Announcements

1. Reworked exam 3 – due today; solutions will be posted
2. Topics for today –

   Review of wave properties of EM radiation

   Notion of interference phenomena

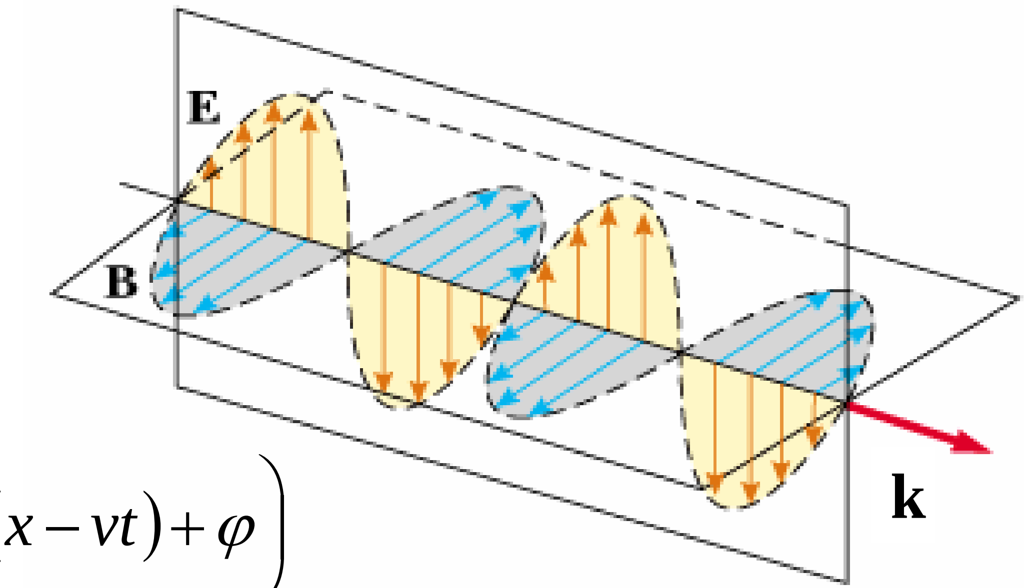
   Young's double slit interference

   Thin film interference effects

   Michelson's interferometer

Wave phenomena associated with light

Plane polarized electromagnetic wave at an instant of time:



$$E_y(x,t) = E_{\max} \sin\left(\frac{2\pi}{\lambda}(x - vt) + \varphi \right)$$

Superposition of light waves: $E_y^{tot}(x,t) = E_y^1(x,t) + E_y^2(x,t)$

Example: $E_y(x,t) = E_{\max} \sin\left(\frac{2\pi}{\lambda}(x - vt) \right) + E_{\max} \sin\left(\frac{2\pi}{\lambda}(x - vt) + \varphi \right)$

$$= 2E_{\max} \sin\left(\frac{2\pi}{\lambda}(x - vt) + \frac{1}{2}\varphi \right) \cos\left(\frac{\varphi}{2} \right)$$

Some details:

$$E_y^{tot}(x,t) = E_y^1(x,t) + E_y^2(x,t)$$

$$E_y(x,t) = E_{\max}\sin\left(\frac{2\pi}{\lambda}(x - vt)\right) + E_{\max}\sin\left(\frac{2\pi}{\lambda}(x - vt) + \varphi\right)$$

$$= 2E_{\max}\sin\left(\frac{2\pi}{\lambda}(x - vt) + \frac{1}{2}\varphi\right)\cos\left(\frac{\varphi}{2}\right)$$

Note that this result follows from the trigonometric identity :

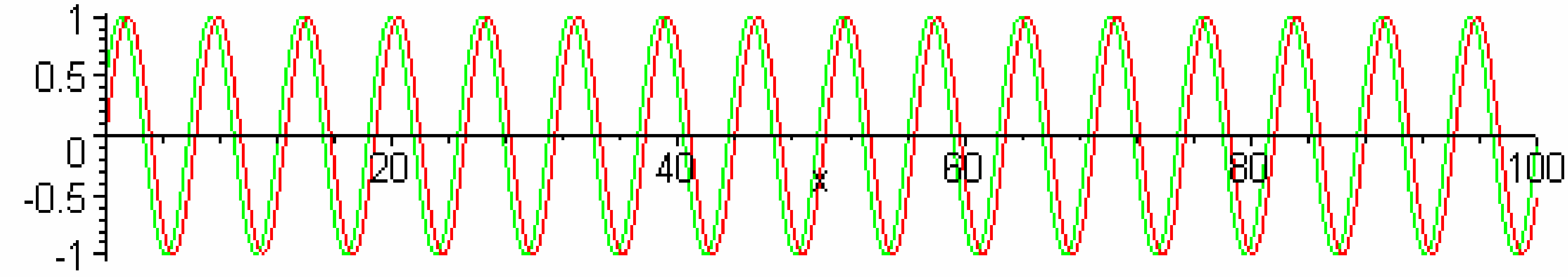
$$\sin(A) + \sin(B) = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

Intensity :

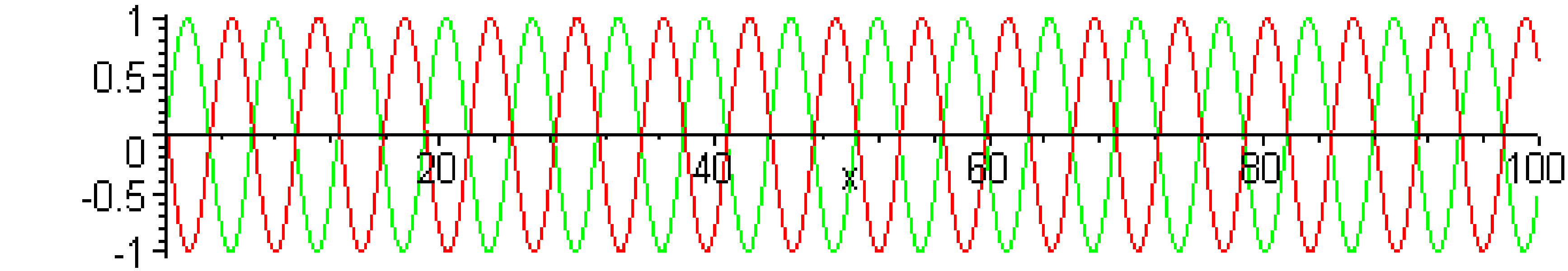
$$I^1 = \frac{1}{2c\mu_0}\left|E_{\max}\right|^2 = I^2$$

$$I^{tot} = \frac{4}{2c\mu_0}\left|E_{\max}\cos\left(\frac{\varphi}{2}\right)\right|^2 = 4I^1\cos^2\left(\frac{\varphi}{2}\right)$$

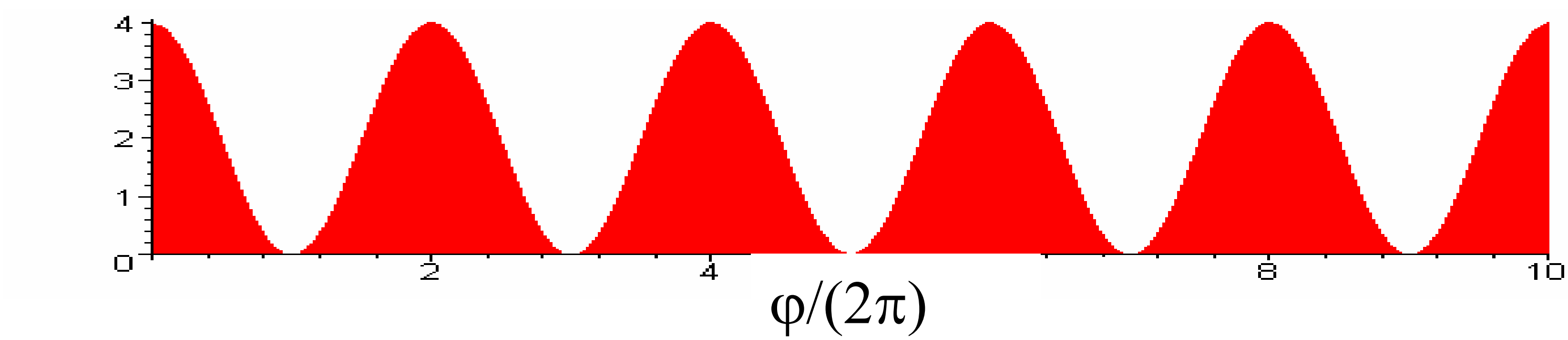
Example: φ=0.5 rad -- plotting snapshot of EM wave



