Announcements

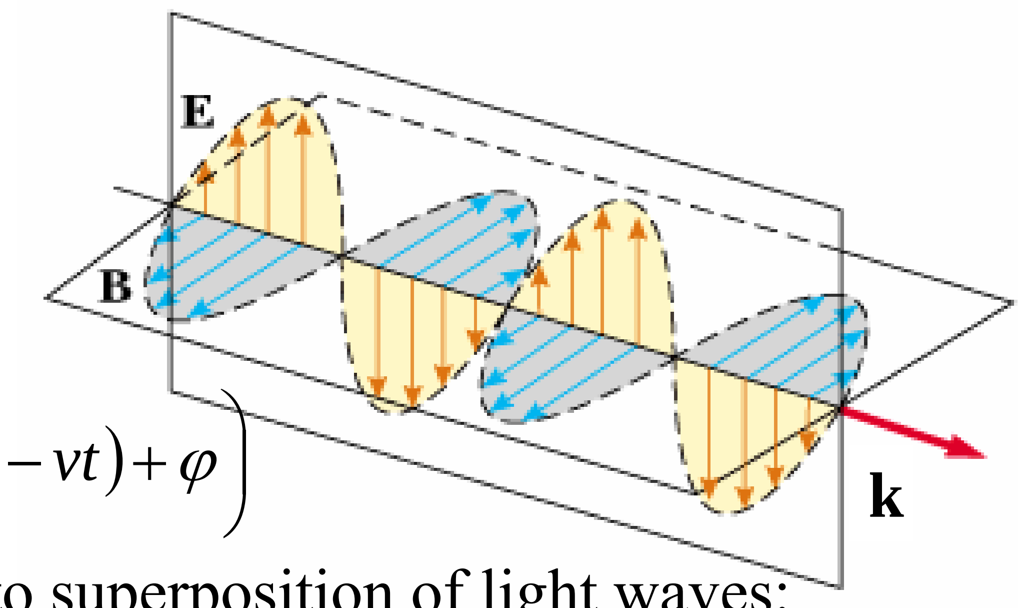
1. New schedule
2. Problem session this evening at 6 PM in Olin 101 ???
3. Extra credit opportunity -- need to find a meeting time for presentations by participants of one of these types:
   - Present detailed solution to one of the exam questions plus another problem of your choosing
   - Present an explanation for a physical phenomenon (such as the optics of a telescope; etc…)
4. Today's topics

   Single slit diffraction

   Diffraction

22	3/28/05	Mirror images	35.1-35.4	35.2,9,10,+	3/30/05
23	3/30/05	Lenses	35.5-35.6	35.18,22,27,30	4/1/05
24	4/1/05	Optical instruments	35.7-35.8	35.32,33,35,37	4/4/05
25	4/4/05	Interference	36.1-36.8	36.6,11,22,57	4/6/05
26	4/6/05	Diffraction	37.1-37.9	37.28,37,39,58	4/8/05
27	4/8/05	Special theory of relativity	38.1-38.9	38.5,16,23,27	4/11/05
28	4/11/05	Special theory of relativity	38.10-38.12	38.31,32,+,+	4/13/05
	4/13/05	Review			
	4/15/05	Exam			
29	4/18/05	Nuclear Physics	43.1-43.8	43.20,26,62,68,69	4/20/05
30	4/20/05	Nuclear Reactions	44.1-44.8	44.3,9,15,44	4/22/05
31	4/22/05	Matter Waves	39.1-39.9	39.51,59,80,82	4/25/05
32	4/25/05	Matter Waves	40.1-40.8	40.1,19,31	4/27/05
	4/27/05	Review			
	5/2/05	Final Exam (2 PM)			

Plane polarized electromagnetic wave at an instant of time:



$$E_y(x,t) = E_{\max} \sin\left(\frac{2\pi}{\lambda}(x - vt) + \varphi\right)$$

Interference effects due to superposition of light waves:

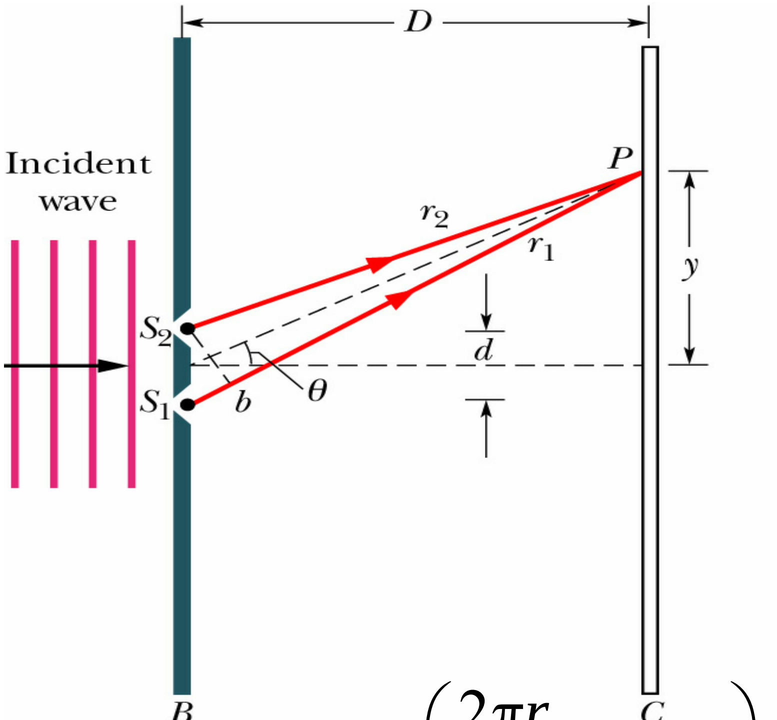
$$E_{tot}(x,t) = E_1(x,t) + E_2(x,t)$$

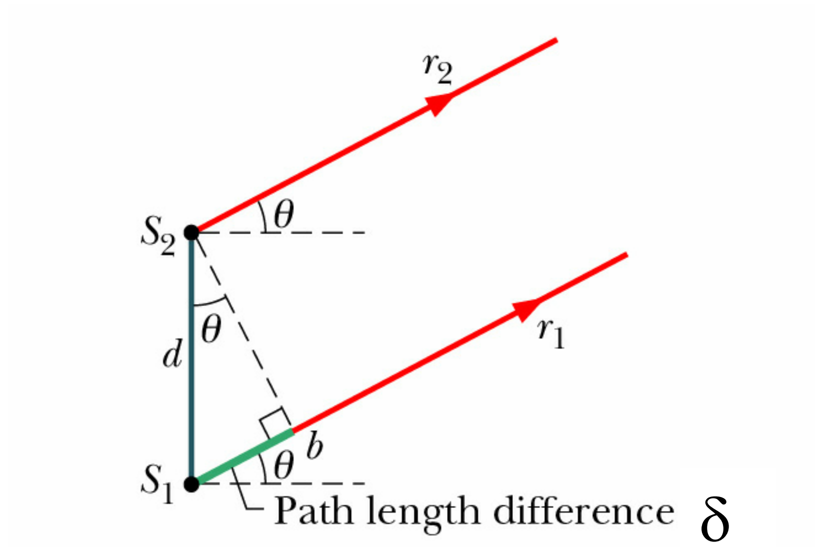
$$= E_{\max} \sin\left(\frac{2\pi}{\lambda}(x - vt)\right) + E_{\max} \sin\left(\frac{2\pi}{\lambda}(x - vt) + \varphi\right)$$

$$= 2E_{\max} \sin\left(\frac{2\pi}{\lambda}(x - vt) + \frac{1}{2}\varphi\right)\cos\left(\frac{\varphi}{2}\right)$$

$$I = |\mathbf{S}|_{av} = \frac{4E_{\max}^2}{2\mu_0 c}\cos^2\left(\frac{\varphi}{2}\right) \equiv I_{\max}\cos^2\left(\frac{\varphi}{2}\right)$$

Young's double slit (idealization)





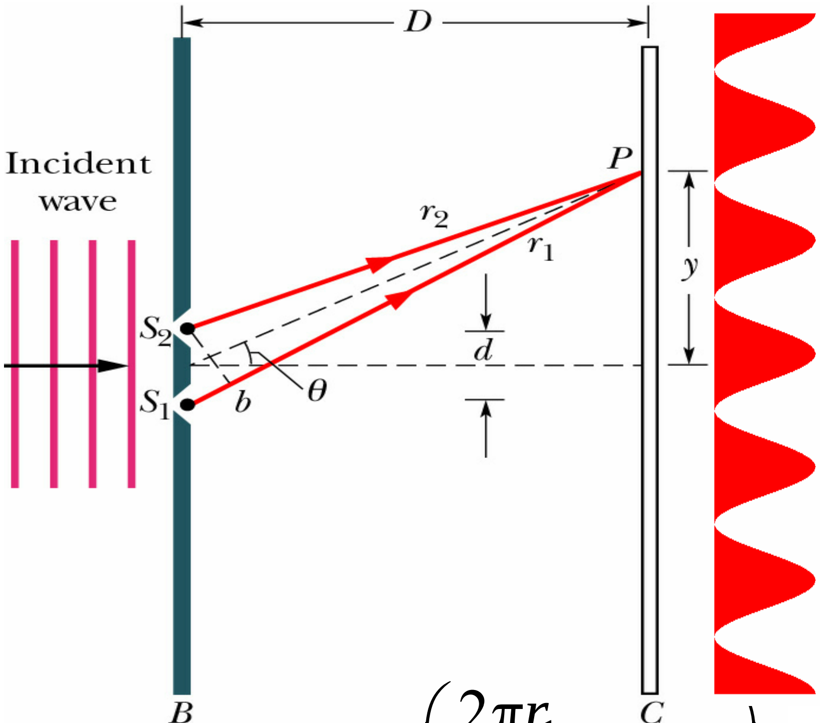
$$r_1 - r_2 = \delta = d \sin \theta$$

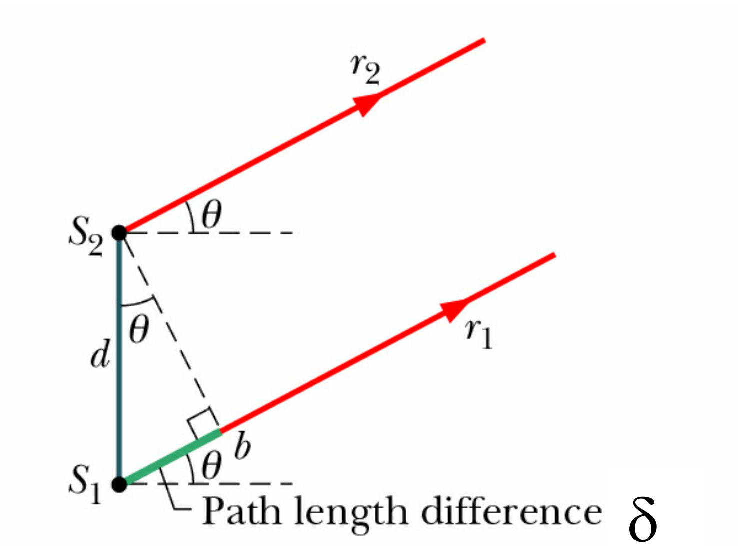
$$E(P,t) = E_{\max} \sin\left(\frac{2\pi r_1}{\lambda} - 2\pi f t\right) + E_{\max} \sin\left(\frac{2\pi r_2}{\lambda} - 2\pi f t\right)$$

$$= 2E_{\max} \sin\left(\frac{\pi(r_1 + r_2)}{\lambda} - 2\pi f t\right) \cos\left(\frac{\pi(r_1 - r_2)}{\lambda}\right)$$

➔ intensity maxima occur for $\frac{\varphi}{2} = \frac{\pi(r_1 - r_2)}{\lambda} = m\pi \Rightarrow d \sin\theta = m\lambda$

Diffraction pattern from a plane wave incident on a double slit:





δ

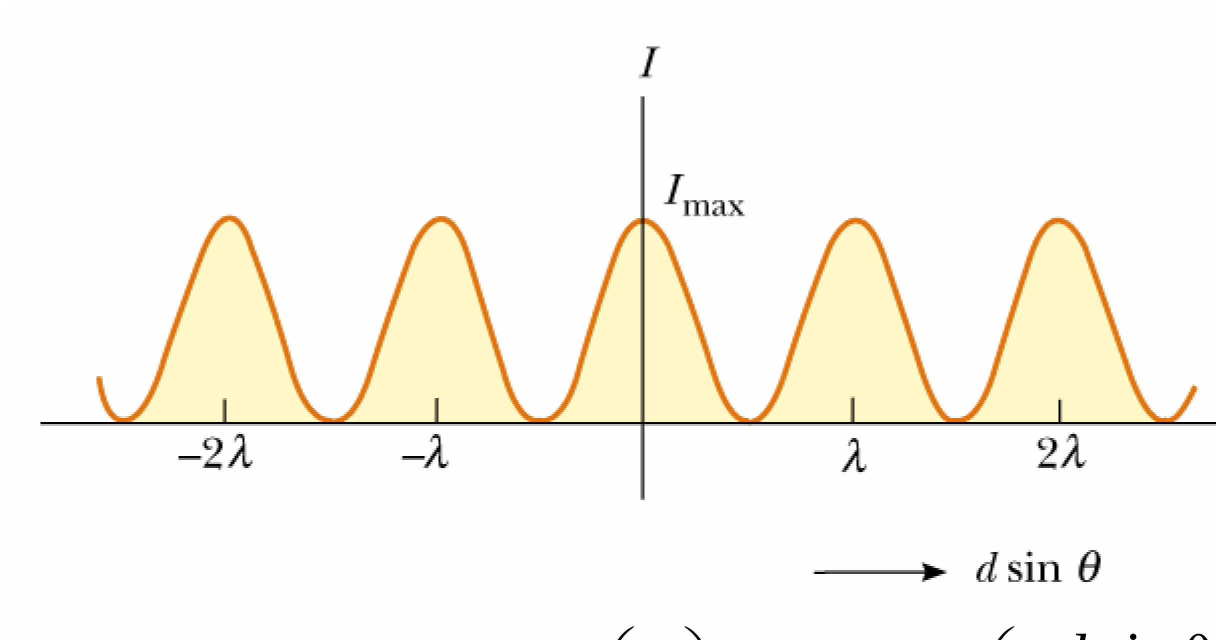
$$r_1 - r_2 = \delta = d \sin\theta$$

$$E(P,t) = E_{\max} \sin\left(\frac{2\pi r_1}{\lambda} - 2\pi f t\right) + E_{\max} \sin\left(\frac{2\pi r_2}{\lambda} - 2\pi f t\right)$$

$$= 2E_{\max} \sin\left(\frac{\pi(r_1 + r_2)}{\lambda} - 2\pi f t\right) \cos\left(\frac{\pi(r_1 - r_2)}{\lambda}\right)$$

➔ intensity maxima occur for $\frac{\pi(r_1 - r_2)}{\lambda} = m\pi \quad \Rightarrow d \sin\theta = m\lambda$

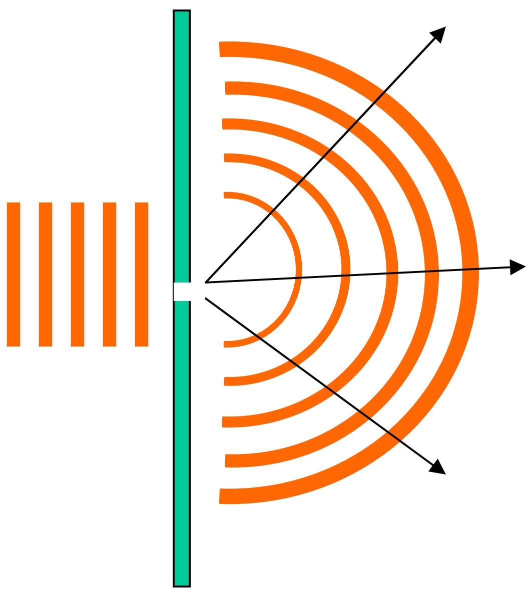
Idealized intensity pattern at screen for double slit:



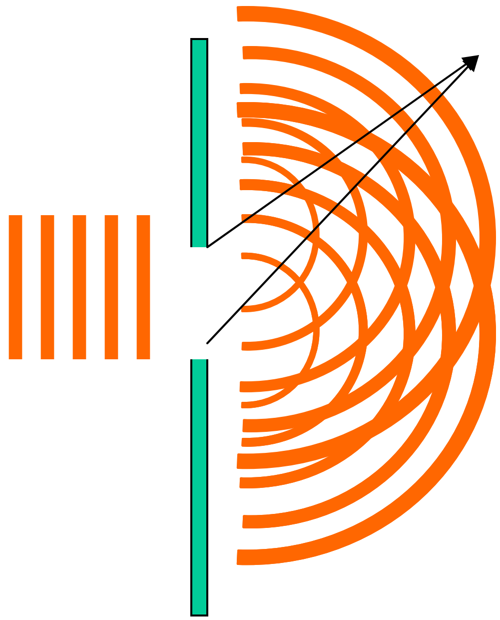
$$I = |\mathbf{S}|_{av} = I_{max} \cos^2\left(\frac{\varphi}{2}\right) = I_{max} \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$$

