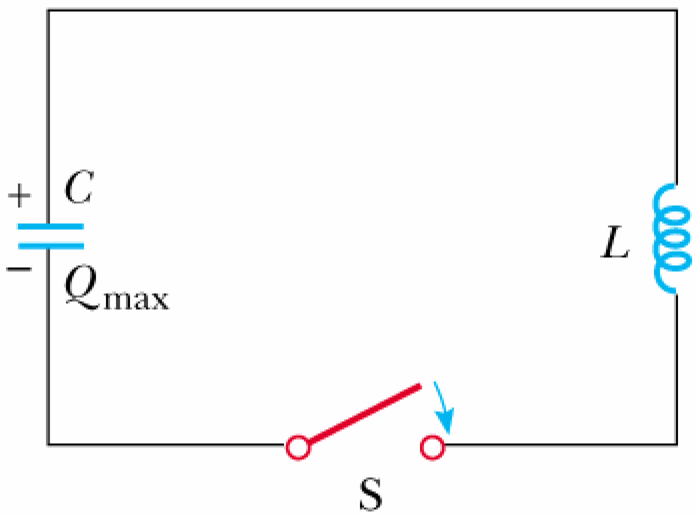
Announcements

1. **Third hour exam – Wednesday, March 23, 2005 – covering Chapters 33-34.**
   - 4 problems – show your work and reasoning for possible partial credit.
   - May bring one 8½" x 11" sheet of paper to the exam (to be turned in with your exam papers).
   - Should also bring clear head
2. Example exam available on website.
3. Today's lecture –
   Advice for studying Systematic review

List of topics

1. Circuits with L, C, and R components and emf source (either constant or oscillating in time)
   a. Current-voltage relationships
   b. Transient and steady-state effects
   c. Frequency dependent effects
   d. Energy stored or dissipated
2. Electromagnetic waves as a consequence of Maxwell's equations
   a. Relationship between Maxwell's equations and the wave equation
   b. Vector and functional relationships of **E**, **B**, and **k** and *c*.
   c. Poynting vector, intensity, and power.
   d. Refractive index
   e. Reflection and refraction of EM waves at surfaces

LC circuit



Resonant frequency:

$$\omega_0 = \sqrt{\frac{1}{LC}} = 2\pi f_0 = \frac{2\pi}{T_0}$$

$$-\frac{q}{C} - L\frac{d^2q}{dt^2} = 0$$

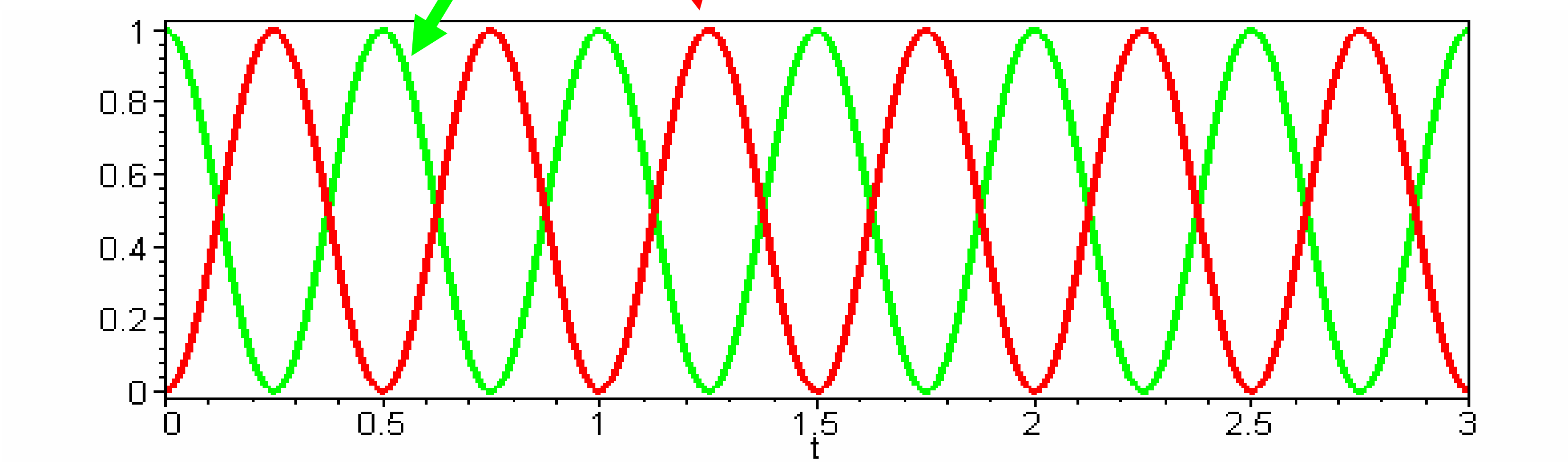
$$q(t) = Q_{max}\cos(\omega_0 t + \varphi)$$

$$i(t) = \frac{dq}{dt} = -\underbrace{\omega_0 Q_{max}}_{I_{max}}\sin(\omega_0 t + \varphi)$$

Energy in LC circuits

Energy in capacitor : $U_E = \frac{q^2}{2C} = \frac{Q_{\max}^2 \cos^2(\omega_0 t + \varphi)}{2C}$

Energy in inductor : $U_B = \frac{Li^2}{2} = \frac{\omega_0^2 L Q_{\max}^2 \sin^2(\omega_0 t + \varphi)}{2}$



Total energy in LC circuit:

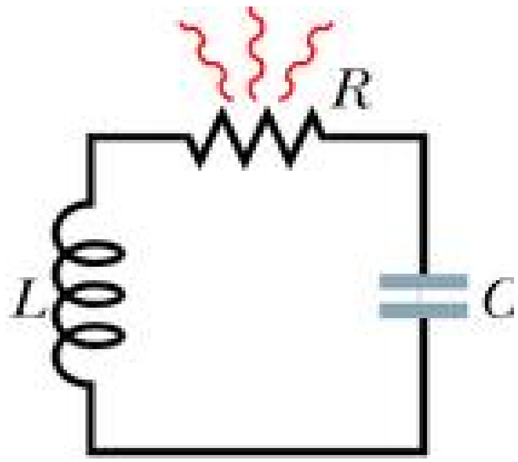
$$U_E + U_B = \frac{q^2}{2C} + \frac{Li^2}{2}$$

$$= \frac{Q_{\max}^2}{2C}\left(\cos^2(\omega_0 t + \varphi) + \sin^2(\omega_0 t + \varphi)\right)$$

$$= \frac{Q_{\max}^2}{2C} = \frac{LI_{\max}^2}{2}$$

$$\text{Note:} \quad \omega_0{}^2 L = \frac{1}{LC} L = \frac{1}{C}$$

Effects of resistance in the circuit



$$-\frac{q}{C} - L\frac{di}{dt} - Ri = 0$$

$$-\frac{1}{C}q - L\frac{d^2q}{dt^2} - R\frac{dq}{dt} = 0$$

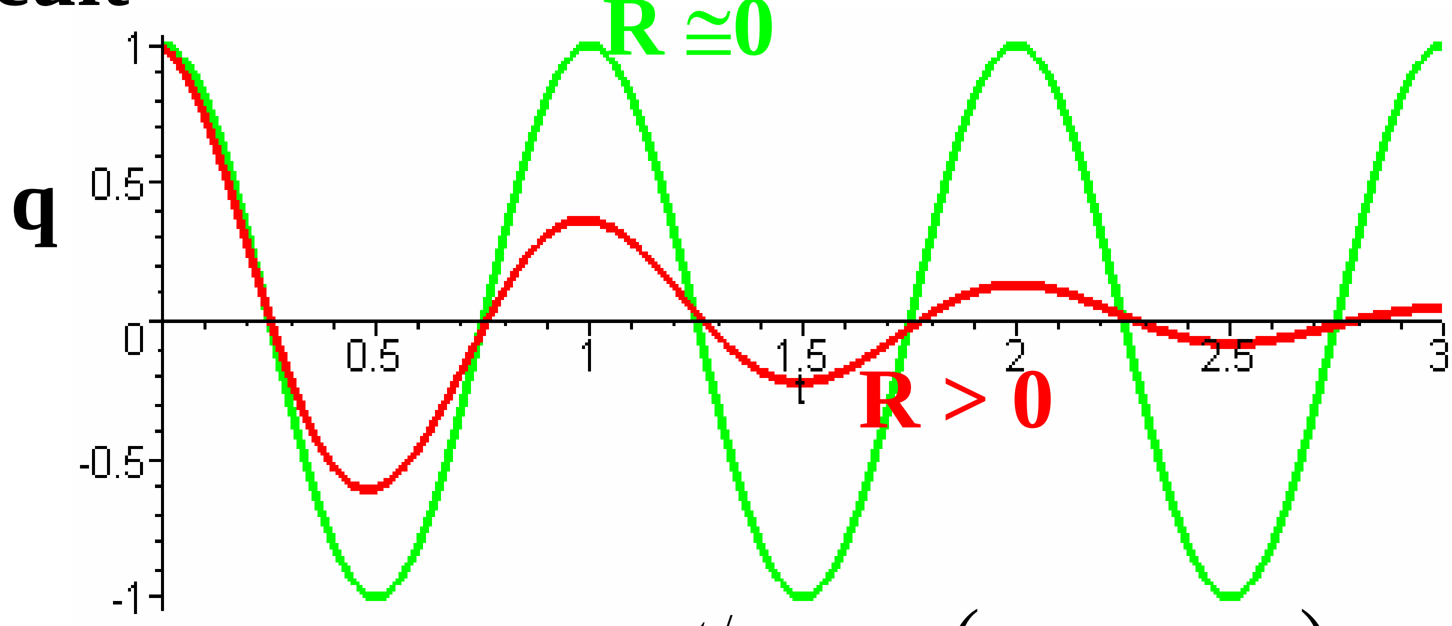
Solution :