Example: φ=3 rad – plotting snapshot of EM wave



Intensity as a function of φ:

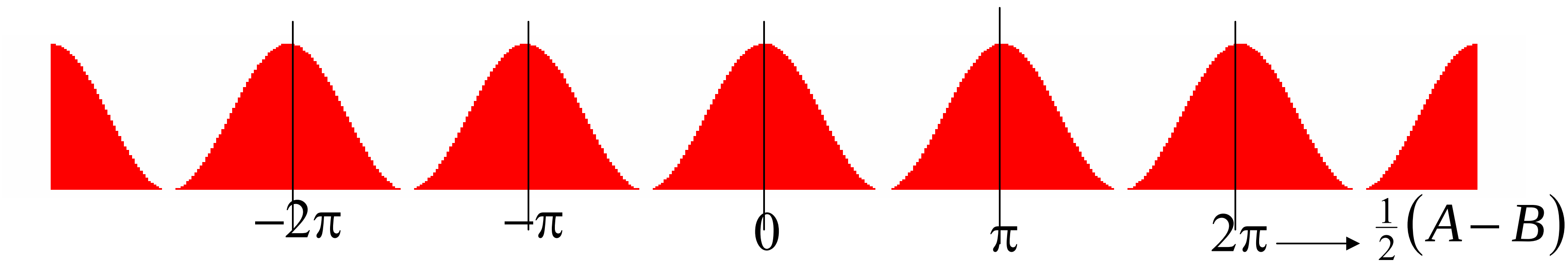


Review of interference phenomena – Occurs when there are two or more electromagnetic waves which combine at a give point *P*.

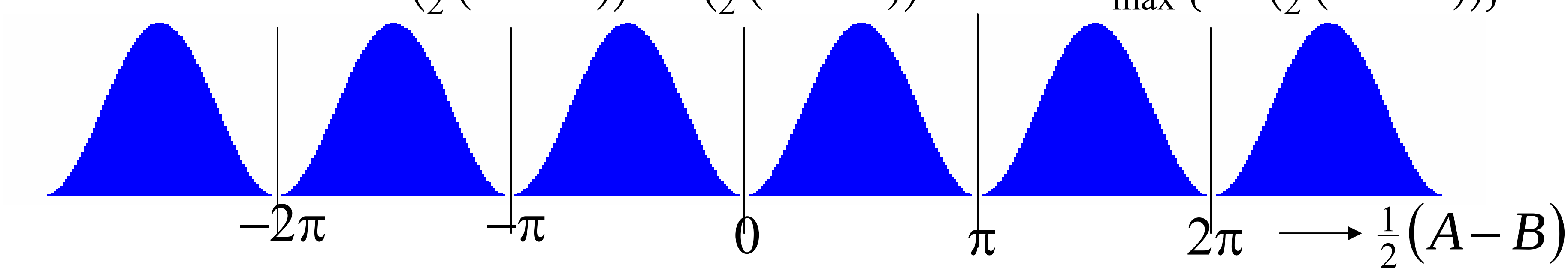
$$E(P,t) = E_{\max}\sin\left(\frac{2\pi r_1}{\lambda} - 2\pi f t\right) \pm E_{\max}\sin\left(\frac{2\pi r_2}{\lambda} - 2\pi f t\right)$$

Trig identity :

$$\sin A + \sin B = 2\sin\left(\tfrac{1}{2}(A+B)\right)\cos\left(\tfrac{1}{2}(A-B)\right) \Rightarrow I = I_{\max}\left\{\cos\left(\tfrac{1}{2}(A-B)\right)\right\}^2$$

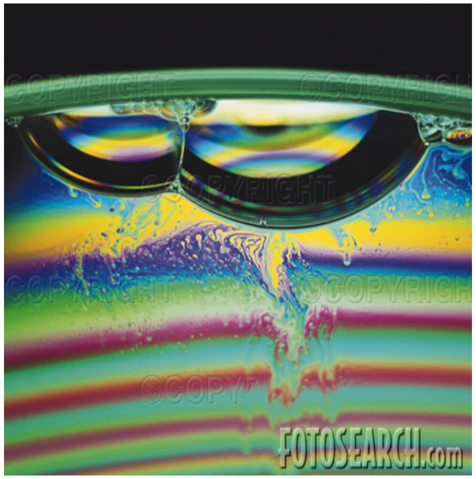


$$\sin A - \sin B = 2\cos\left(\tfrac{1}{2}(A+B)\right)\sin\left(\tfrac{1}{2}(A-B)\right) \Rightarrow I = I_{\max}\left\{\sin\left(\tfrac{1}{2}(A-B)\right)\right\}^2$$

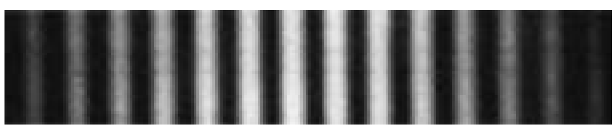


Two examples of superposed radiation:

Interference from refraction and reflection of thin films



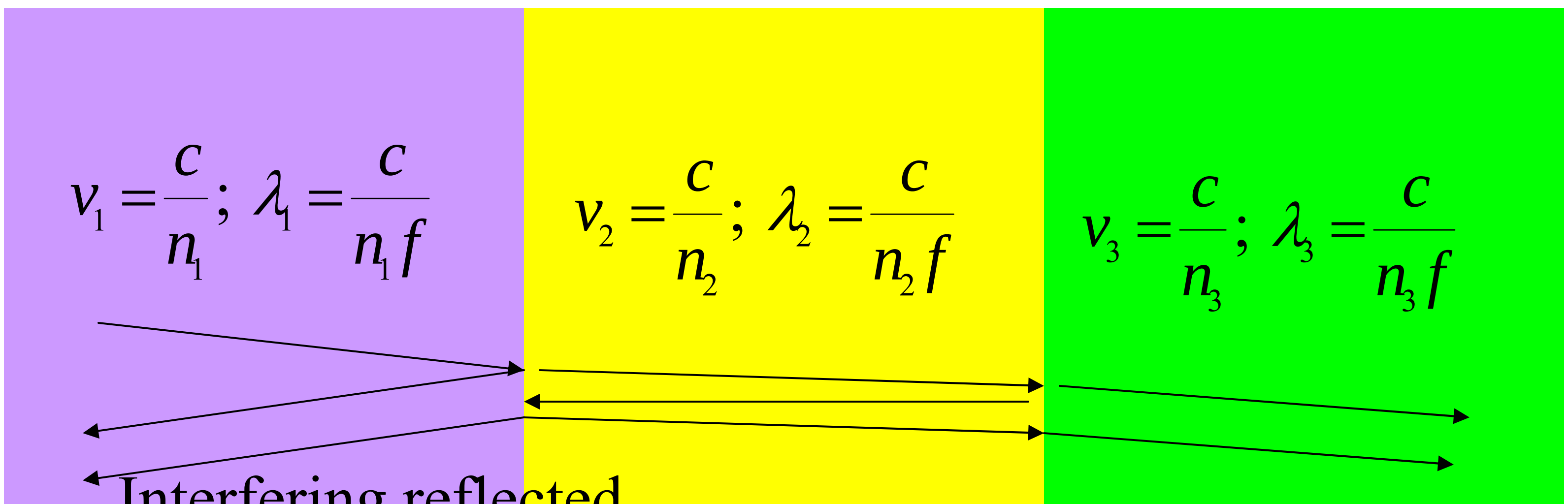
Young’s double slit



Interference in thin films

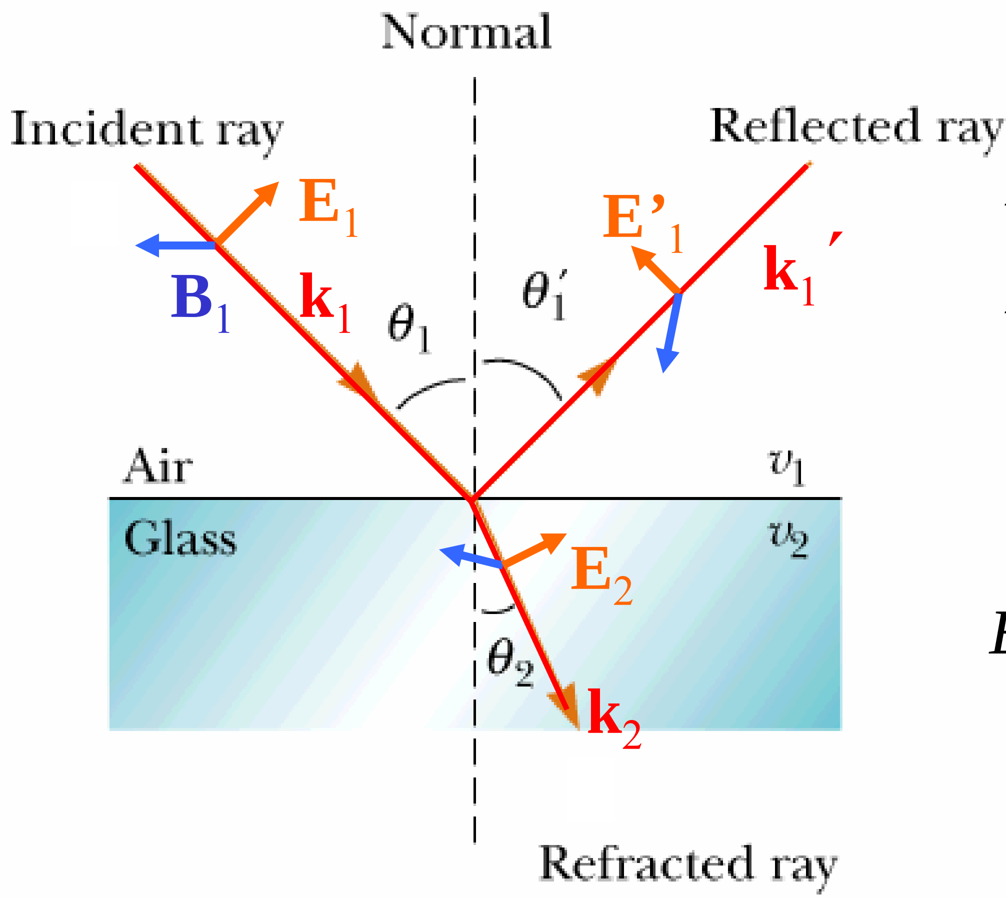
$$E_y(x,t) = E_{\max}\sin\left(\frac{2\pi}{\lambda}(x - vt) + \varphi\right)$$

$$v = \frac{c}{n}; \qquad \lambda = \frac{c}{nf}$$



Interference between reflected waves:

Recall -- the behavior of a plane-polarized electromagnetic wave near the surface of two materials:



Periodic waves:

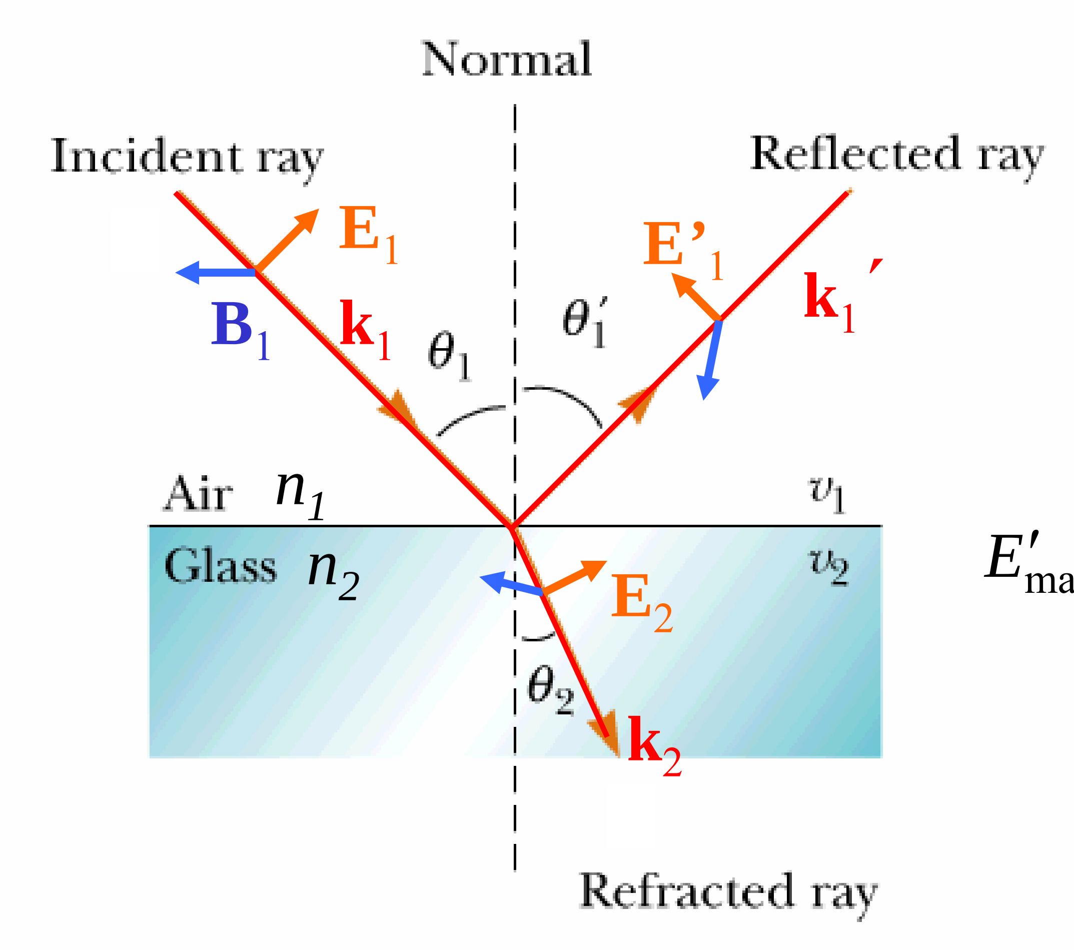
$$E_1 = E_{max_1} \sin(\mathbf{k}_1 \cdot \mathbf{r} - \omega_1 t)$$