Actual interference effects within a single finite size slit

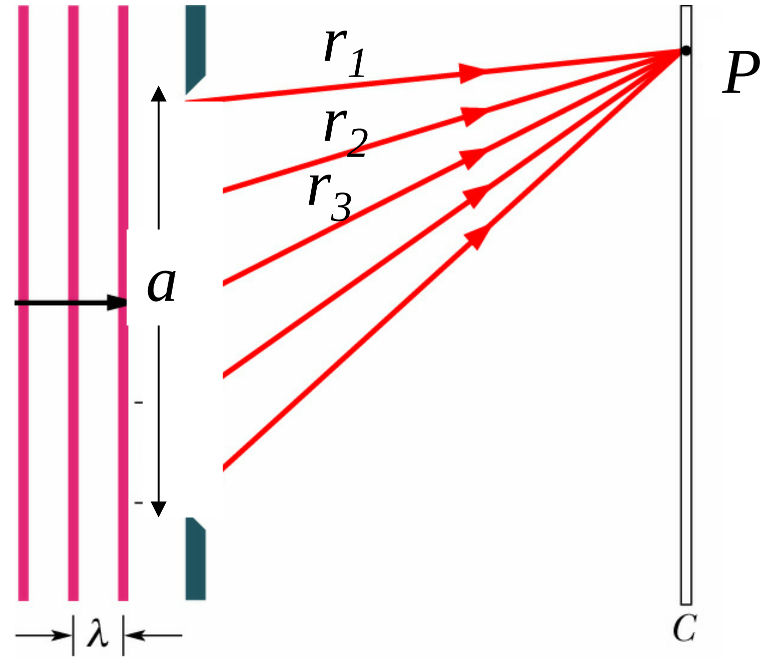
thin slit



thicker slit



Mathematical description of single slit diffraction



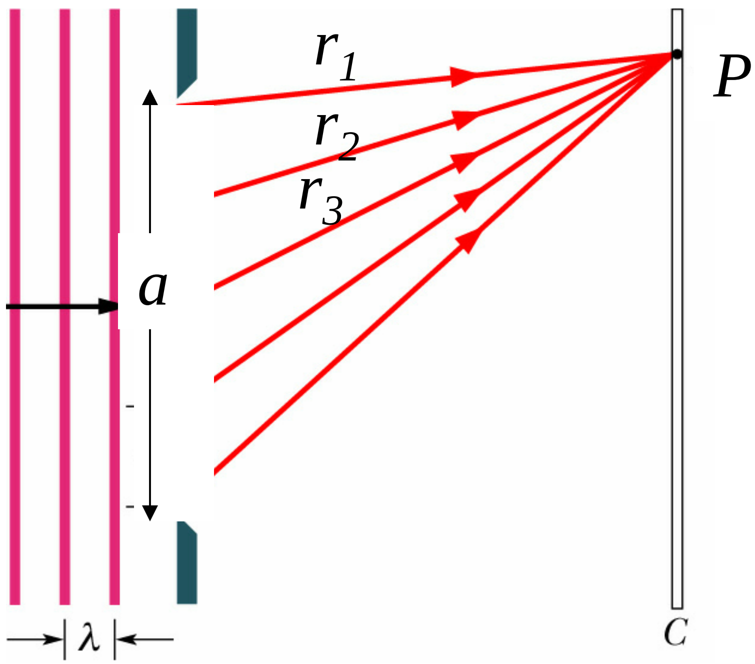
$$E(P,t) = \sum_i E_{\max} \sin\left(\frac{2\pi r_i}{\lambda} - 2\pi f t\right)$$

$$\approx (\text{Constant}) E_{\max} \int_{-a/2}^{a/2} \sin\left(\frac{2\pi\left(r_{av} - x\sin\theta\right)}{\lambda} - 2\pi f t\right) dx$$

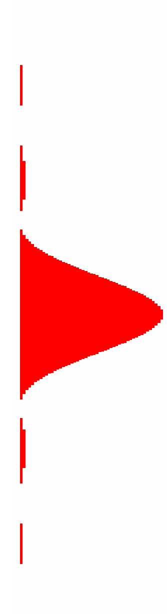
$$= 2(\text{Constant}) E_{\max} \sin\left(\frac{2\pi r_{av}}{\lambda} - 2\pi f t\right)\left\{\frac{\sin\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\left(\frac{\pi a \sin\theta}{\lambda}\right)}\right\}$$

Single slit intensity pattern:

$$\langle I \rangle_{av} = I_{\max} \left\{ \frac{\sin\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\left(\frac{\pi a \sin\theta}{\lambda}\right)} \right\}^2$$

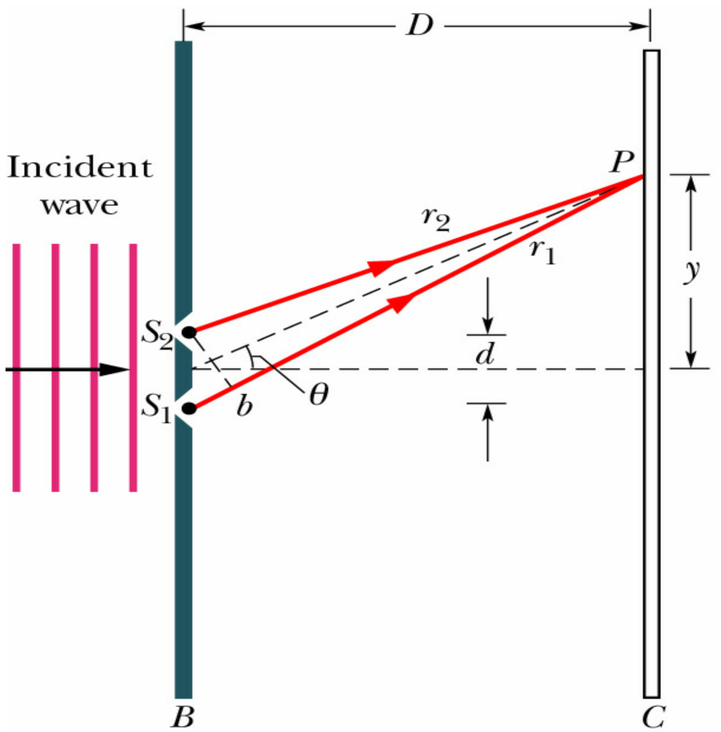


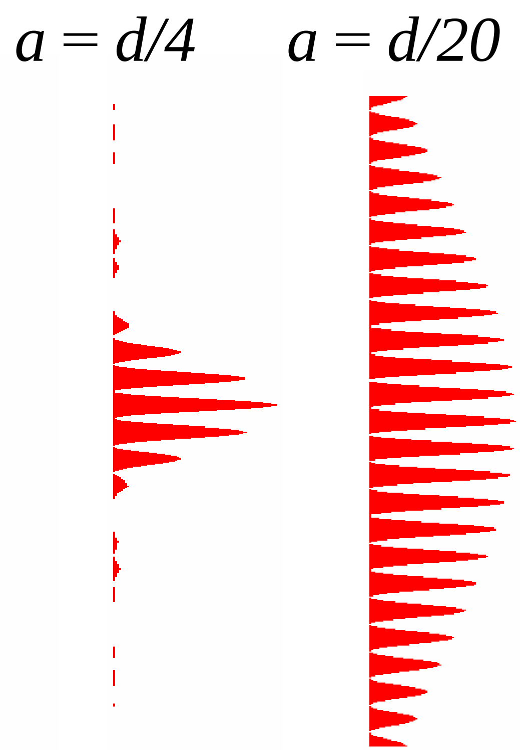
small *a*



smaller *a*

Effect of slit size on double slit pattern





$$I = I_{\max}\left[\cos\left(\frac{\pi d \sin\theta}{\lambda}\right)\right]^2 \left[\frac{\sin\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\frac{\pi a \sin\theta}{\lambda}}\right]^2$$

1. [HRW6 37.P.028.] For d = 6a in Fig. 37-38, how many bright interference fringes lie in the central diffraction envelope?