$$q(t) = Q_{\max} e^{-t/\tau} \cos(\varpi_0 t + \varphi)$$

constants

$$\tau = \frac{2L}{R} \qquad \varpi_0 = \sqrt{\omega_0^2 - 1/\tau^2}$$

LCR circuit

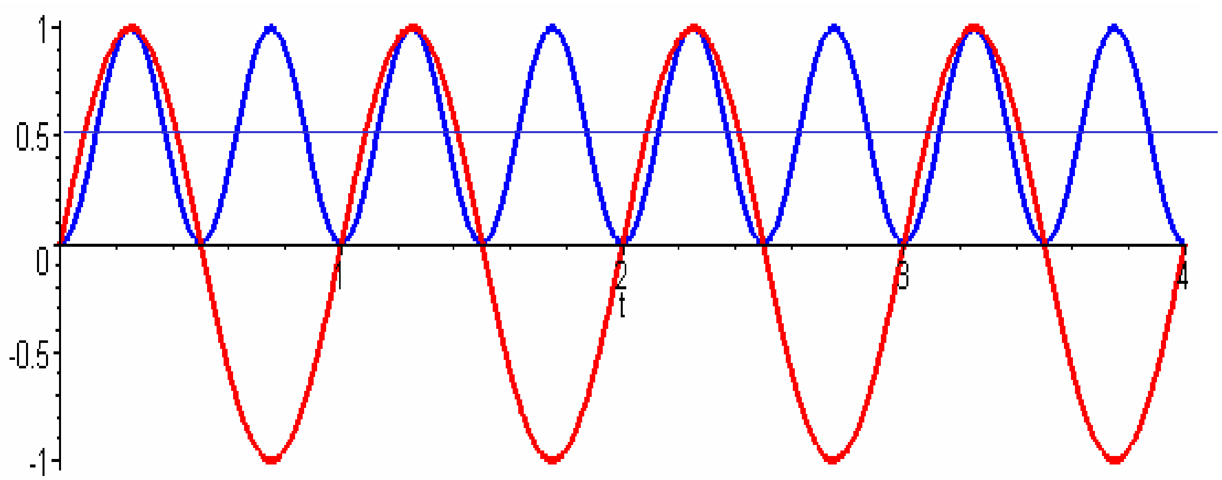


$$q(t) = Q_{\max} e^{-t/\tau} \cos(\varpi_0 t + \varphi)$$

$$\tau = \frac{2L}{R} \qquad \varpi_0 = \sqrt{\omega_0^2 - 1/\tau^2}$$

Properties of AC circuits

$$\mathcal{E}=\mathcal{E}_{max} \sin \omega t \quad \text{or} \quad \mathcal{E}_{max} \cos \omega t$$



$[(\sin \omega t)^2]_{avg} = ½$

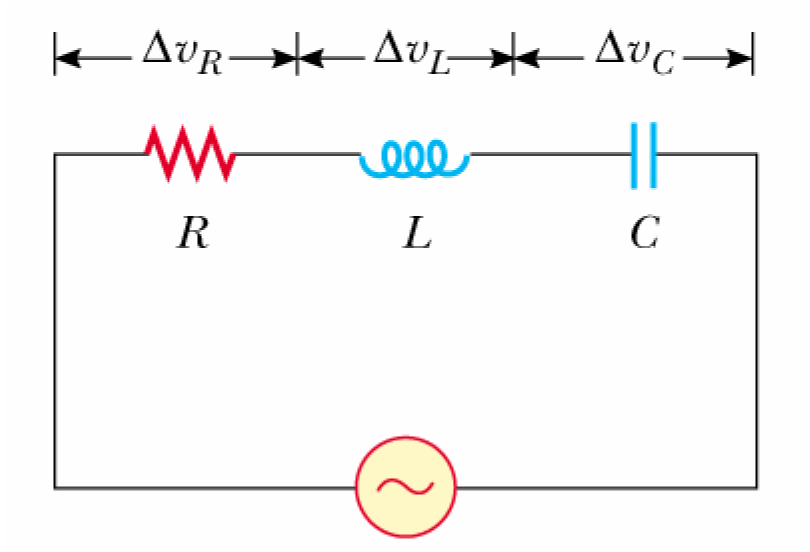
$[\sin \omega t]_{avg} = 0$

$$\left\langle |\mathcal{E}|^2 \right\rangle_{avg} = \tfrac{1}{2} |\mathcal{E}_{max}|^2 \qquad \mathcal{E}_{rms} = \sqrt{\left\langle |\mathcal{E}|^2 \right\rangle_{avg}}$$

$$\mathcal{E}_{rms} = \sqrt{\tfrac{1}{2}}\ \mathcal{E}_{max} \qquad \text{similarly,} \qquad I_{rms} = \sqrt{\tfrac{1}{2}}\ I_{max}$$

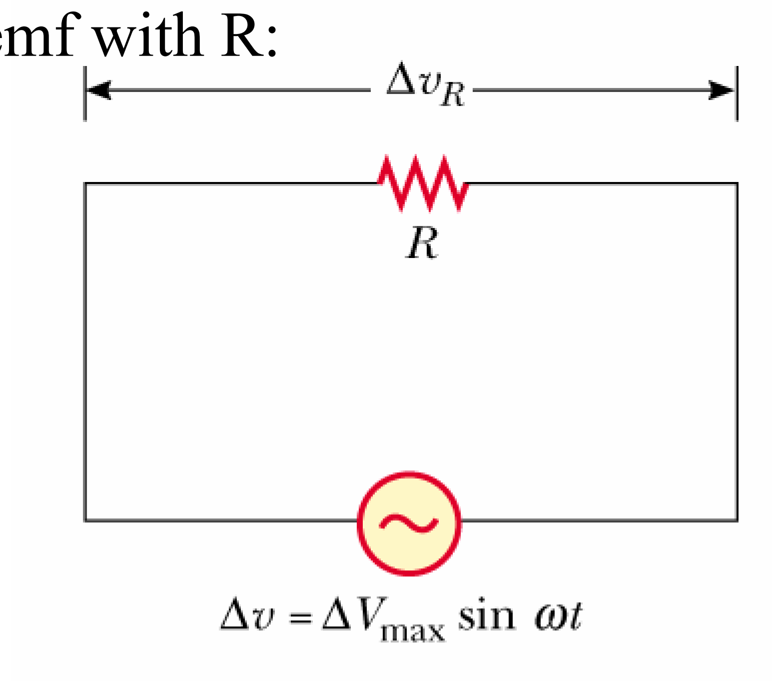
DC circuits	AC circuits
$\mathcal{E}$ = constant in time	$\mathcal{E} = \mathcal{E}_{max}$ sin ωt or $\mathcal{E}_{max}$ cos ωt
$I = \begin{cases} \text{constant in time} \\ \text{constant} + I_0 e^{-t/\tau} \\ \text{damped oscillations} \end{cases}$	I= transients + I_0 sin (ωt−ϕ)
Kirchhoff's rules apply	Kirchhoff's rules apply

Analysis of LCR circuits with AC emf:



Consider each component separately with AC emf --

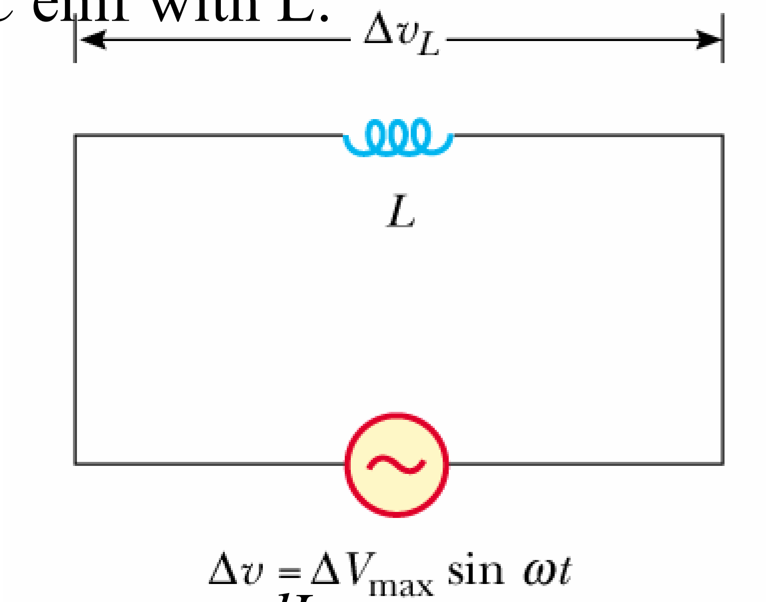
AC emf with R:



$$-IR + \Delta V_{\max} \sin \omega t = 0 \qquad \text{Note:} \quad \mathcal{E}_{\max} \leftrightarrow \Delta V_{\max}$$

$$I = \frac{\Delta V_{\max}}{R} \sin \omega t$$

AC emf with L:

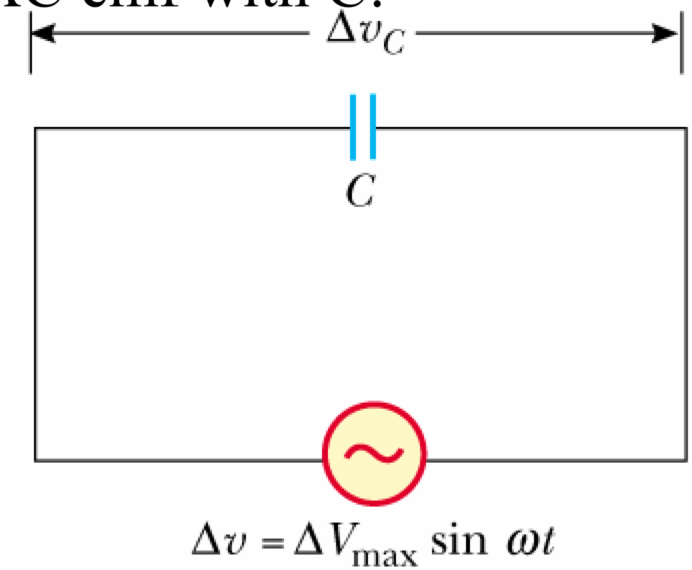


$$-L\frac{dI}{dt} + \Delta V_{\max} \sin \omega t = 0$$

$$\frac{dI}{dt} = \frac{\Delta V_{\max}}{L} \sin \omega t$$

$$I = -\frac{\Delta V_{\max}}{\omega L} \cos \omega t$$

AC emf with C:

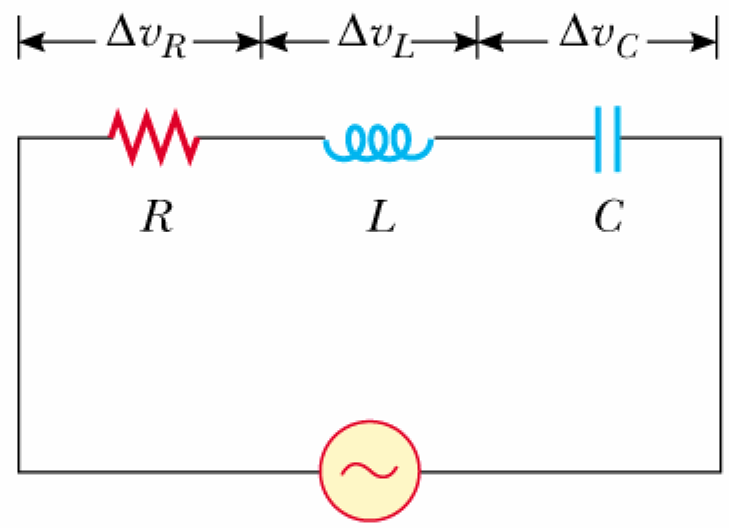


$$-\frac{q}{C} + \Delta V_{\max} \sin \omega t = 0$$

$$q = C\Delta V_{\max} \sin \omega t$$

$$I = \omega C\Delta V_{\max} \cos \omega t$$

AC emf with series LCR circuit:



$$\text{Differential eq}: \quad -RI - L\frac{dI}{dt} - \frac{q}{C} + \mathcal{E}_{\max}\sin\omega t = 0$$

Steady-state term

Transient term

$$\text{Solution for } I(t): \quad I = I_{\max}\sin(\omega t - \varphi) + Ke^{-t/\tau}\cos(\varpi_0 t + \alpha)$$

$$\tau = \frac{2L}{R}$$

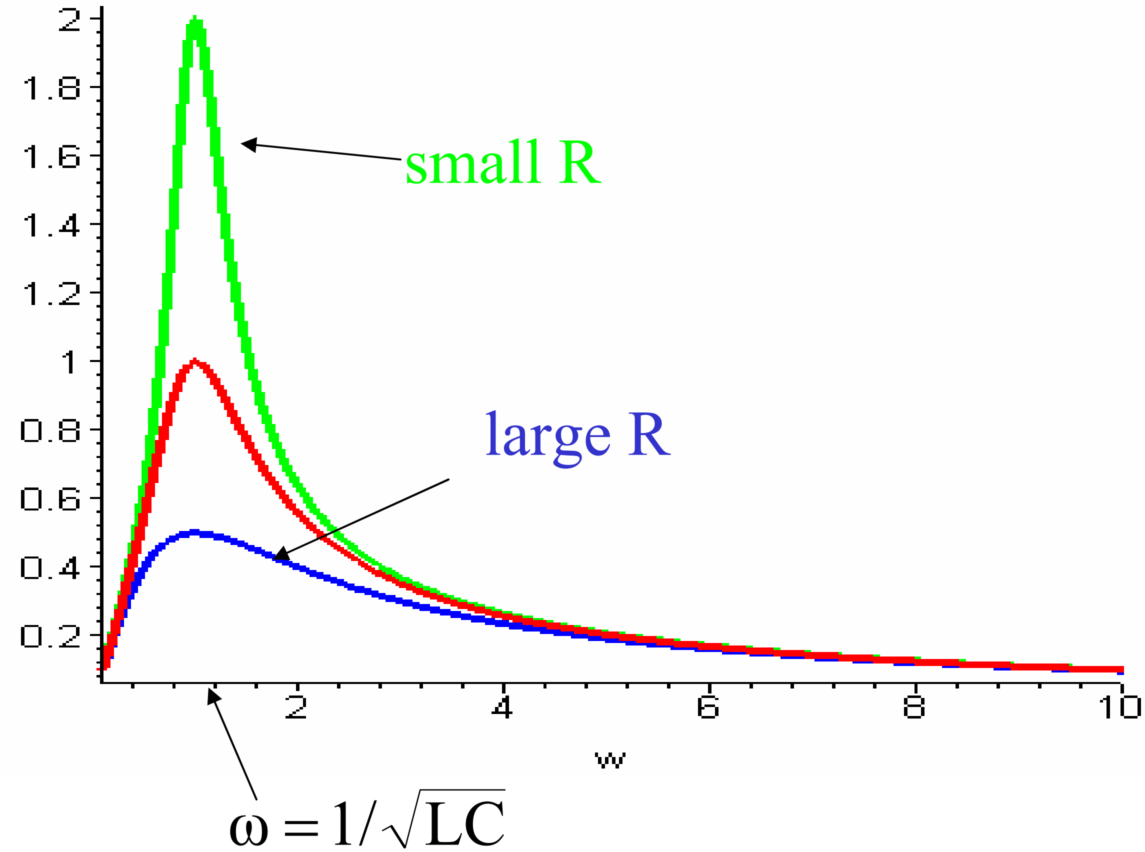
$$\varpi_0 = \sqrt{\omega_0^2 - 1/\tau^2}$$

$$\text{where}: \quad I_{\max} = \frac{\mathcal{E}_{\max}}{Z} \qquad Z \equiv \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$

