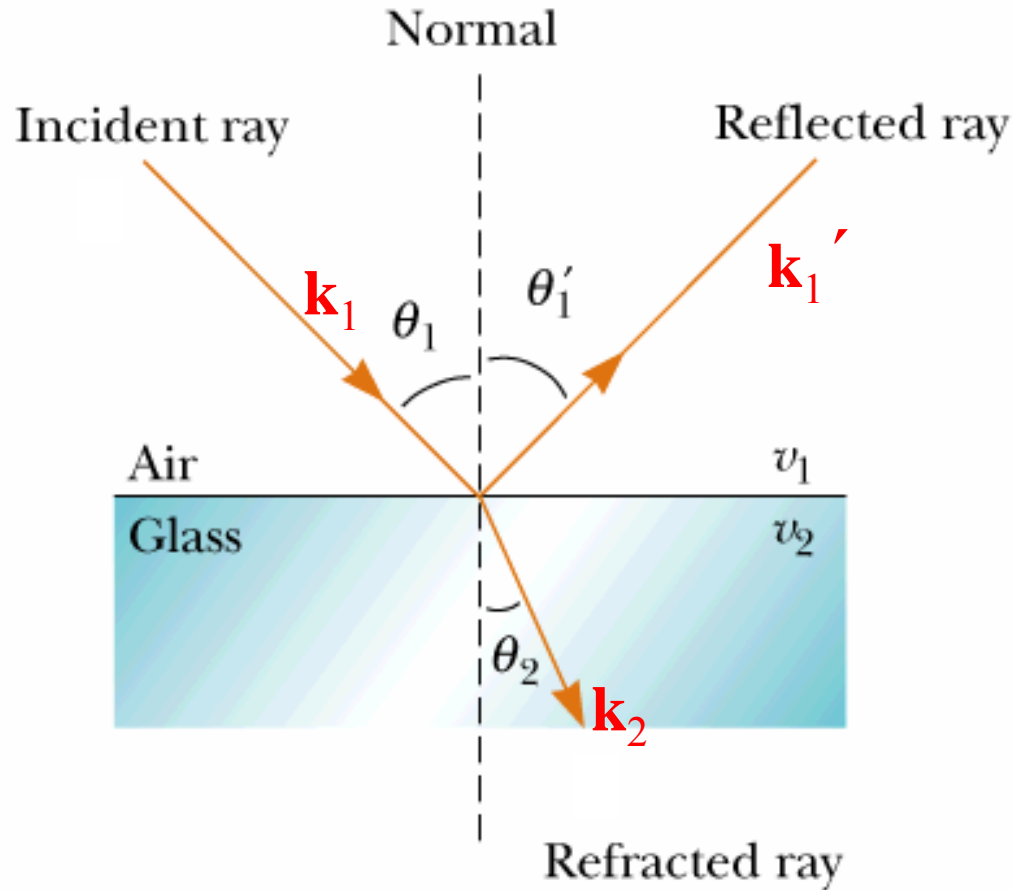$$\frac{\omega_1}{k_1} = \frac{\omega_1'}{k_1'} = v_1$$