$$I = I_{max}\left[\cos\left(\frac{\pi d \sin\theta}{\lambda}\right)\right]^2\left[\frac{\sin\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\frac{\pi a \sin\theta}{\lambda}}\right]^2$$

$$-\pi < \frac{\pi a \sin\theta}{\lambda} < \pi$$

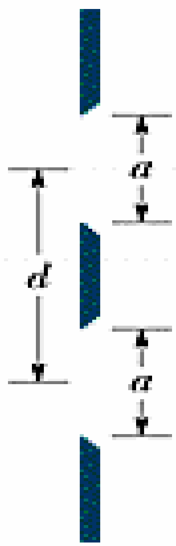
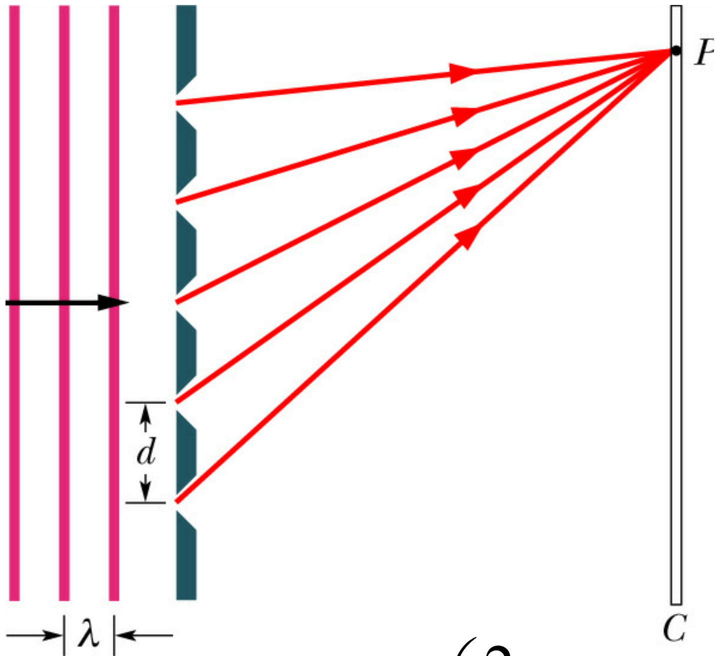
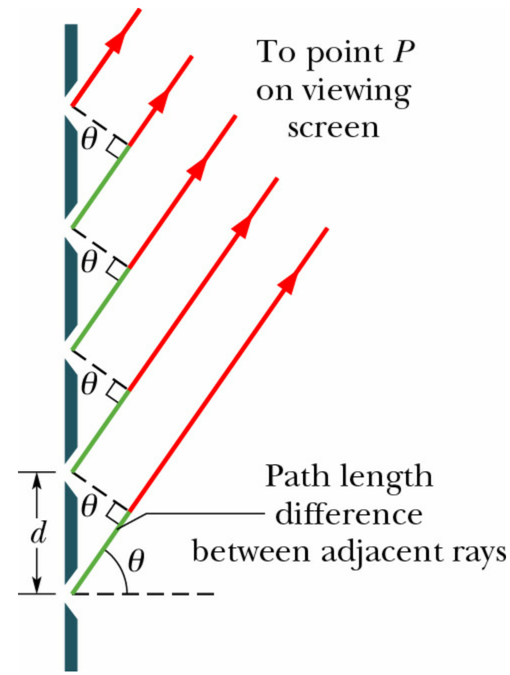


Figure 37-38.

Diffraction pattern for multiple slits – diffraction grating

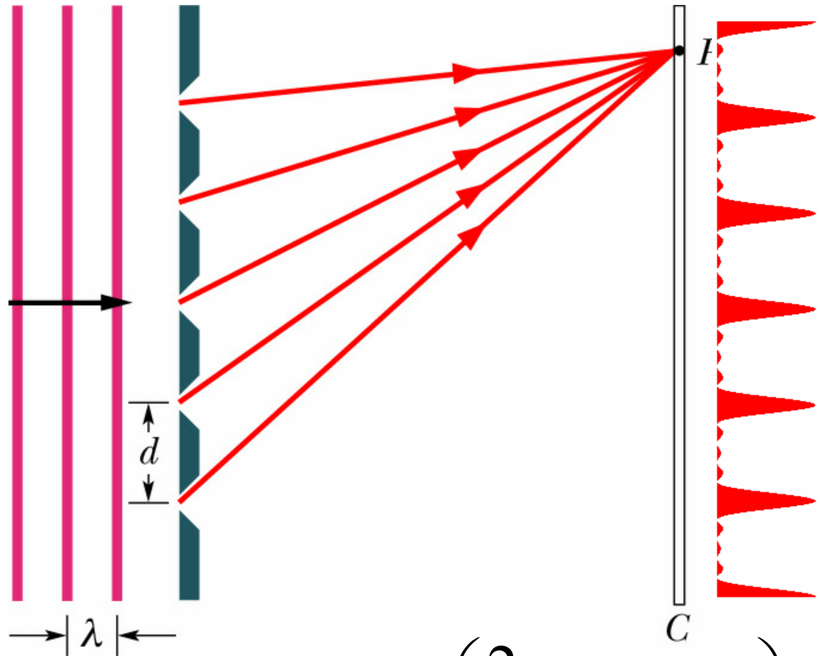


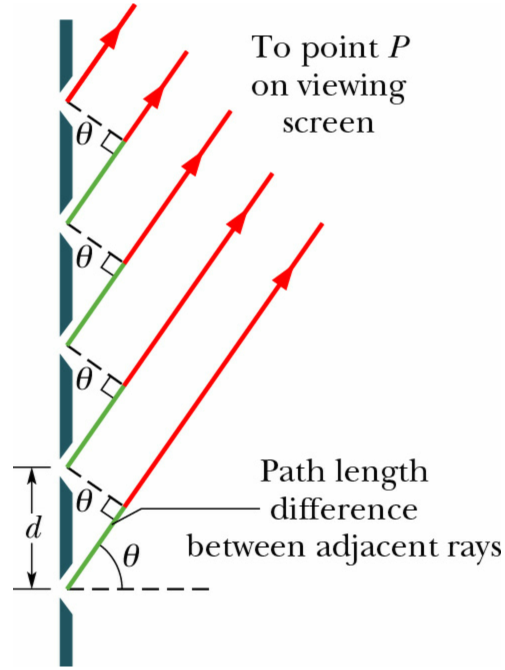


$$E(P,t) = \sum_i E_{\max} \sin\left(\frac{2\pi r_i}{\lambda} - 2\pi f t\right)$$

$$= E_{\max} \sin\left(\frac{2\pi r_{av}}{\lambda} - 2\pi f t\right) \frac{\sin\left(\frac{N\pi d \sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi d \sin\theta}{\lambda}\right)}$$