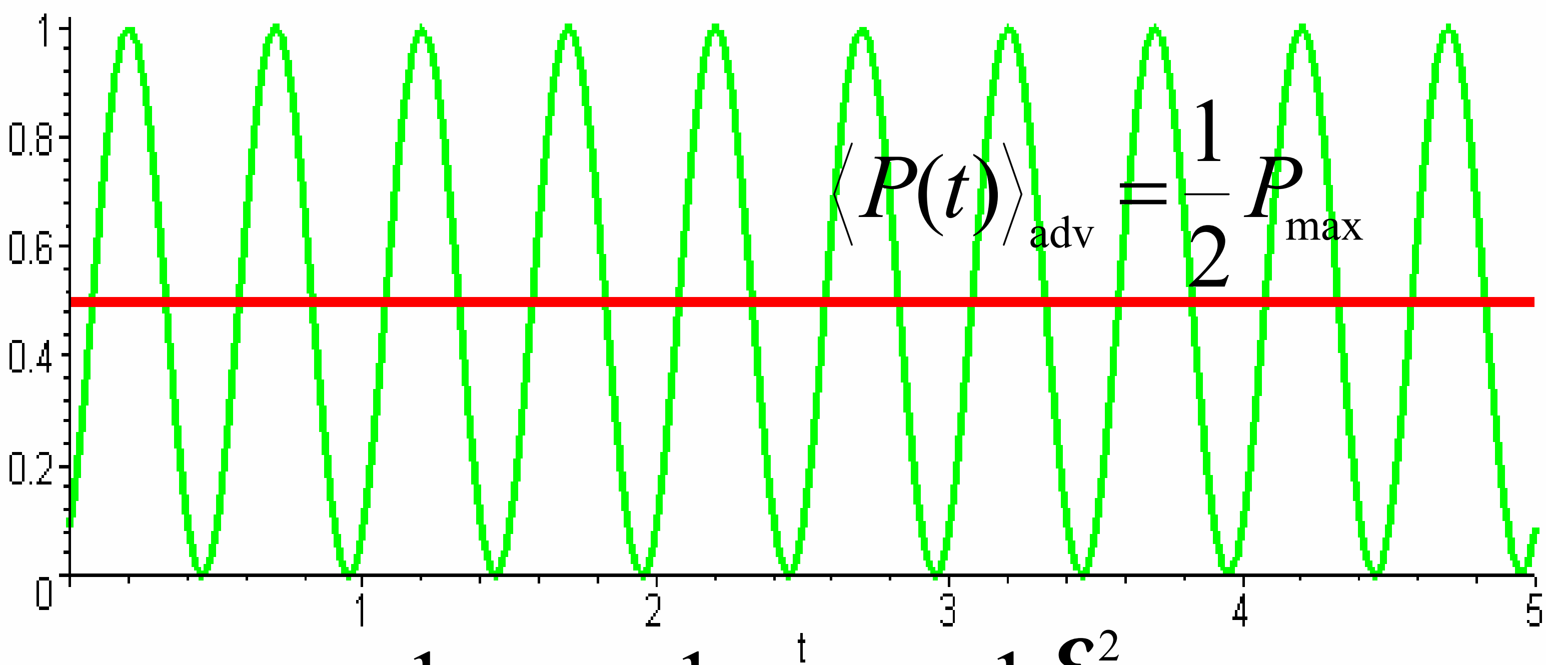
$$\tan\varphi = \frac{\omega L - 1/\omega C}{R}$$

Behavior of I_{max} in steady-state term --

$$I_{\max} = \frac{\Delta V_{\max}}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$



Power in AC circuit: $P = I^2 R = \left(I_{\max} \sin(\omega t - \varphi)\right)^2 R$

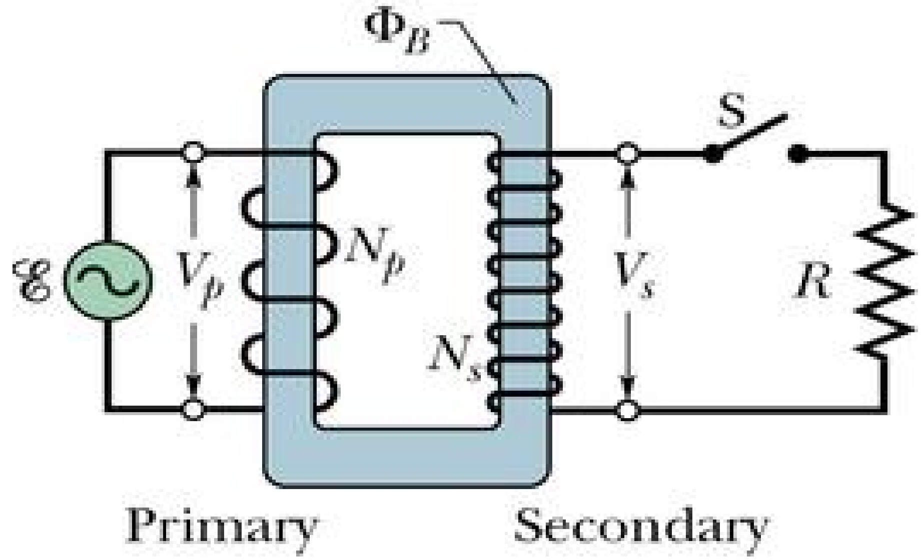


$$\langle P(t) \rangle_{\text{adv}} = \frac{1}{2} P_{\max}$$

$$\langle P(t) \rangle_{\text{adv}} = \frac{1}{2} P_{\max} = \frac{1}{2} I_{\max}^2 R = \frac{1}{2} \frac{\mathcal{E}_{\max}^2}{Z^2} R$$

$$I_{rms} = \sqrt{\frac{1}{2} I_{\max}^2} = \sqrt{\frac{1}{2}} I_{\max}$$

AC transformer



Faraday's law for single wire loop: $\mathcal{E} = -\frac{d\Phi_B}{dt}$

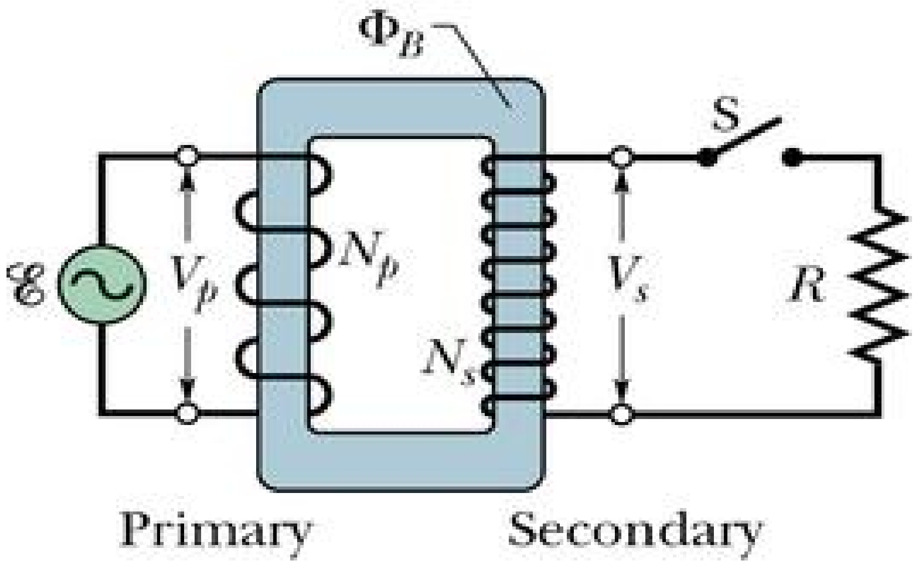
For transformer:

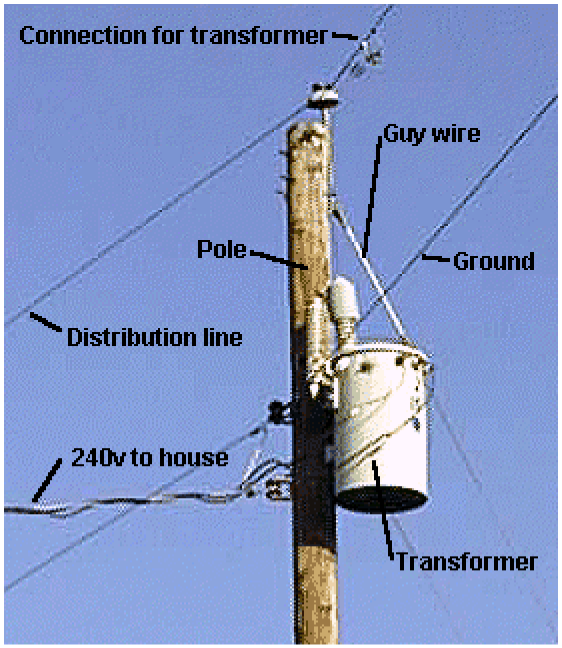
$$\mathcal{E}_P = -N_P \frac{d\Phi_B}{dt} \qquad \mathcal{E}_S = -N_S \frac{d\Phi_B}{dt}$$

$$-\frac{d\Phi_B}{dt} = \frac{\mathcal{E}_P}{N_P} = \frac{\mathcal{E}_S}{N_S} \qquad \Rightarrow \mathcal{E}_S = \frac{N_S}{N_P}\mathcal{E}_P$$

