Announcements

1. Fourth hour exam scheduled for Friday 4/15/05 covering material in Chapters 35-38 (alternate exam date ~4/19 or 4/20)
   Prepare equation sheet
   Review homework and class examples
   Previous exam
2. Remaining classes will cover nuclear physics and some topics in materials physics
3. Extra credit??
4. Today's topic Einstein's special theory of relativity
   Relationships between frames of reference
   Doppler effect for EM waves

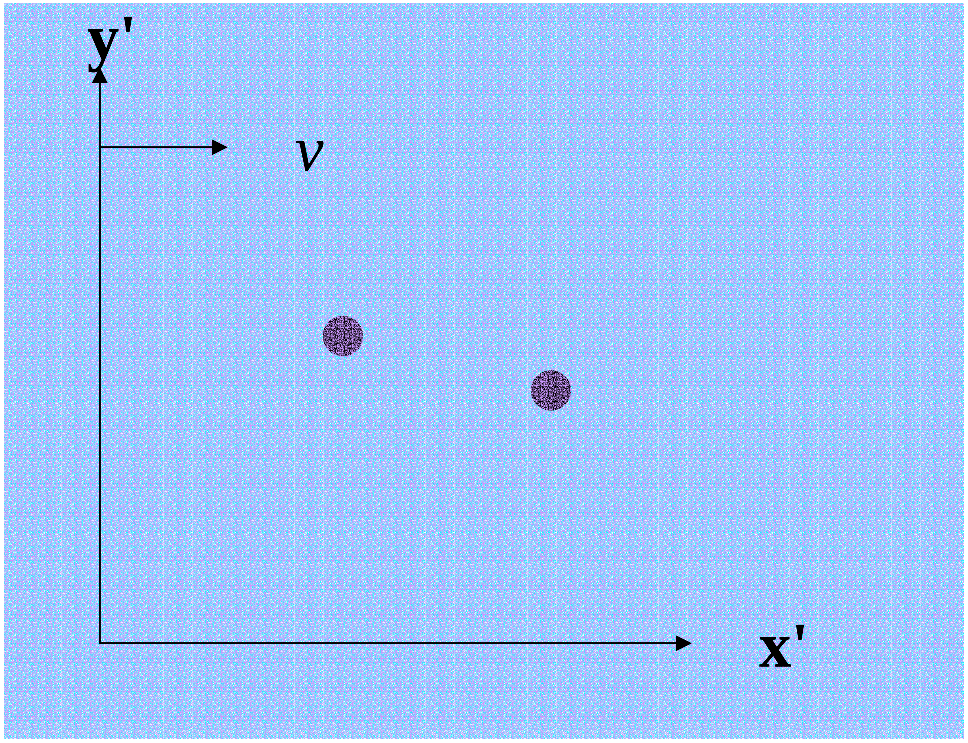
Some results from the special theory of relativity

Postulates:

- The fundamental laws of physics (Newton's laws, Maxwell's equations, etc.) are the same in all inertial reference frames. ("inertial reference frame" == reference frame moving at a constant velocity)
- The speed of light in vacuum c = 299792458 m/s is measured to be the same in all inertial reference frames.

The effects:

- c is the limiting speed in vacuum.
- The Lorentz transformation describes position and time relationships between frames of reference
- New formulations of momentum, and energy.



Lorentz transformation: $x' = \gamma(x - vt)$ $\qquad x = \gamma(x' + vt')$

$$\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad y' = y \qquad y = y'$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \qquad t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$

Properties of γ factor: $$\gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \qquad 1 \le \gamma \le \infty$$

For example:

If $v = 0.5c, \gamma = 1.1547$

$v = 0.994c, \gamma = 9.14243$

$v = 0.99994c, \gamma = 91.2885$

$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$

$$y' = y \qquad y = y'$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \qquad t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$