Diffraction pattern for *N* slits – diffraction grating





$$E(P,t) = \sum_i E_{\max} \sin\left(\frac{2\pi r_i}{\lambda} - 2\pi f t\right)$$

$$= E_{\max} \sin\left(\frac{2\pi r_{av}}{\lambda} - 2\pi f t\right) \frac{\sin\left(\frac{N\pi d \sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi d \sin\theta}{\lambda}\right)}$$

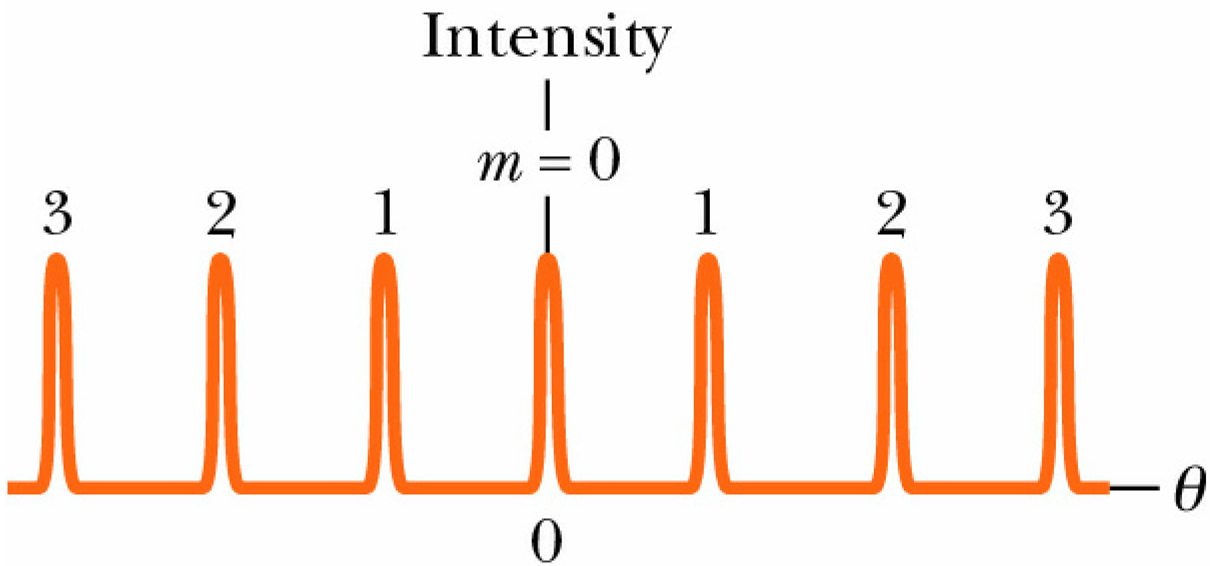
Intensity maxima at

$d \sin\theta = m\lambda$

Intensity pattern for *N* slits

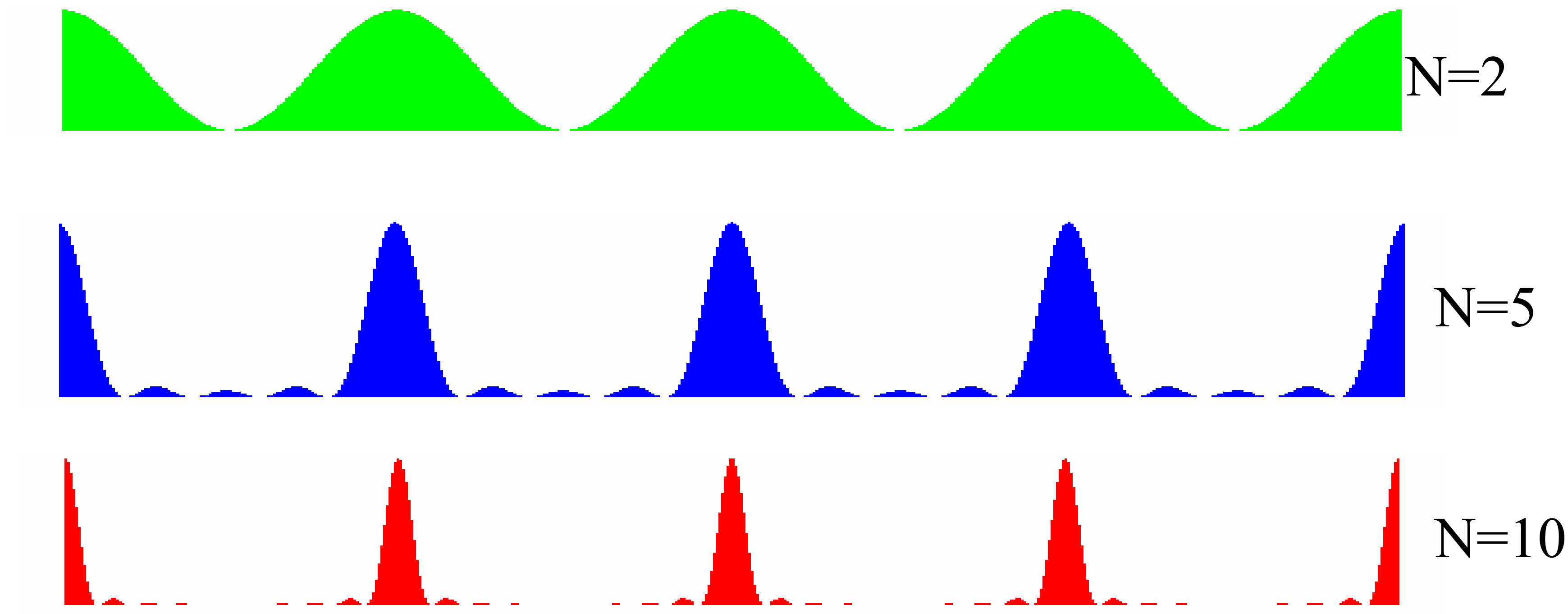
$$I = I_{max}\left[\frac{\sin\left(\frac{N\pi d\sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi d\sin\theta}{\lambda}\right)}\right]^2$$

Peaks at $d\sin\theta = m\lambda$ (same as for double slit but peak widths much narrower)