Practical use of transformer:

$$\mathcal{E}_S = \frac{N_S}{N_P}\mathcal{E}_P$$





Example :

$$\mathcal{E}_S = \frac{N_S}{N_P}\mathcal{E}_P = \frac{100}{6000}7200V = 120V$$

http://science.howstuffworks.com/power9.htm

The mathematical form of the "wave equation":

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$$

velocity of wave

Solution form for a periodic solution:

$$y(x,t) = A\sin\left(\frac{2\pi}{\lambda}(x - vt) + \varphi\right) \equiv A\sin\left(\frac{2\pi x}{\lambda} - 2\pi f t + \varphi\right) = A\sin(kx - \omega t + \varphi)$$

amplitude

phase

$$v = \lambda f = \frac{\omega}{k}$$

Comparison of mechanical and electromagnetic waves

Mechanical	Electromagnetic
Satisfy wave equation	Satisfy wave equation
v depends upon propagation material.	v depends upon propagation material (or vacuum).
Can be transverse or longitudinal	Can only be transverse
Can only propagate within materials (solids, liquids, gases, strings, etc.)	Can propagate within a vacuum and within some materials.
Doppler effect: $f' = f\dfrac{1 \pm u_O / v}{1 \mp u_S / v}$	Doppler effect: $f' = f\sqrt{\dfrac{1+u/v}{1-u/v}}$

Maxwell's equations:

$$\oiint \mathbf{E} \cdot d\mathbf{A} = Q / \varepsilon_0$$

Coulomb-Gauss law

$$\oiint \mathbf{B} \cdot d\mathbf{A} = 0$$

Gauss's for magnetic field

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Ampere-Maxwell law

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

Faraday's law

Maxwell's equations far away from sources ($Q=0$, $I=0$):

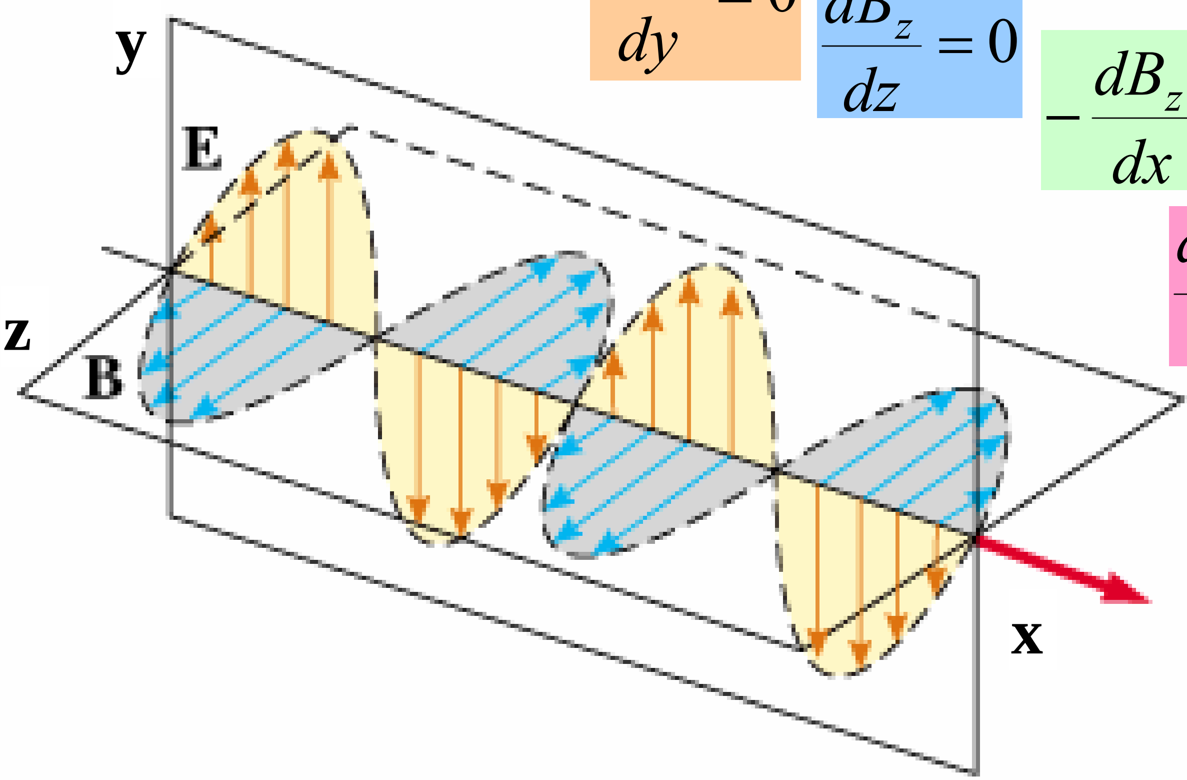
$$\oint\!\!\oint \mathbf{E}\cdot d\mathbf{A} = 0 \qquad \oint\!\!\oint \mathbf{B}\cdot d\mathbf{A} = 0 \qquad \oint \mathbf{B}\cdot d\mathbf{s} = \mu_0\varepsilon_0 \frac{d\left(\iint \mathbf{E}\cdot d\mathbf{A}\right)}{dt} \qquad \oint \mathbf{E}\cdot d\mathbf{s} = -\frac{d\left(\iint \mathbf{B}\cdot d\mathbf{A}\right)}{dt}$$

Plane-polarized solution:

$$\frac{dE_y}{dy} = 0 \qquad \frac{dB_z}{dz} = 0$$

$$-\frac{dB_z}{dx} = \mu_0\varepsilon_0 \frac{dE_y}{dt}$$

$$\frac{dE_y}{dx} = -\frac{dB_z}{dt}$$



Results:

$$-\frac{dB_z}{dx} = \mu_0 \varepsilon_0 \frac{dE_y}{dt} \qquad \frac{dE_y}{dx} = -\frac{dB_z}{dt}$$

Wave equations for E_y and B_z:

$$\frac{\partial^2 E_y}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \frac{\partial^2 E_y}{\partial x^2} \qquad \frac{\partial^2 B_z}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \frac{\partial^2 B_z}{\partial x^2}$$

$$\Rightarrow v \equiv c = \sqrt{\frac{1}{\mu_0 \varepsilon_0}} = 2.99792458 \times 10^8 \, m/s$$

Periodic solution:

$$E_y(x,t) = E_{\max} \sin\left(\frac{2\pi}{\lambda}(x - ct)\right) = E_{\max} \sin(kx - \omega t)$$

$$B_z(x,t) = \frac{E_{\max}}{c} \sin\left(\frac{2\pi}{\lambda}(x - ct)\right) = \frac{E_{\max}}{c} \sin(kx - \omega t)$$

Summary of significant properties of electromagnetic waves:

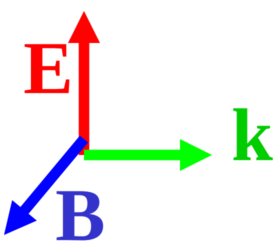
- "Self-sustaining" electric and magnetic fields which can propagate in vacuum at a velocity of $c = 2.99792458 \times 10^8$ m/s (or within matter at a velocity of $v=c/n$) .
- **E** and **B** fields are perpendicular to each other and perpendicular to propagation direction (transverse waves).
- $\left|\mathbf{E}(x,t)\right| = \dfrac{\left|\mathbf{B}(x,t)\right|}{v}$
- Periodic waves have the form:

$$E_y(x,t) = E_{\max} \sin(kx - \omega t)$$

$$B_z(x,t) = \frac{E_{\max}}{v} \sin(kx - \omega t)$$

$$\frac{\omega}{k} = v$$

🡐 polarization direction (**y**)



Radiation power

Poynting vector: $\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$ **units: Watts/area**