$$\frac{\omega_2}{k_2} = v_2$$

If $\quad \omega_1 = \omega_1' = \omega_2$

$\quad\quad k_1 v_1 = k_1' v_1 = k_2 v_2$

Define: $\quad n_1 \equiv c / v_1$

$\quad\quad\quad n_2 \equiv c / v_2$

Results from solving this boundary-value problem --

components of wave vectors in the plane of the surface must be equal $\quad k_1 \sin \theta_1 = k_1' \sin \theta_1' = k_2 \sin \theta_2$
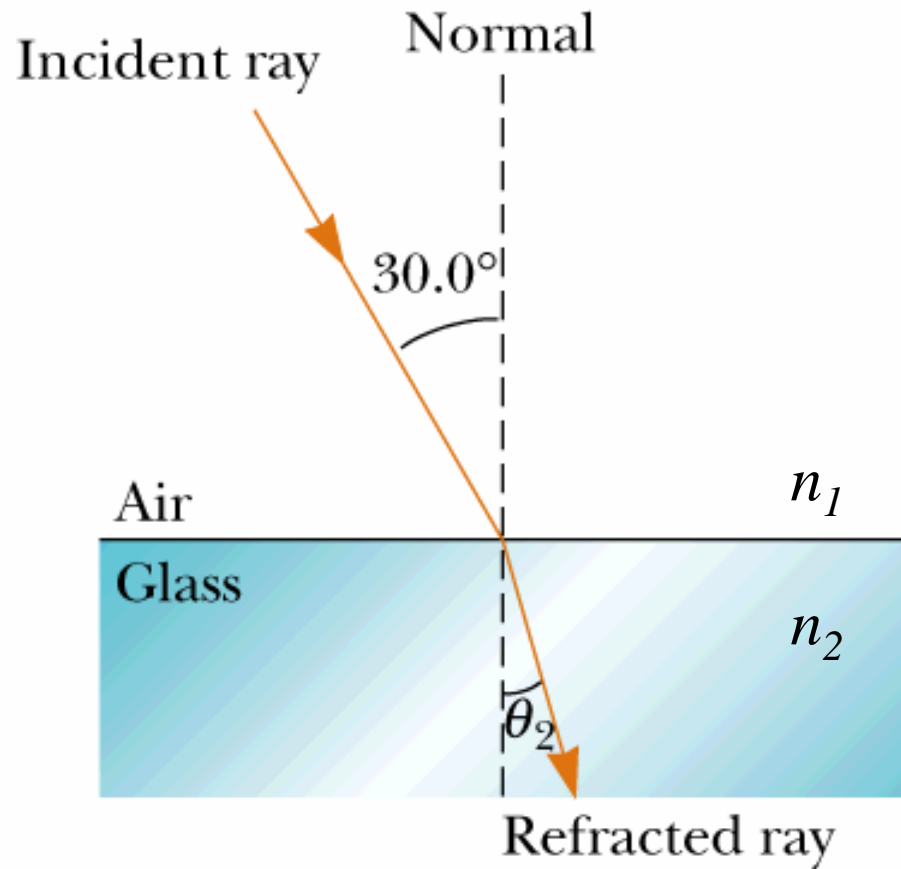

$$k_1 = \frac{n_1 \omega}{c} = k_1'$$

$$k_2 = \frac{n_2 \omega}{c}$$

$$\Rightarrow \theta_1 = \theta_1'$$

$$\Rightarrow n_1 \sin \theta_1 = n_2 \sin \theta_2$$
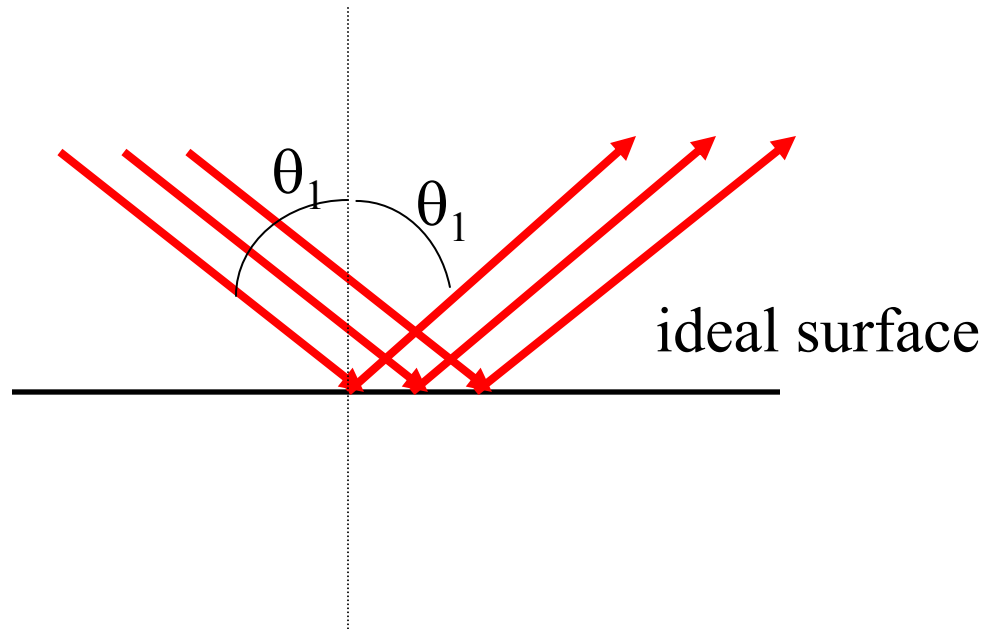
Refraction