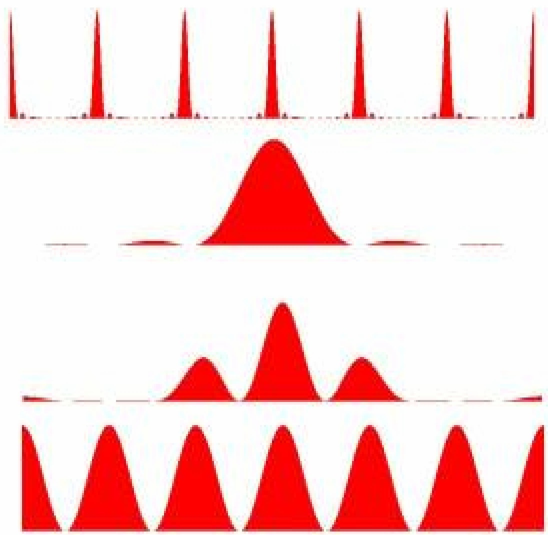


Intensity pattern from multiple slit grating:

$$I = I_{max} \left[\frac{\sin\left(\frac{N \pi d \sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi d \sin\theta}{\lambda}\right)} \right]^2$$



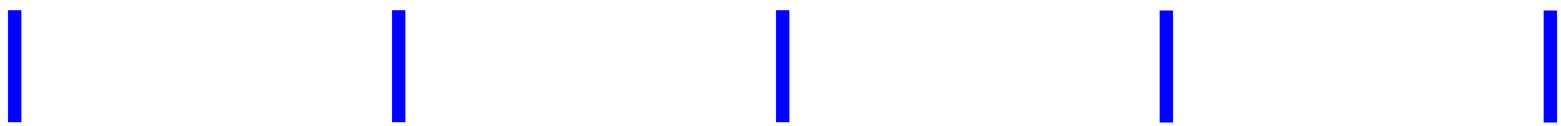
Online Quiz for Lecture 26
Diffraction patterns -- Apr. 6, 2005



1. thin multiple slits
2. fat single slit
3. fat double slits
4. thin double slits

Consider the 4 plots which represent the intensity of monochromatic light on a screen a large distance away from various slit configurations. For each of the plots identify the type of slits -- thin double slits, fat double slits, thin multiple slits, fat multiple slits, 1 thin slit, 1 fat slit, etc. Write your answers in the space provided. The computer won't be able to grade your responses.

Peer instruction question



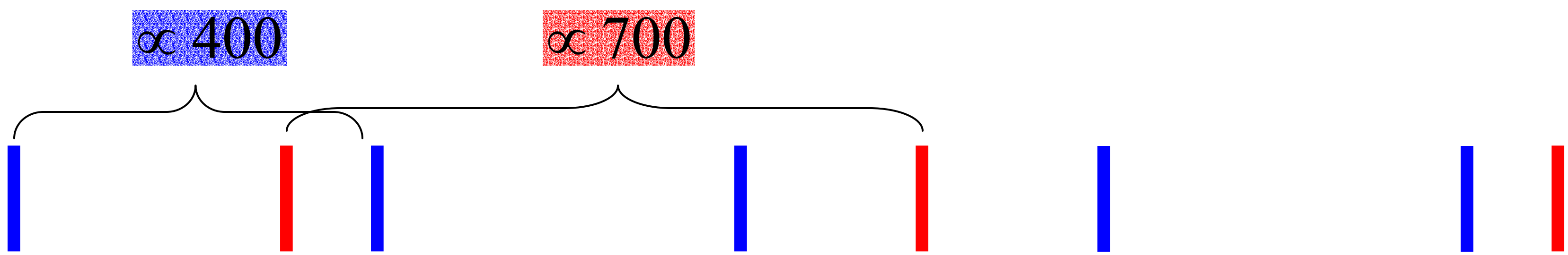
The above lines show the diffraction pattern of blue light from a given grating. Where would you expect red light to be diffracted?

(A) Approximately midway between each blue line

(B) Separation between red lines is greater than that of blue lines

(C) Separation between red lines is smaller than that of blue lines

Question from last time --



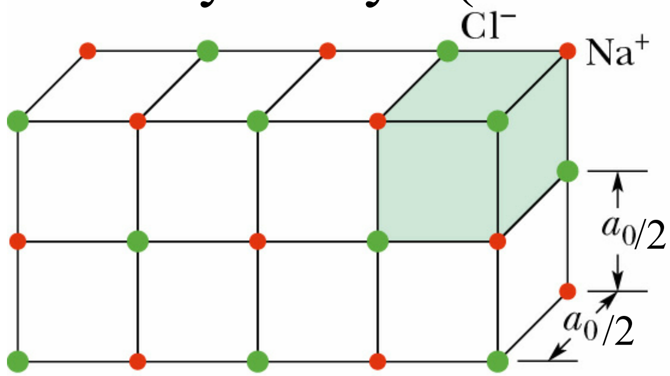
The above lines show the diffraction pattern of blue light from a given grating. Where would you expect red light to be diffracted?

(A) Approximately midway between each blue line

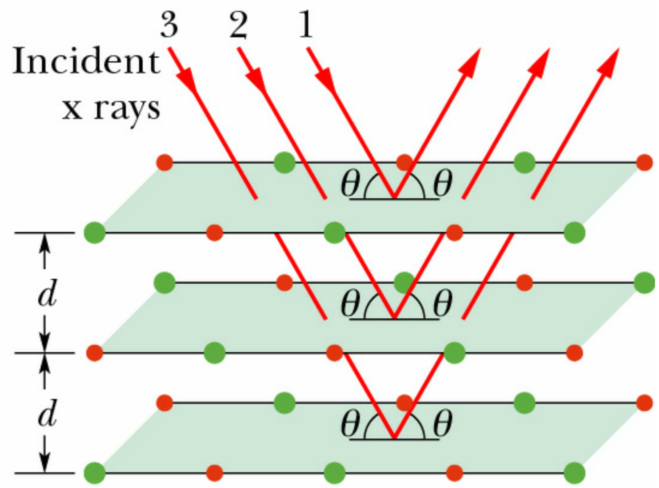
(B) Separation between red lines is greater than that of blue lines

(C) Separation between red lines is smaller than that of blue lines

Diffraction by X-rays ($\lambda \approx 0.1$ nm)