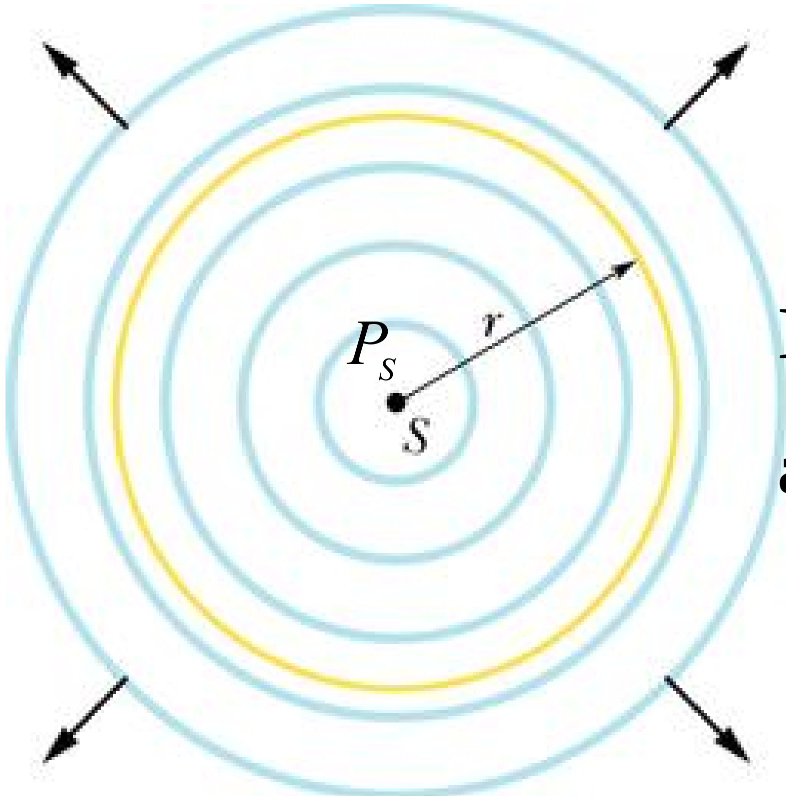
pointing in propagation direction

Intensity : $I \equiv \left\langle \left| \mathbf{S} \right| \right\rangle_{avg}$

Intensity for plane wave : $I = \frac{E_{\max}^2}{2c\mu_0}$

due to averaging

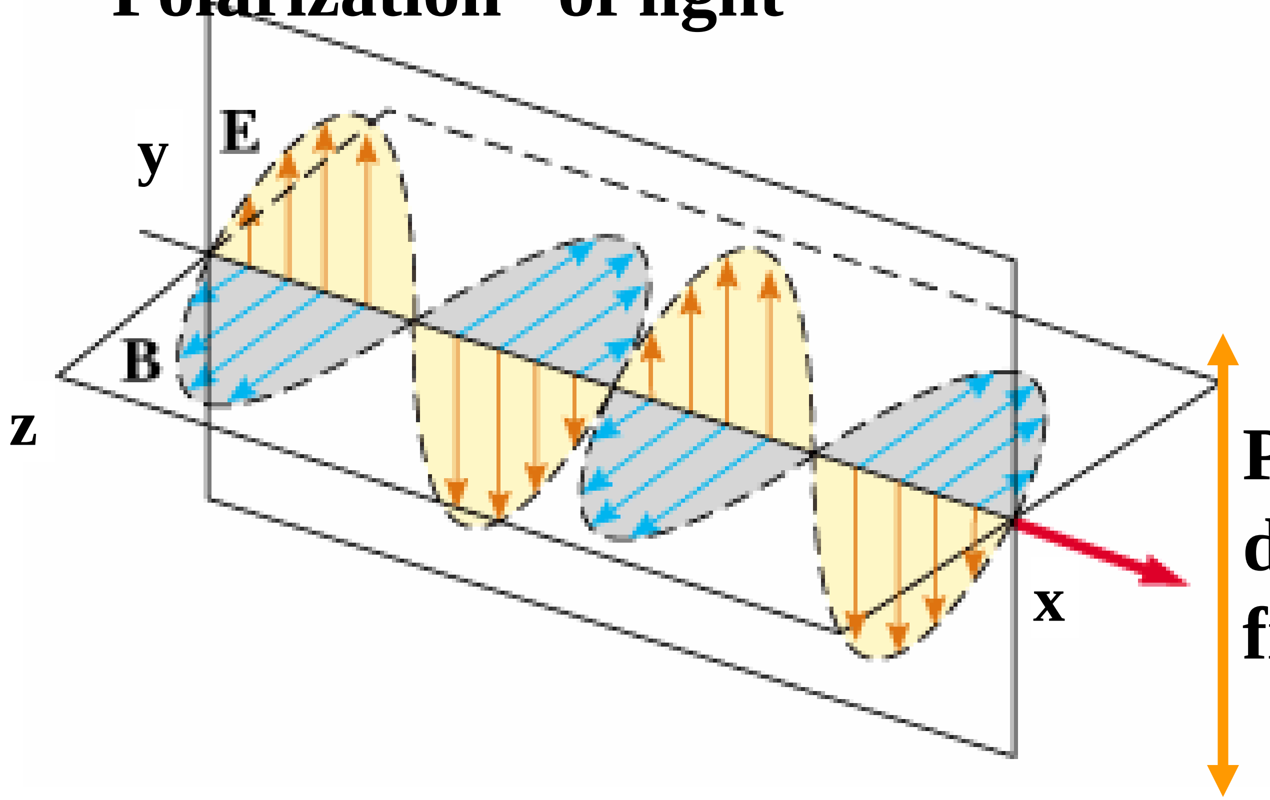
Radiation power from point source



Intensity from point source at a distance r**:**

$$I = \frac{P_S}{4\pi r^2}$$

"Polarization" of light

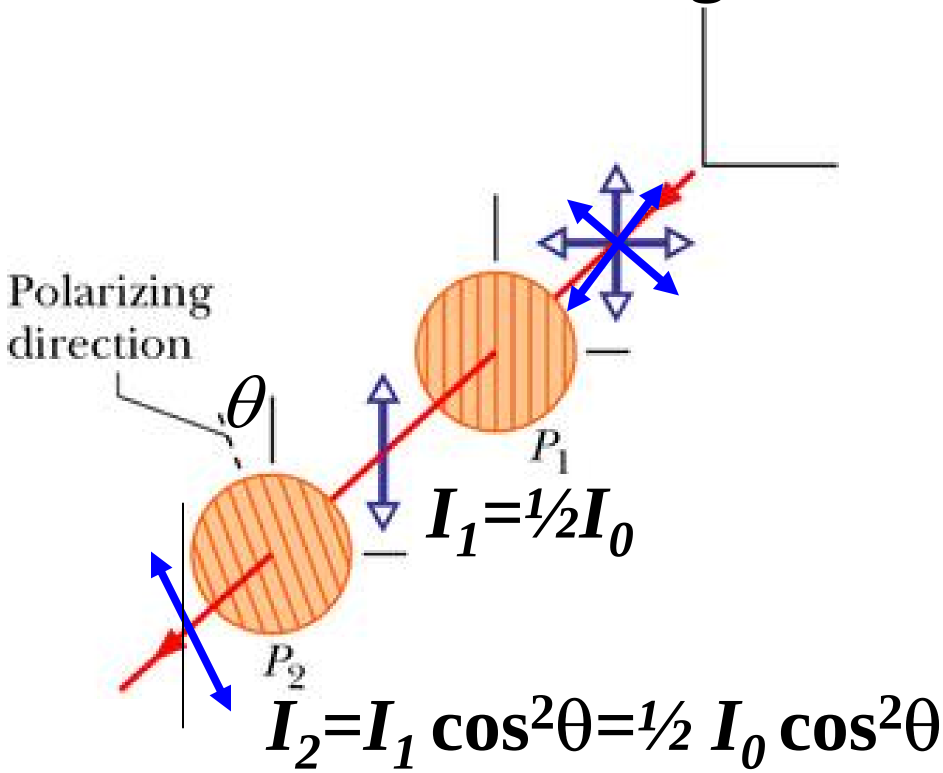


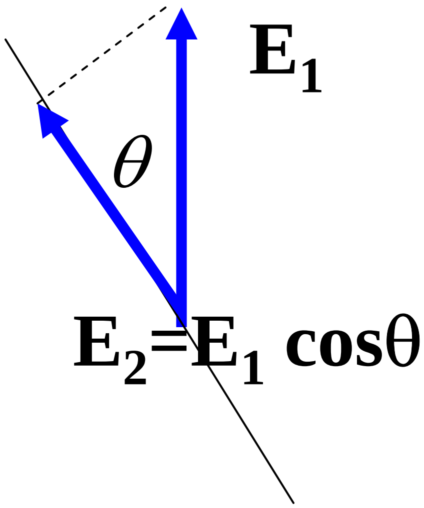
Polarization direction ≡ E field direction