Snell's law:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

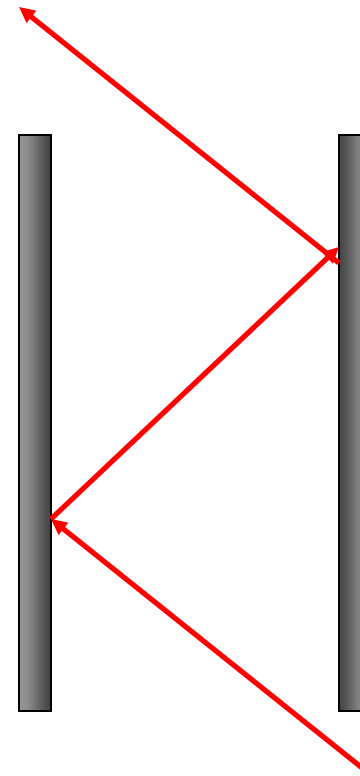$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2} \sin \theta_1\right)$$
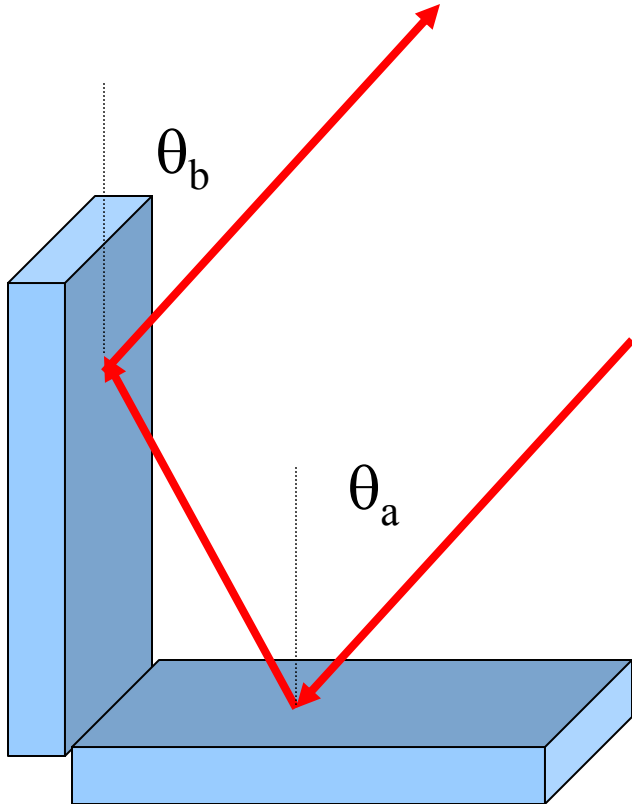
$$= 17.5^o$$

$$\text{for } n_1 = 1, n_2 = 1.66$$

Reflection

Ray tracing

Example: 2 mirrors

θ_1 θ_1

ideal surface

Peer instruction question

What can you say about the relationship between θ_a and θ_b?

(A) $\theta_a = \theta_b$

(B) $\theta_a = 90^o - \theta_b$

(C) Insufficient information.