$$E_1' = E_{max_1}' \sin(\mathbf{k}_1' \cdot \mathbf{r} - \omega_1' t)$$

$$\frac{\omega_1}{k_1} = \frac{\omega_1'}{k_1'} = v_1$$

$$E_2 = E_{max_2} \sin(\mathbf{k}_2 \cdot \mathbf{r} - \omega_2 t)$$

$$\frac{\omega_2}{k_2} = v_2$$



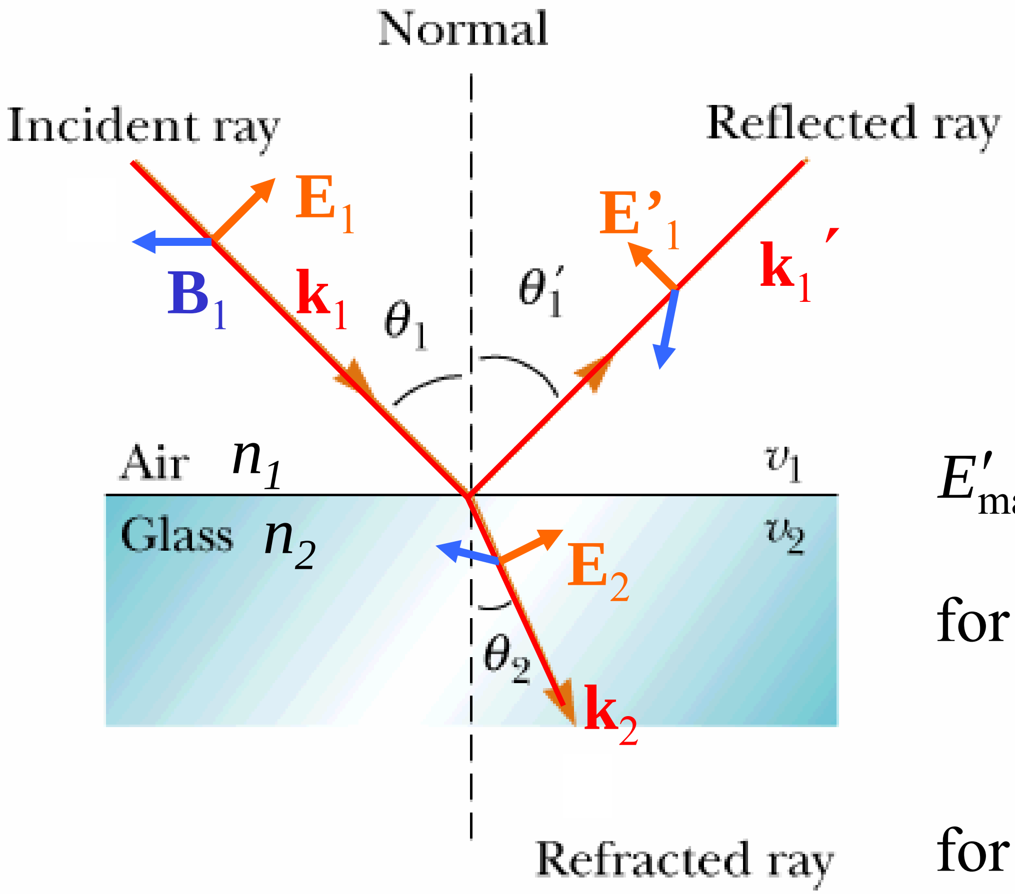
Matching electric and magnetic fields at boundary:

$$E'_{\max_1} = E_{\max_1} \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2}$$

or

$$E'_{\max_1} = E_{\max_1} \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

Plane waves reflected and refracted at surface:



Matching electric and magnetic fields at boundary:

For reflected waves:

$$E'_{\max_1} = E_{\max_1} \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2}$$

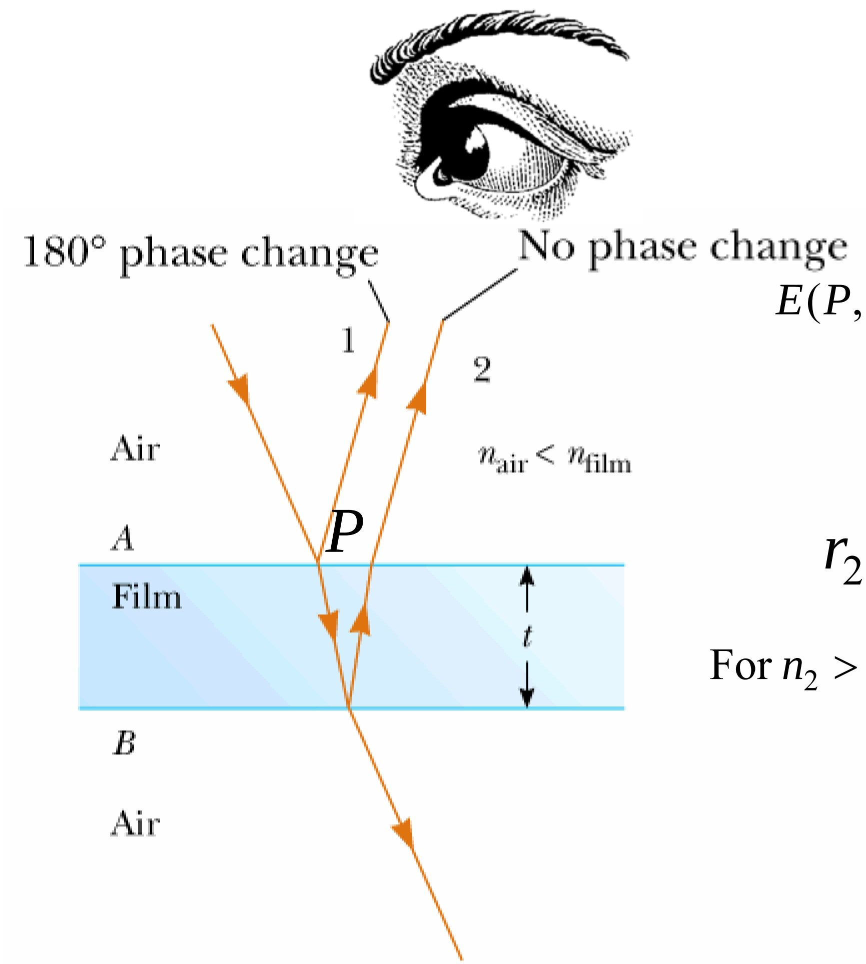
for E in plane of incidence

or

for E out of plane of incidence:

$$E'_{\max_1} = E_{\max_1} \frac{n_2 \cos\theta_2 - n_1 \cos\theta_1}{n_2 \cos\theta_2 + n_1 \cos\theta_1}$$

Multiple refractions and reflections in a thin film



$$E(P,t) = \pm E_{max} \sin\left(\frac{2\pi r_1}{\lambda_1} - 2\pi f t\right) \pm E'_{max} \sin\left(\frac{2\pi r_2}{\lambda_2} - 2\pi f t\right)$$

$$\approx 2E_{max} \begin{Bmatrix} \sin \\ \cos \end{Bmatrix} \left(\frac{\pi(r_1 + r_2)}{\lambda_{av}} - 2\pi f t\right) \begin{Bmatrix} \cos \\ \sin \end{Bmatrix} \left(\frac{\pi(r_2 - r_1)}{\lambda_2}\right)$$

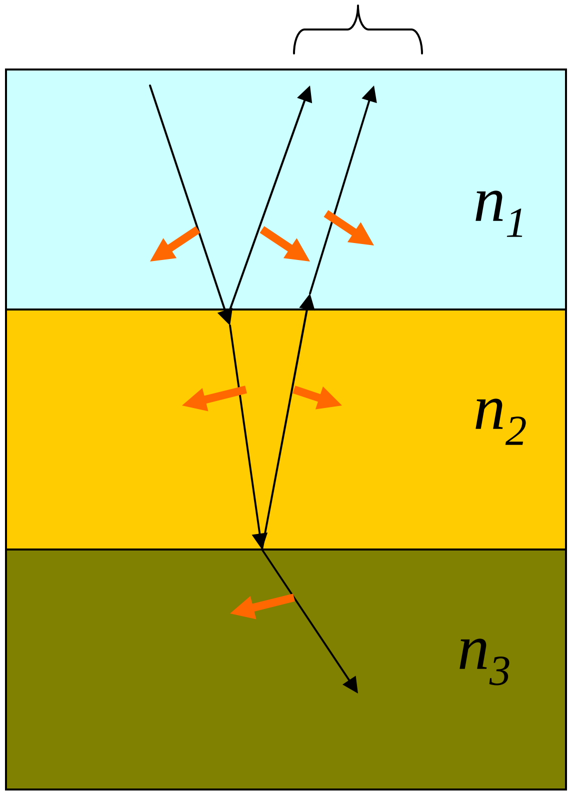
$$r_2 - r_1 \approx 2t$$

For $n_2 > n_1$ $\quad E_{max_1} = -; E'_{max_1} = +$

$$\Rightarrow \sin\left(\frac{\pi(2t)}{\lambda_2}\right) \quad \Rightarrow \text{max at } \; 2nt = (m + \tfrac{1}{2})\lambda$$

$$\Rightarrow \text{min at } \; 2nt = m\lambda$$

Example of interference with "+"