Some radiation sources produce polarized light –

For example: lasers, antennas

Filtration of polarized light as a function of angle





Electromagnetic waves in materials

$$-\frac{dB_z}{dx} = \mu\varepsilon \frac{dE_y}{dt}$$

$$\frac{dE_y}{dx} = -\frac{dB_z}{dt}$$

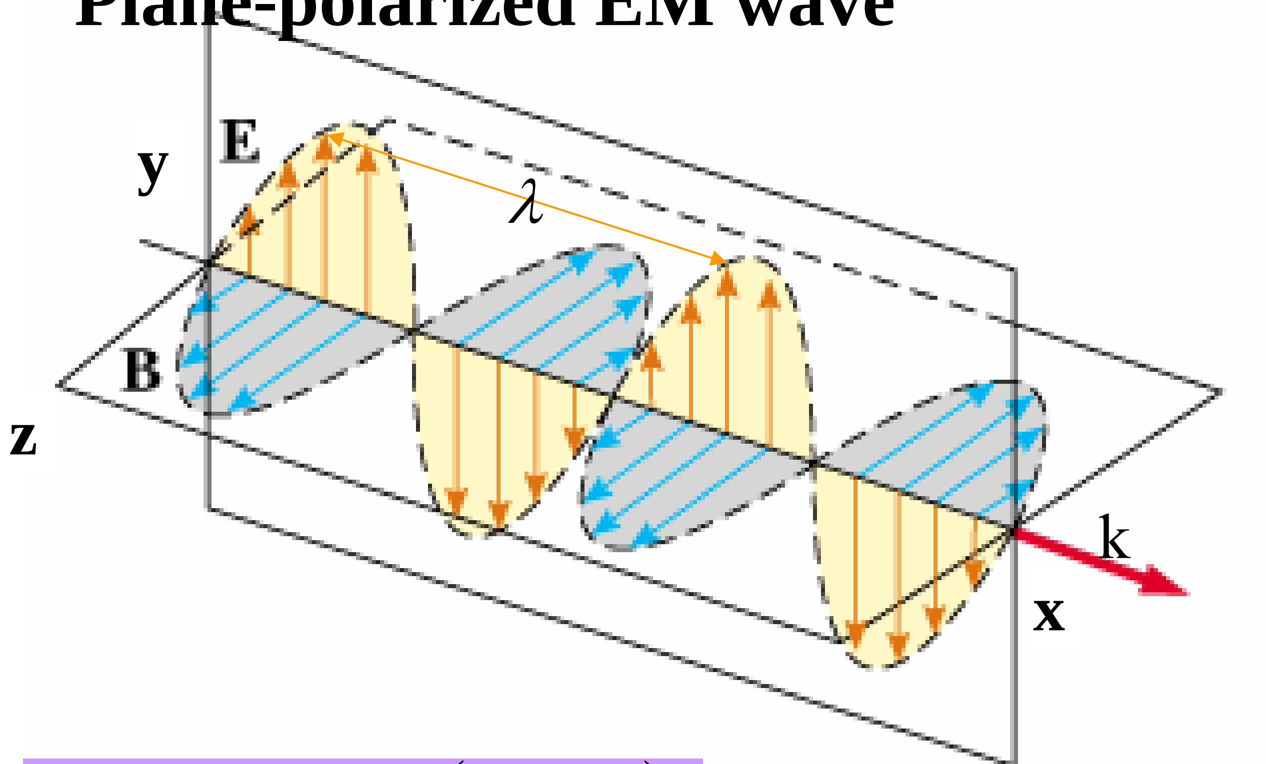
$$\frac{\partial^2 E_y}{\partial t^2} = \frac{1}{\mu\varepsilon}\frac{\partial^2 E_y}{\partial x^2}$$

$$\frac{\partial^2 B_z}{\partial t^2} = \frac{1}{\mu\varepsilon}\frac{\partial^2 B_z}{\partial x^2}$$

$$\Rightarrow v = \sqrt{\frac{1}{\mu\varepsilon}} = \sqrt{\frac{\mu_0\varepsilon_0}{\mu\varepsilon}}\sqrt{\frac{1}{\mu_0\varepsilon_0}} \equiv \frac{c}{n}$$

refractive index

Plane-polarized EM wave



$$\mathbf{E}(x,t) = E_{\max}\sin(kx-\omega t)\hat{\mathbf{y}}$$

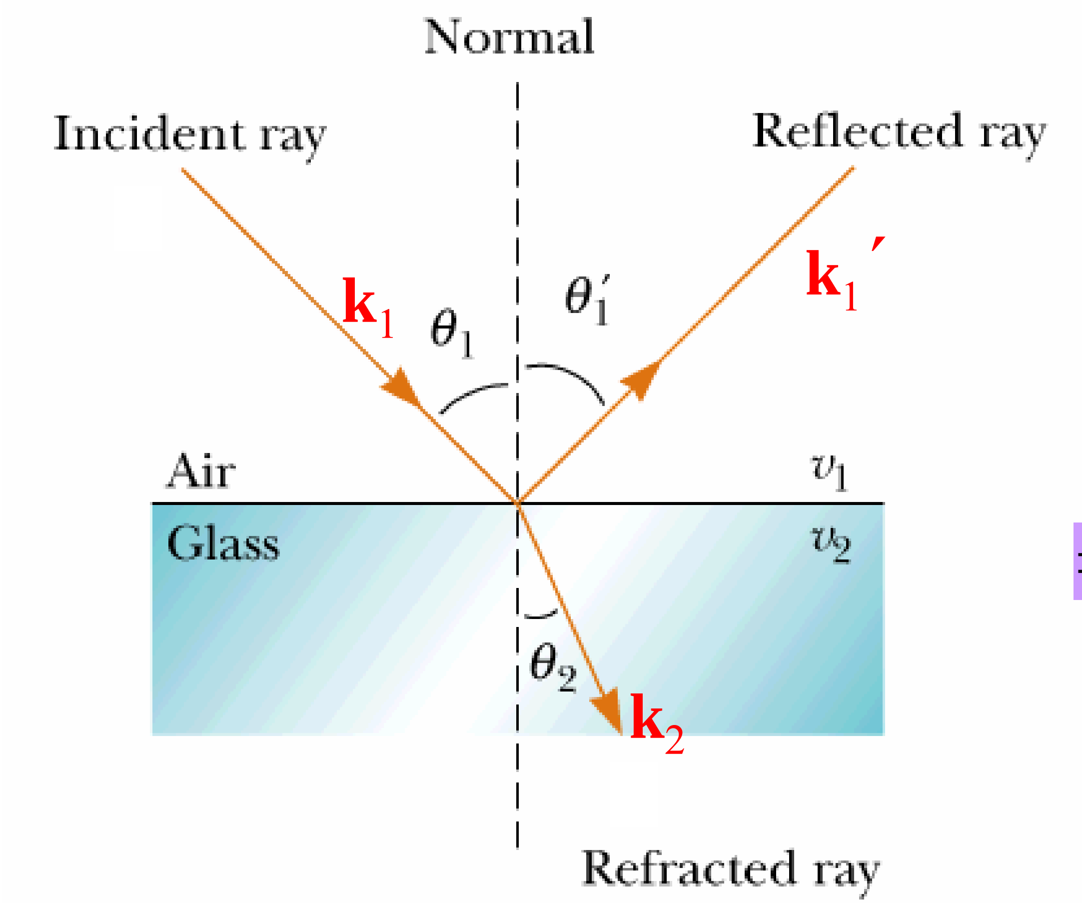
$$\mathbf{B}(x,t) = \frac{E_{\max}}{c}\sin(kx-\omega t)\hat{\mathbf{z}}$$

$$\frac{\omega}{k} = \frac{c}{n} = \frac{\text{speed of light in vacuum}}{\text{refractive index of medium}}$$

$$\Rightarrow k = \frac{n\omega}{c} \quad \text{or} \quad \lambda = \frac{c}{nf}$$