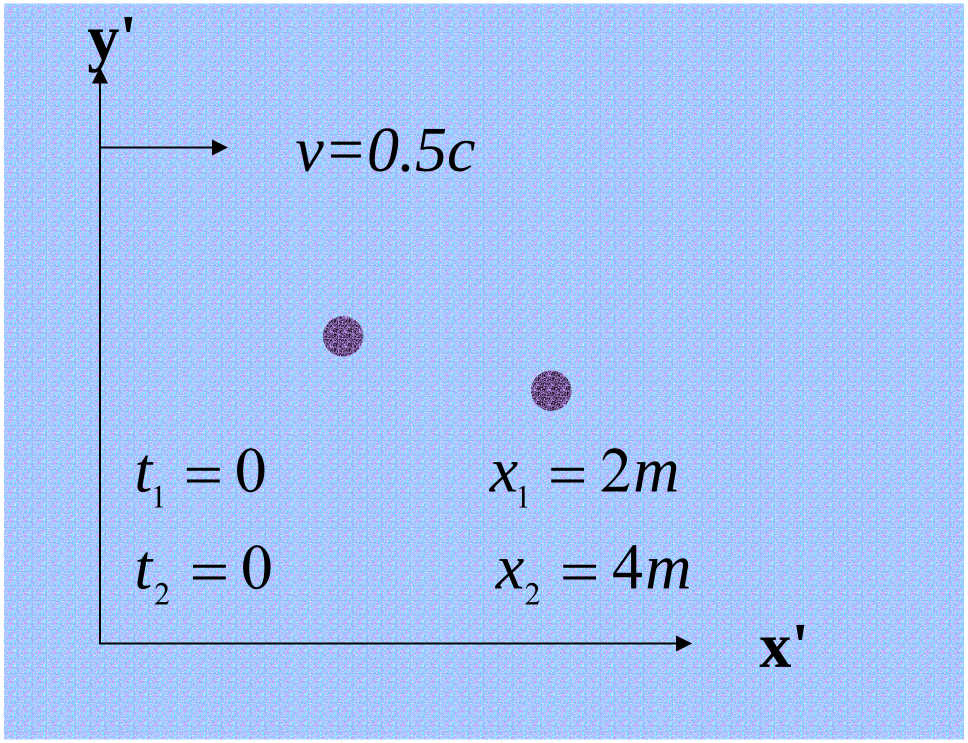
Note that:

$\Delta x = \gamma \Delta x'$ if $\Delta t' = 0$

$\Delta x' = \gamma \Delta x$ if $\Delta t = 0$

$\Delta t = \gamma \Delta t'$ if $\Delta x' = 0$

$\Delta t' = \gamma \Delta t$ if $\Delta x = 0$



$$t_1' = \gamma\left(t_1 - \frac{v}{c^2}x_1\right) = \gamma\left(-0.5\frac{2m}{3\times10^8 m/s}\right) = -3.849\times10^{-9}s$$

$$t_2' = \gamma\left(-0.5\frac{4m}{3\times10^8 m/s}\right) = -7.698\times10^{-9}s$$

$$x_1' = \gamma(x_1 - vt_1) = \gamma(2m) = 2.3094m$$

$$x_2' = \gamma(4m) = 4.6188m$$

Example:

Consider some measurable process such as a decay of a cosmic ray particle $\mu \rightarrow e + \nu + \bar{\nu}$ which is known to the follow the relationship, $N_\mu(t) = N_0 e^{-t/\tau}$ with τ=2.2μs.

In a classic experiment, Rossi and Hall (*Phys. Rev.* **59**,223 (1941)), measured μ particles traveling with *v*=0.994*c* on the top and bottom of a mountain with Δ*x* = 2000 m .

Δ*t*=2000m/0.994*c*=6.7μs ➔ expect $\frac{N_\mu(6.7\mu s)}{N_0} = e^{-6.7/2.2} = 0.048$

➔ found $\frac{N_\mu}{N_0} \approx 0.72 = e^{-0.7/2.2}$

$$\text{Infer}: \quad \underbrace{\Delta t_\mu}_{0.7\ \mu\text{s}} = \underbrace{\Delta t_{Earth}}_{6.7\ \mu\text{s}} / \gamma \qquad \gamma = \frac{1}{\sqrt{1-(0.994)^2}} = 9.1$$

Peer instruction question

How can you explain this "time dilation" from the point of view of the meson?