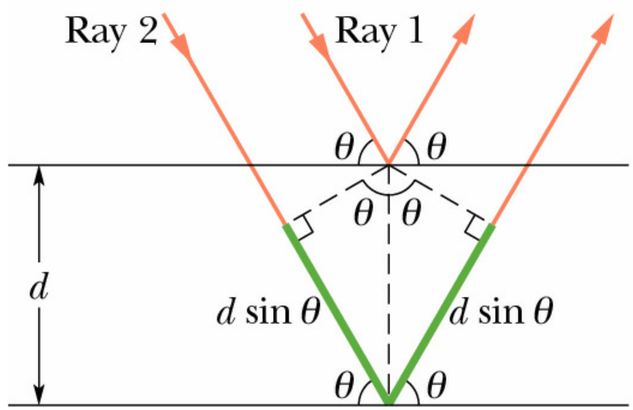


(a)

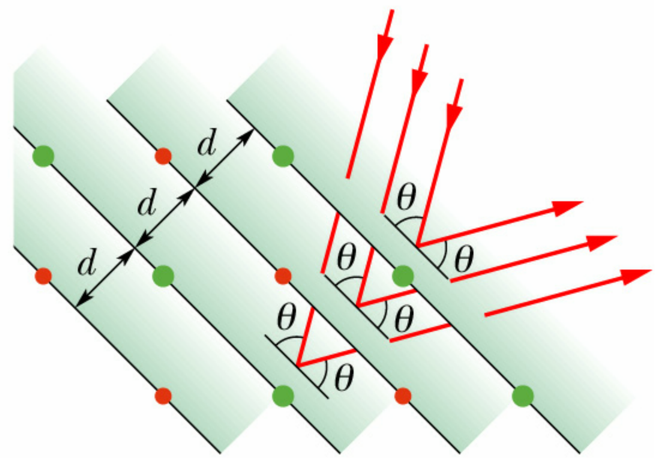


(b)

NaCl $a_0 \approx 0.56$ nm



(c)



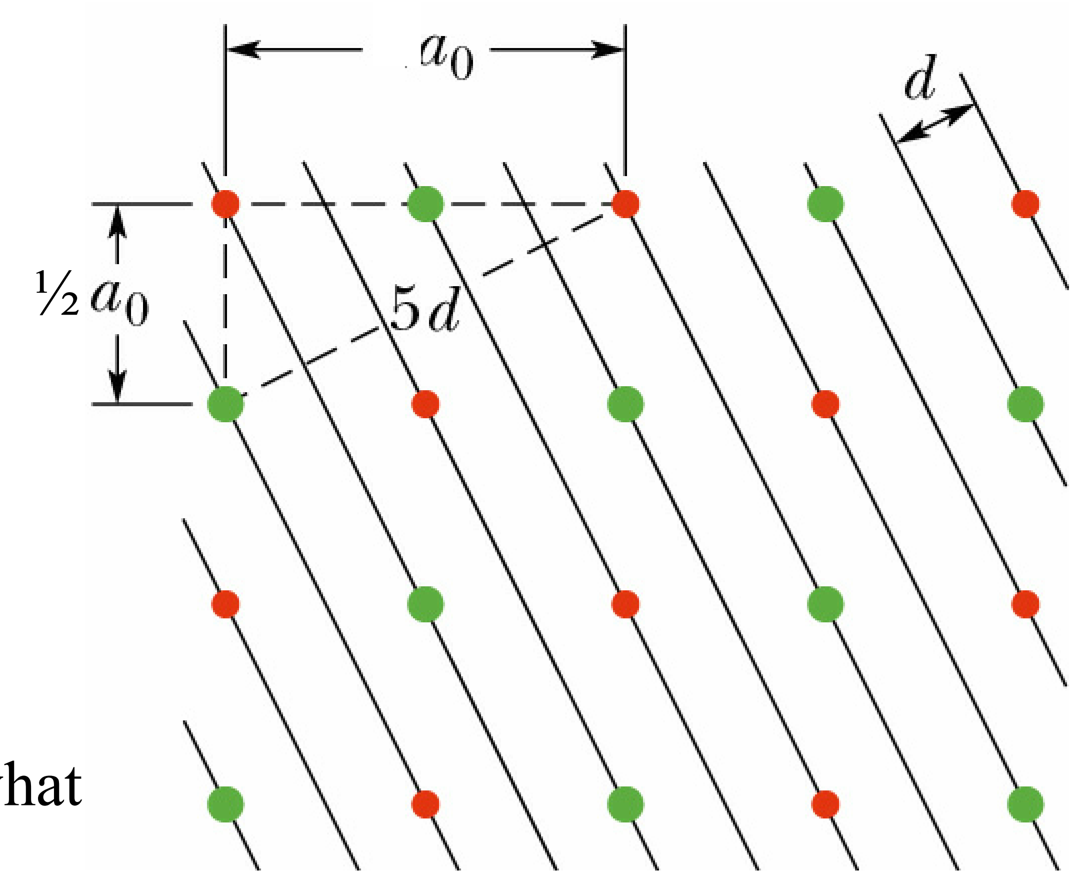
(d)

Bragg condition:

$$2d \sin\theta = m\lambda$$

Bragg condition:

$$2d \sin \theta = m\lambda$$



Example $\lambda = 0.154$ A°

$$d = \frac{a_0}{2\sqrt{5}}$$

If $\theta = 37.95°$ for m=1, what is a_0?

$$2d \sin\theta = 2\frac{a_0}{2\sqrt{5}} \sin\theta = \lambda$$

$$a_0 = \frac{\lambda\sqrt{5}}{\sin\theta} = 0.56 \text{ nm}$$

4. [HRW6 37.P.058.] In Fig. 37-44, first-order reflection from the reflection planes shown occurs when an x-ray beam of wavelength 0.270 nm makes an angle of 63.8° with the top face of the crystal. What is the unit cell size a_0?

[ ] nm

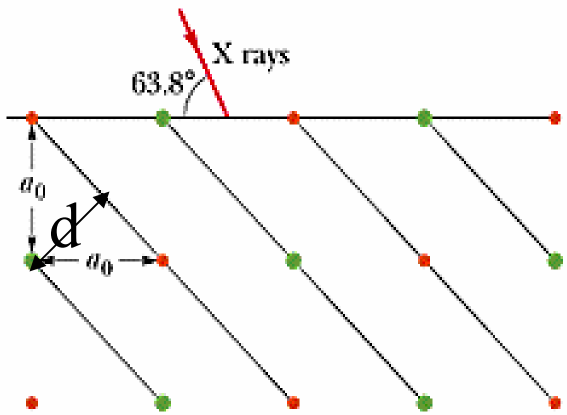


Figure 37-44.

For this case: $d = a_0 / \sqrt{2}$

Note: strictly speaking, lattice constant = $2a_0$.