Consider the behavior of a plane-polarized electromagnetic wave near the surface of two materials:

$$k_1 \sin\theta_1 = k_1' \sin\theta_1' = k_2 \sin\theta_2$$



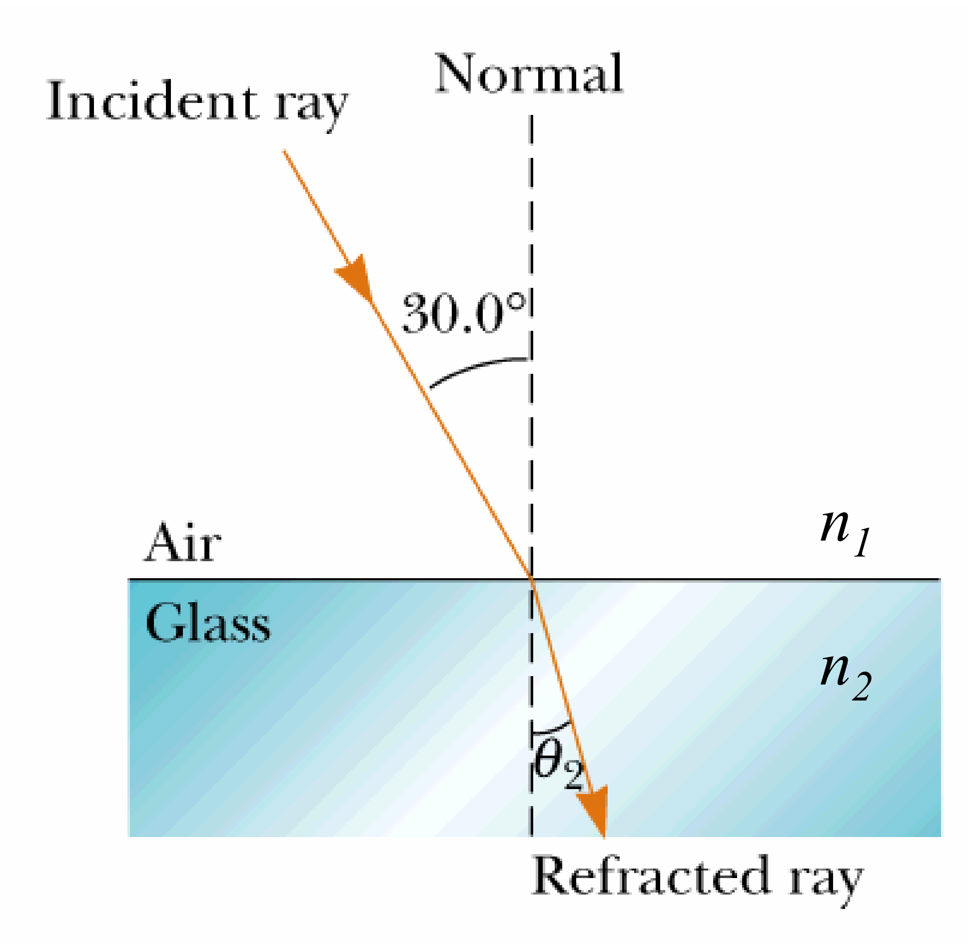
$$k_1 = \frac{n_1\omega}{c} = k_1'$$

$$k_2 = \frac{n_2\omega}{c}$$

$$\Rightarrow \theta_1 = \theta_1'$$

$$\Rightarrow n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Refraction



Snell's law:

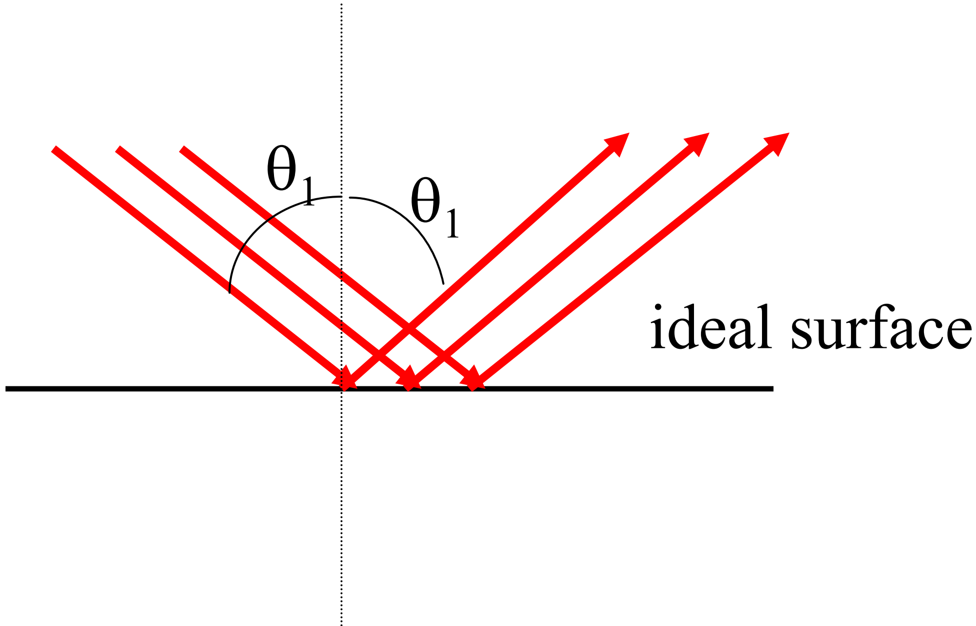
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right)$$

$$= 17.5^o$$

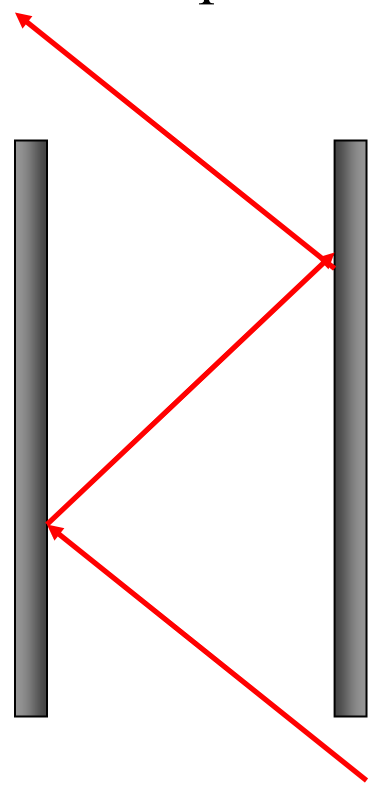
for $n_1 = 1, n_2 = 1.66$

Reflection

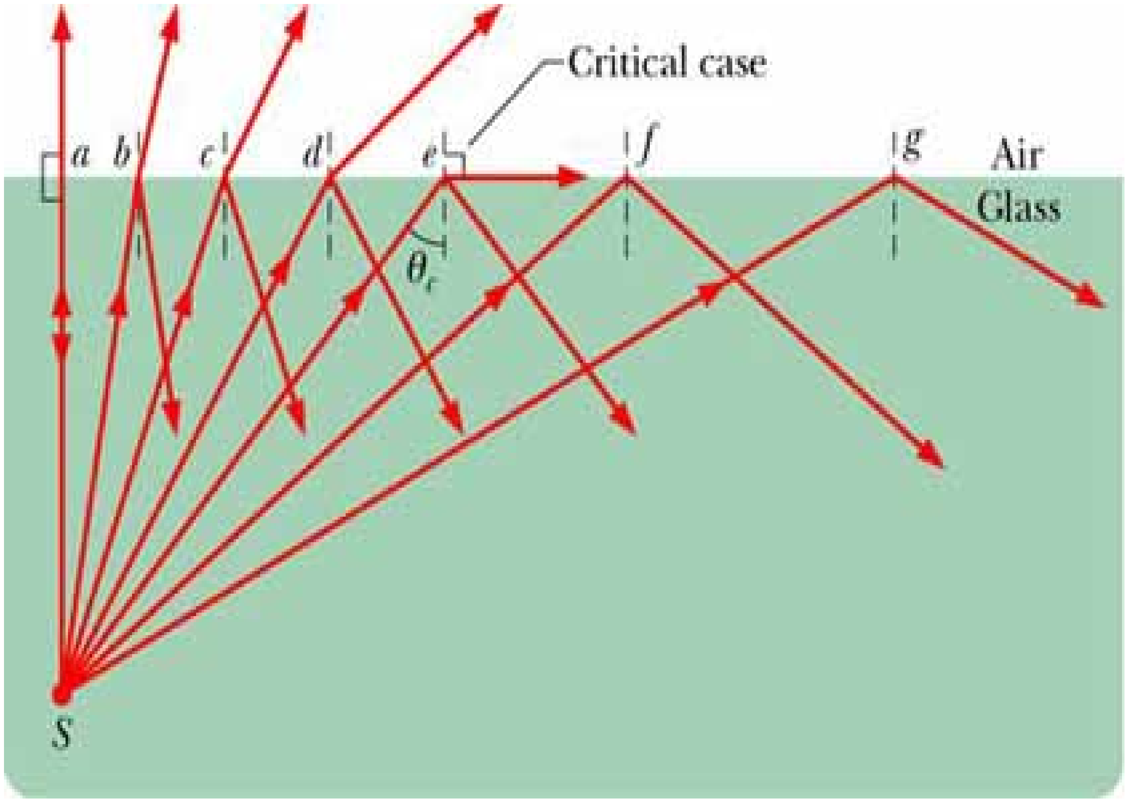


Ray tracing

Example: 2 mirrors



Total internal reflection



$$\sin\theta_1 = \frac{n_2}{n_1}\sin\theta_2 \le 1$$

$$\theta_{2|\max} \equiv \theta_c = \sin^{-1}\left(\frac{n_1}{n_2}\right)$$