$n_1 < n_2 < n_3$

For each surface with $n_1 < n_2$

$E_2 = -E_1$ for reflected beam

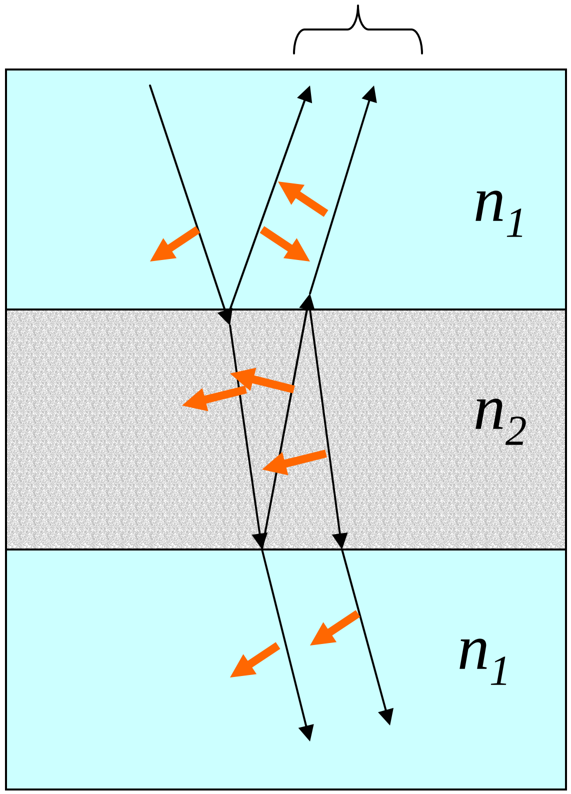
$$\Rightarrow I = I_{\max}\left\{\cos\left(\tfrac{1}{2}(A-B)\right)\right\}^2$$

peaks at

$$\tfrac{1}{2}(A-B) \equiv \frac{\pi(r_2 - r_1)}{\lambda} = m\pi$$

Example of interference with "–"

$n_1 > n_2$



$$\Rightarrow I = I_{\max}\left\{\sin\left(\tfrac{1}{2}(A-B)\right)\right\}^2$$

peaks at

$$\tfrac{1}{2}(A-B) \equiv \frac{\pi(r_2 - r_1)}{\lambda} = \left(m + \tfrac{1}{2}\right)\pi$$

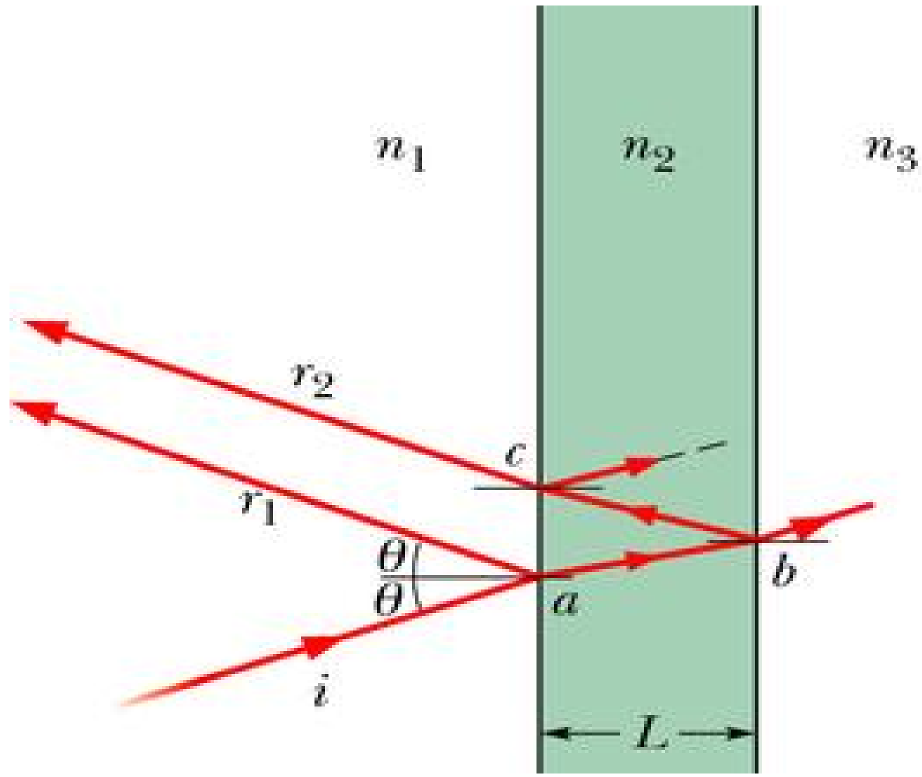
Example of interference with "+"

$$\Rightarrow I = I_{\max}\left\{\cos\left(\tfrac{1}{2}(A-B)\right)\right\}^2$$

peaks at

$$\tfrac{1}{2}(A-B) \equiv \frac{\pi(r_2 - r_1)}{\lambda} = m\pi$$

Summary



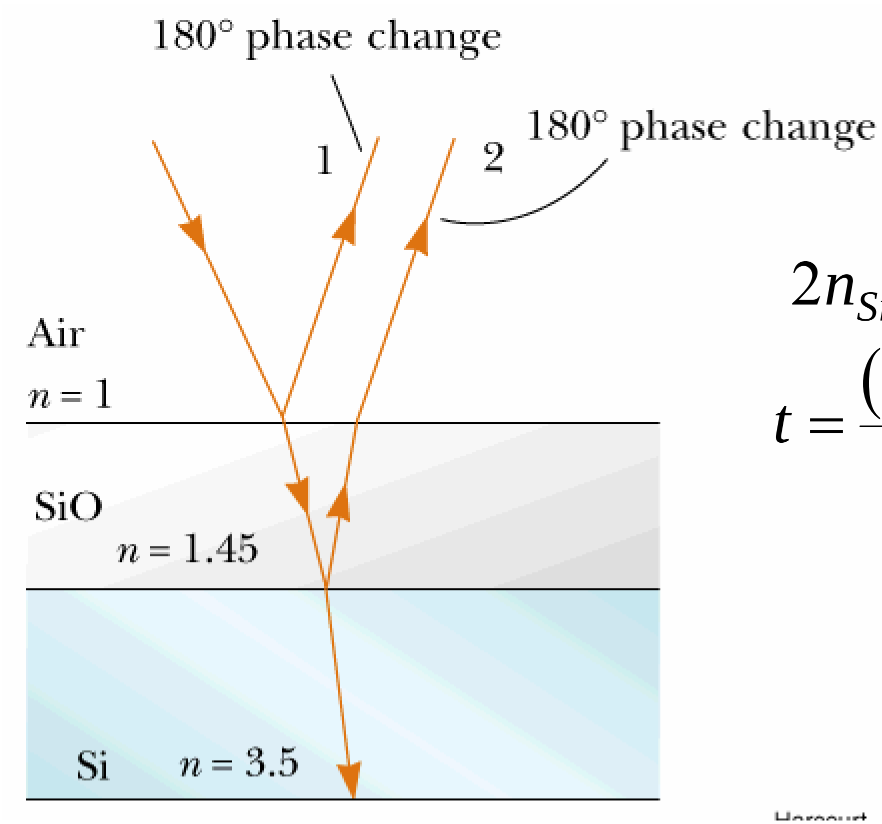
$$\varphi \approx 2\pi \frac{2L}{\lambda_2} = 2\pi \frac{2L}{\lambda} n_2$$

$$I = 4I_0 \cos^2\left(\frac{\varphi}{2}\right) \quad \Rightarrow \text{ maxima at } \quad 2L = m\frac{\lambda}{n_2}$$

$$I = 4I_0 \sin^2\left(\frac{\varphi}{2}\right) \quad \Rightarrow \text{ maxima at } \quad 2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2}$$

Example:

Suppose you want to prepare a surface so that it has minimal reflection such as in a solar cell where it is desirable to optimize refraction and minimize reflection at λ=550nm.



$$2n_{SiO}t = (m + \tfrac{1}{2})\lambda$$

$$t = \frac{(m + \tfrac{1}{2})\lambda}{2n_{SiO}} = \frac{(m + \tfrac{1}{2})550\text{ nm}}{2 \cdot 1.45}$$

$$= 94.8\text{ nm} \quad (\text{for } m = 0)$$

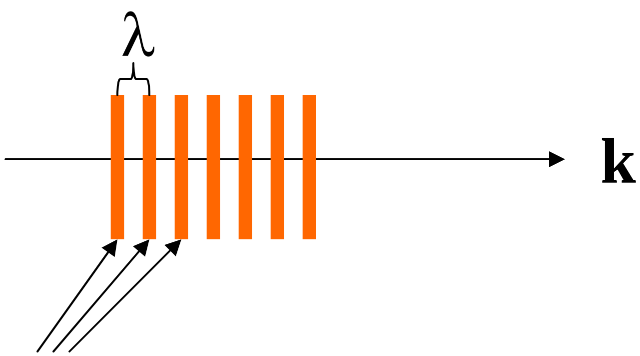
Example:

Suppose you see a rainbow pattern for oil on a pavement. What is the the approximate thickness of the oil in the red (λ=700nm) region. Assume that n_{oil}=1.4.

$$2n_{oil}t = (m + \tfrac{1}{2})\lambda$$

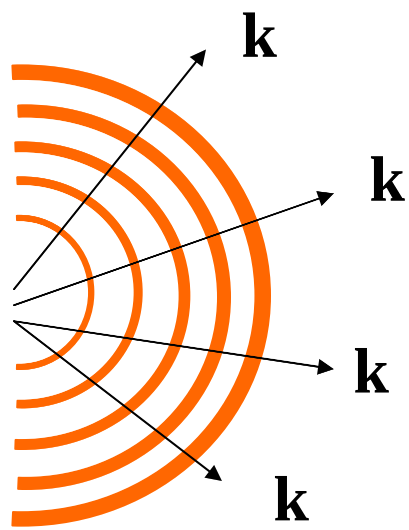
$$t = \frac{(m + \tfrac{1}{2})\lambda}{2n_{oil}} = \frac{(m + \tfrac{1}{2})700\text{ nm}}{2 \cdot 1.4} = 125\text{ nm},\ 375\text{ nm, etc.}$$

Top view of plane EM wave

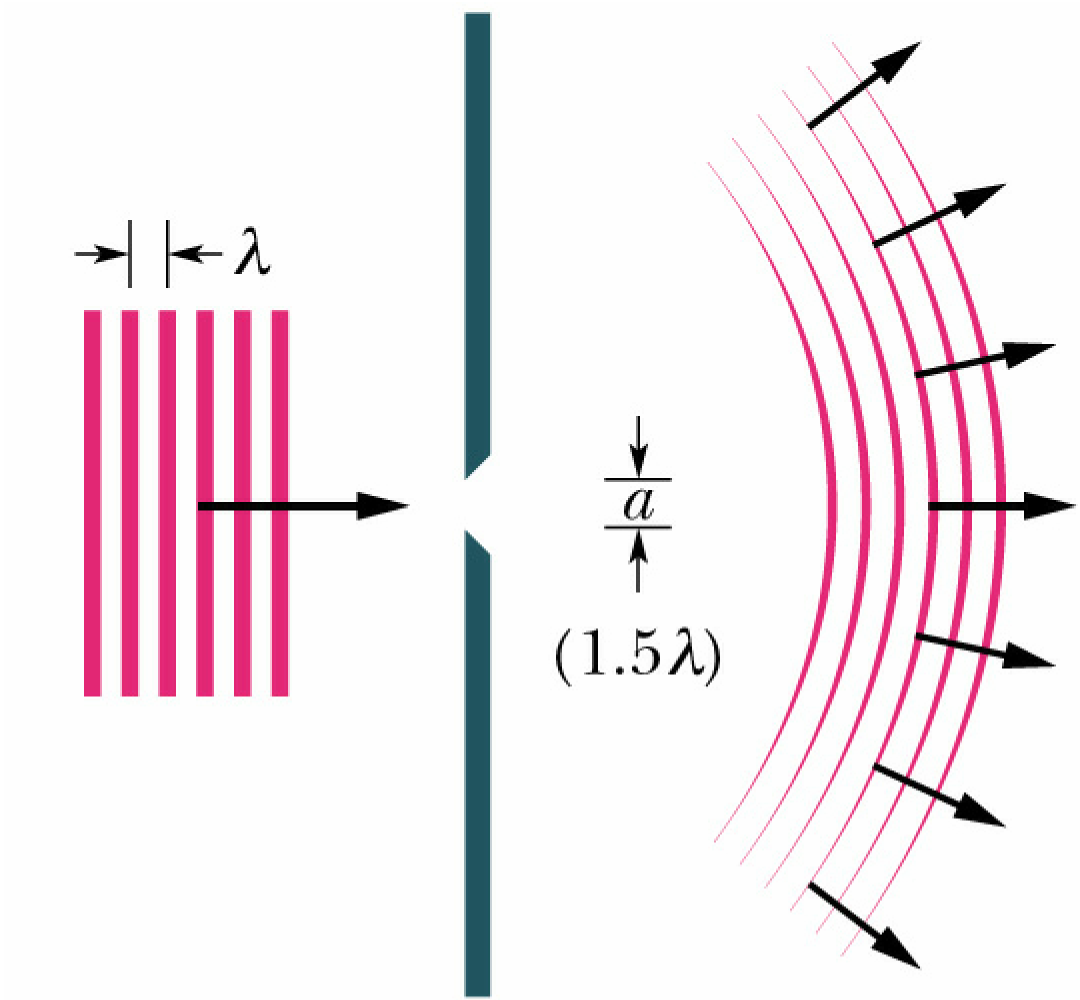


peaks of E field (perpendicular to screen)

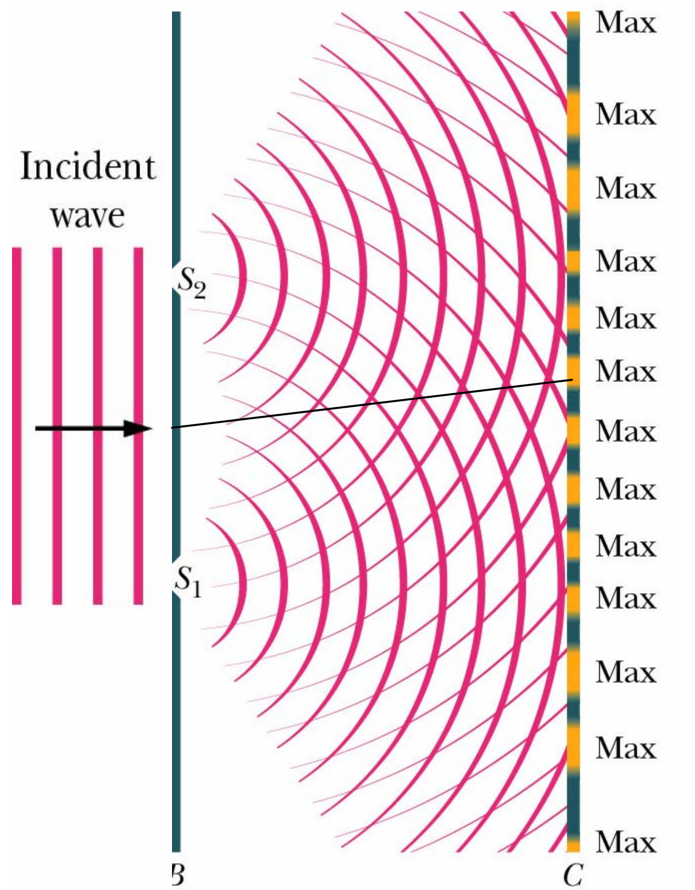
Top view of spherical EM wave



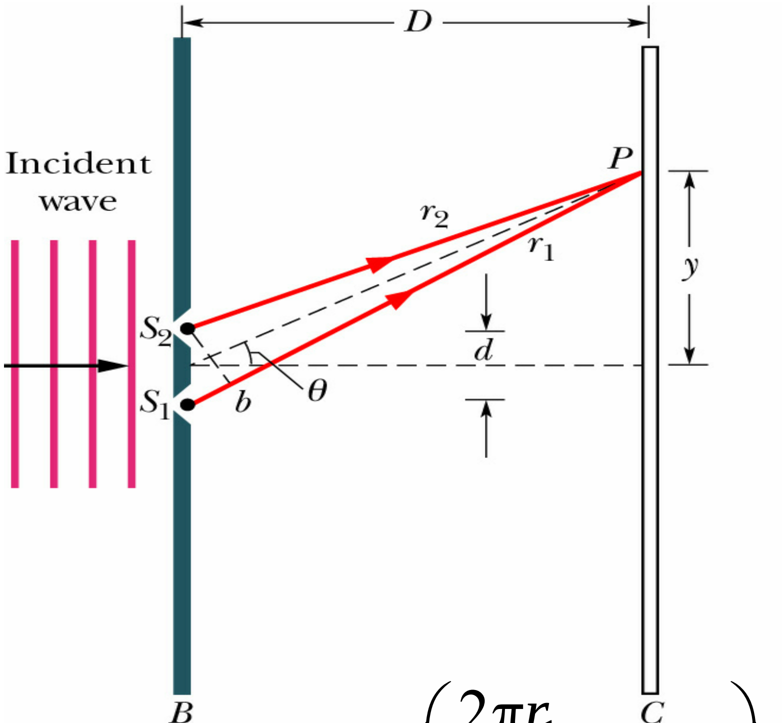
Production of a spherical wave from a plane wave using a slit