(A) Who cares what the meson thinks.

(B) Meson doesn't know and therefore is not effected by the fact that a crazy scientist measures a time interval of 9.1 times longer than 0.7 μs.

(C) Meson has a good reason to think that it traveled from the top to the bottom of the mountain in 0.7 μs.

What the meson thinks:

$$\Delta t'_{\mu} = \frac{\Delta x'_{\mu}}{v}$$

$$\Delta x'_{\mu} = \frac{\Delta x_{Earth}}{\gamma}$$

$$\Delta t'_{\mu} = \frac{\Delta x_{Earth}}{\gamma v} = \frac{\Delta t_{Earth}}{\gamma}$$ ➔ agrees with scientist

Moving clocks run slowly: $\Delta t' = \dfrac{\Delta t}{\gamma}$

Moving rulers are shortened: $\Delta x' = \dfrac{\Delta x}{\gamma}$

Lorentz transformation of velocities

$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$

$$y' = y \qquad y = y'$$

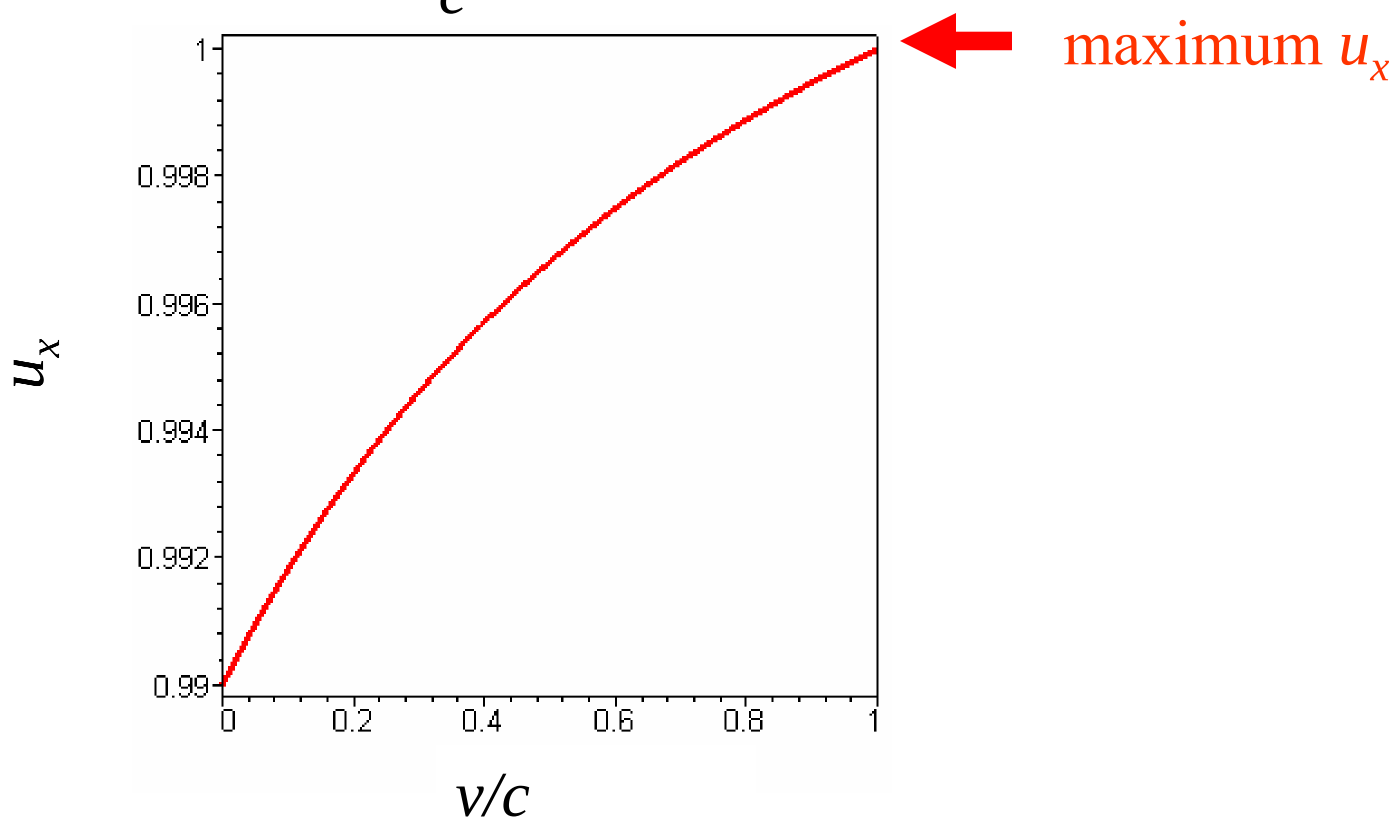
$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \qquad t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$