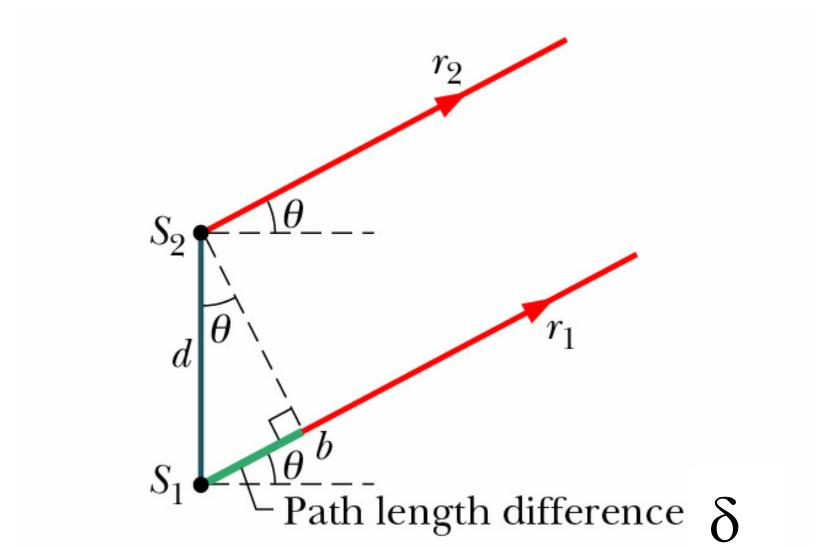


Interference of spherical waves in phase:



Mathematical analysis of bright fringes:





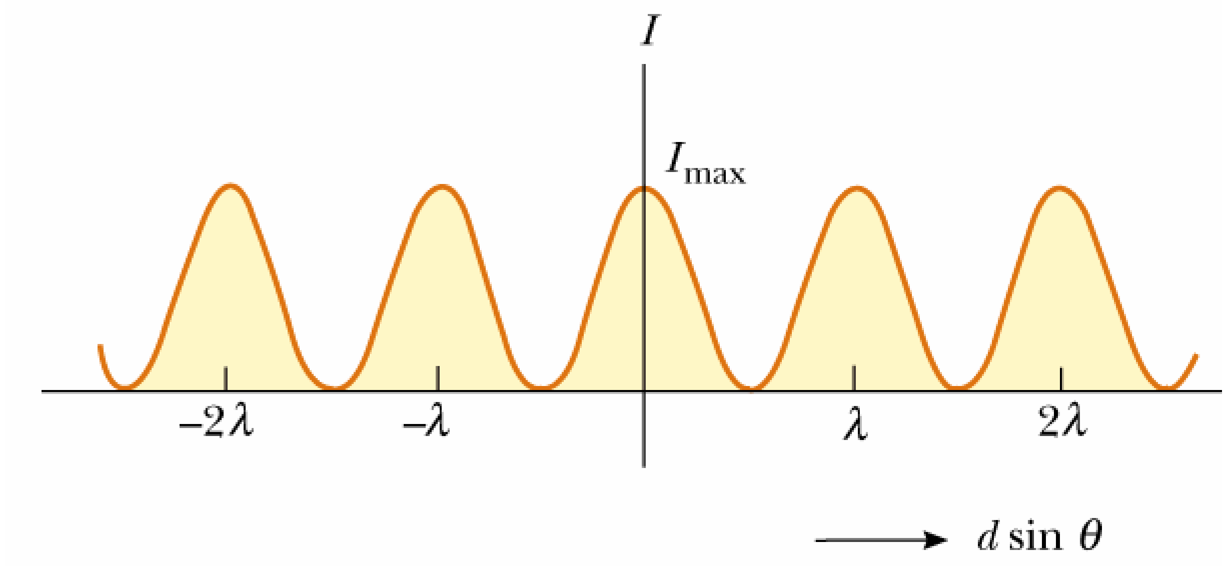
$$r_1 - r_2 = \delta = d\sin\theta$$

$$E(P,t) = E_{\max}\sin\left(\frac{2\pi r_1}{\lambda} - 2\pi f t\right) + E_{\max}\sin\left(\frac{2\pi r_2}{\lambda} - 2\pi f t\right)$$

$$= 2E_{\max}\sin\left(\frac{\pi(r_1 + r_2)}{\lambda} - 2\pi f t\right)\cos\left(\frac{\pi(r_1 - r_2)}{\lambda}\right)$$

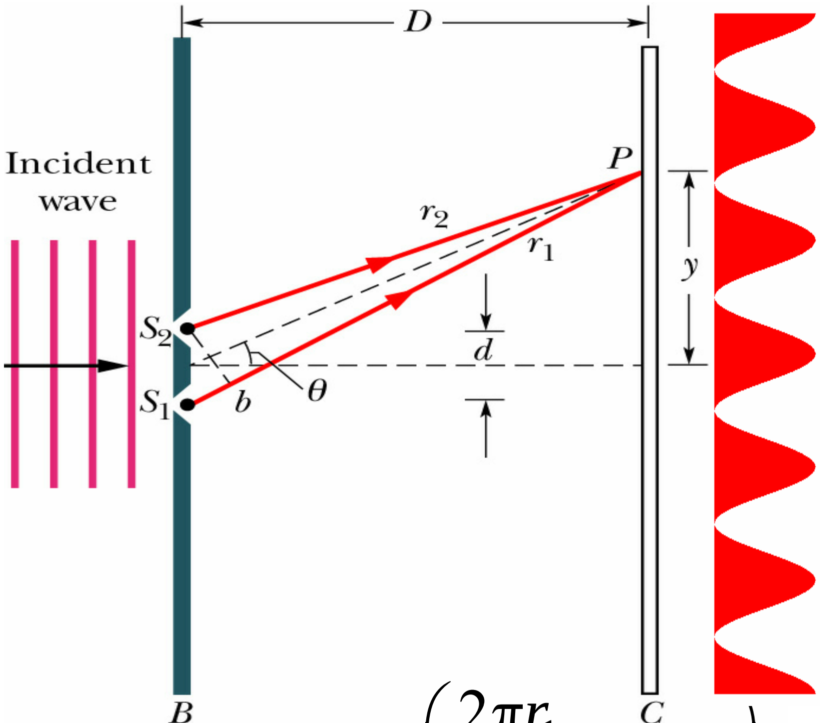
➔ intensity maxima occur for $\frac{\pi(r_1 - r_2)}{\lambda} = m\pi \quad \Rightarrow d\sin\theta = m\lambda$

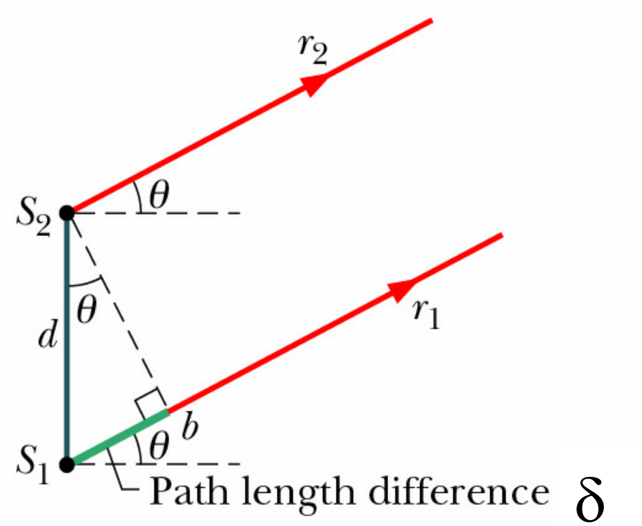
Intensity pattern at screen for double slit:



$$I = |\mathbf{S}|_{av} = \underbrace{\frac{4E_{max}^2}{2\mu_0 c}}_{I_{max}} \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$$

Diffraction pattern from a plane wave incident on a double slit:





δ

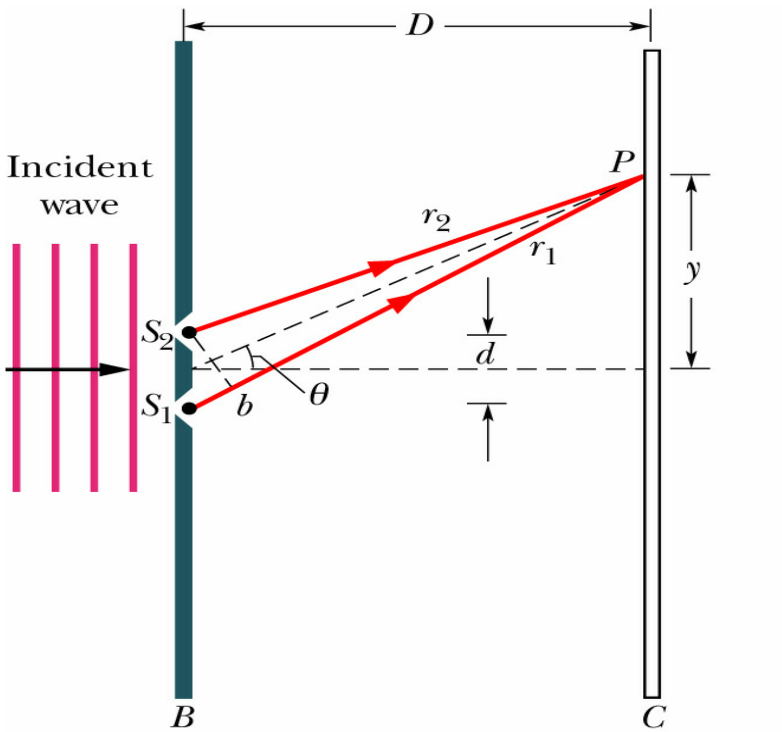
$$r_1 - r_2 = \delta = d \sin\theta$$

$$E(P,t) = E_{\max} \sin\left(\frac{2\pi r_1}{\lambda} - 2\pi f t\right) + E_{\max} \sin\left(\frac{2\pi r_2}{\lambda} - 2\pi f t\right)$$

$$= 2E_{\max} \sin\left(\frac{\pi(r_1 + r_2)}{\lambda} - 2\pi f t\right) \cos\left(\frac{\pi(r_1 - r_2)}{\lambda}\right)$$

➔ intensity maxima occur for $\frac{\pi(r_1 - r_2)}{\lambda} = m\pi \quad \Rightarrow d \sin\theta = m\lambda$

Summary of results:



Constructive interference:

$$d \sin\theta = m\lambda$$

Image on screen:

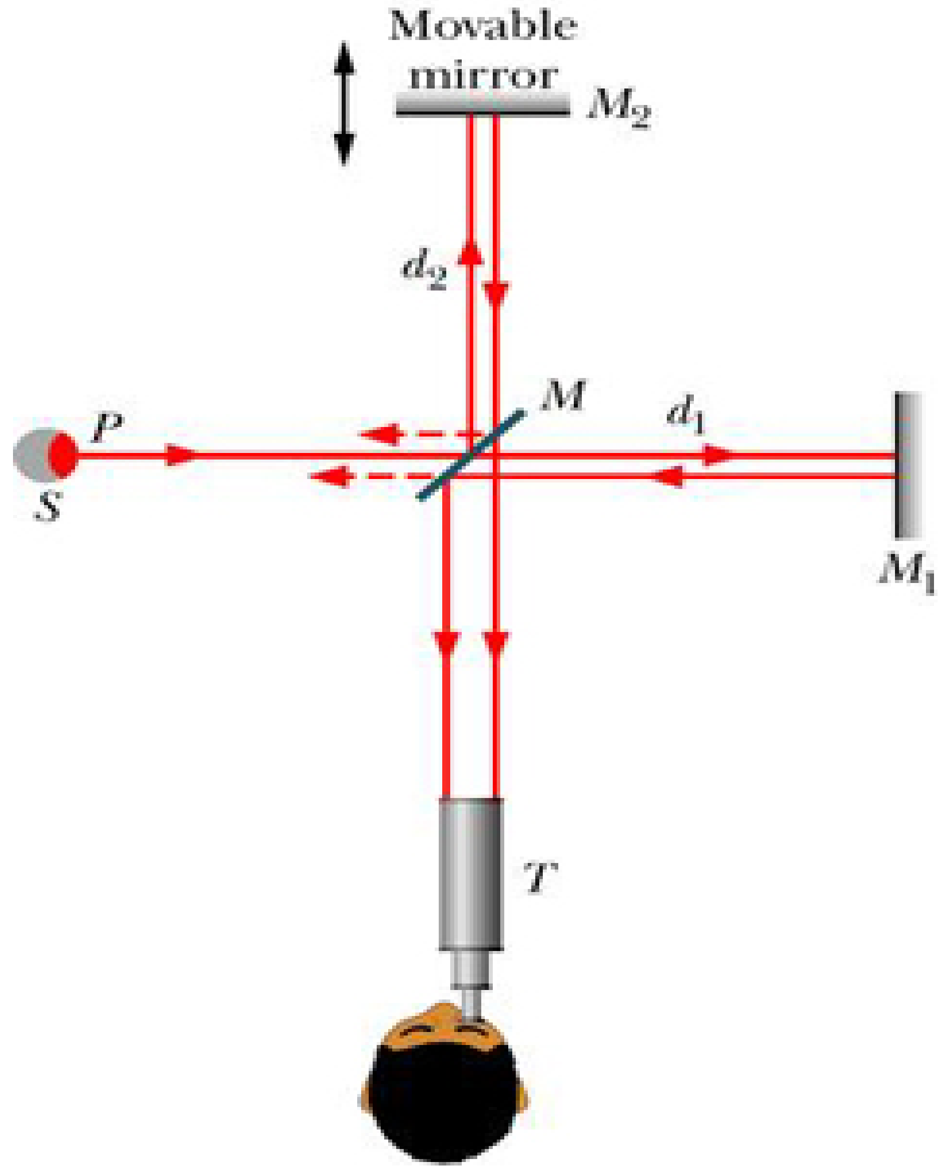
$$y \approx m\lambda \frac{D}{d}$$

Michelson interferometer

$$\varphi = \frac{2\pi}{\lambda}(d_1 - d_2)$$

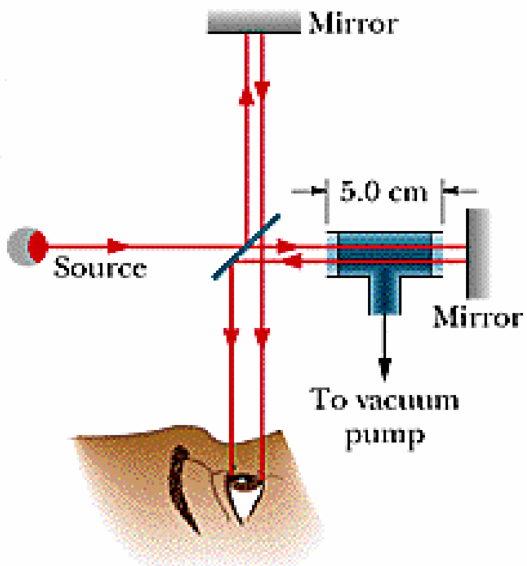
Intensity :

$$I = I_{\max} \sin^2\left(\frac{\varphi}{2}\right)$$



4. [HRW6 36.P.057.] An airtight chamber 5.0 cm long with glass windows is placed in one arm of a Michelson interferometer as indicated in Fig. 36-44. Light of wavelength λ = 500 nm is used. When the air has been completely evacuated from the chamber, there has been a shift of 60 fringes. From these data, find the index of refraction of air at atmospheric pressure.

$$\varphi = \frac{2\pi}{\lambda} 2L\left(n_{air} - 1\right)$$



Online Quiz for Lecture 25

Interference phenomena in light waves -- Apr. 4, 2005

If you hold a compact disk up to a white light, you will see the colors separated. Is this phenomenon more like

(A) an interference pattern produced by reflection from a thin transparent film

or

(B) an interference pattern produced by a grating.

Use the box to explain more details.