$$u'_x = \frac{dx'}{dt'} = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} \qquad u_x = \frac{dx}{dt} = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}}$$

$$u'_y = \frac{dy'}{dt'} = \frac{u_y}{\gamma\left(1 - \frac{u_x v}{c^2}\right)} \qquad u_y = \frac{dy}{dt} = \frac{u'_y}{\gamma\left(1 + \frac{u'_x v}{c^2}\right)}$$

Behavior of transformation for u_x'$=0.99c$

$$u_x = \frac{dx}{dt} = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}}$$



3. [HRW6 38.P.023.] Galaxy A is reported to be receding from us with a speed of 0.49*c*. Galaxy B, located in precisely the opposite direction, is also found to be receding from us at this same speed. What recessional speed would an observer on Galaxy A find

(a) for our galaxy and

c

(b) for Galaxy B?

c

Lorentz transformations for electromagnetic waves

$$E = E_{\max} \sin(kx - \omega t) = E_{\max} \sin(k'x' - \omega' t')$$

$$x' = \gamma(x - vt) \qquad k' = \gamma\left(k - \frac{v\omega}{c^2}\right)$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \qquad \omega' = \gamma\left(\omega - \frac{v}{c}k\right)$$

$$|k'| = \frac{\omega'}{c} \qquad \Rightarrow \frac{\omega'}{c} = \gamma\frac{\omega}{c}\left(1 - \frac{v}{c}\right)$$

$$\omega' = \omega\sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}} \qquad \text{or} \qquad \boxed{f' = f\sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}}}$$

Doppler effect for electromagnetic waves

$$f' = f\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$

c == velocity of light

$-$ ➔ away

$+$ ➔ towards

Doppler effect for sound waves :

$$f' = f\,\frac{1 \pm v_D/s \;\; {}^{\text{toward}}_{\text{away}}}{1 \mp v_S/s \;\; {}^{\text{toward}}_{\text{away}}} \quad \begin{matrix}\text{(detector)}\\ \text{(source)}\end{matrix}$$

s == velocity of sound

Online Quiz for Lecture 27
Diffraction patterns -- Apr. 8, 2005

Suppose a police person is using a radar detector (f_0=1x10^9 Hz) to find speeders by detecting the frequency of radar reflected from the speeding car f. These questions ask you to find the frequency shift $f - f_0$.

1. What is the shift if the speeding car is moving 30 m/s away from the radar detector?
   A. -100 Hz
   B. -200 Hz
   C. +200 Hz
   D. +100 Hz
2. What is the shift if the speeding car is moving 30 m/s toward the radar detector?
   A. -100 Hz
   B. -200 Hz
   C. +200 Hz
   D. +100 Hz

Incident radar

v

$$f' = f_0 \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$

f_0

$$f'' = f' \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$

Reflected radar

$$f'' = f_0 \frac{1-\frac{v}{c}}{1+\frac{v}{c}} \approx f_0\left(1-2\frac{v}{c}\right)$$