Announcements

1. Fourth hour exam scheduled for Friday 4/15/05 covering material in Chapters 35-38 (alternate exam date ~4/19 or 4/20 ➔ **make special arrangements**)
   Prepare equation sheet
   Review homework and class examples
   Formulate and find answers to your questions
   Previous exam

2. Topics after exam: nuclear physics and some topics in materials physics

3. Today's topic: -- continue discussion of Einstein's special theory of relativity
   Review Lorentz transformations
   Review Doppler effect for EM waves
   New relations for energy and momentum

Review of some results from the special theory of relativity

Postulates:

- The fundamental laws of physics (Newton's laws, Maxwell's equations, etc.) are the same in all inertial reference frames. ("inertial reference frame" == reference frame moving at a constant velocity)

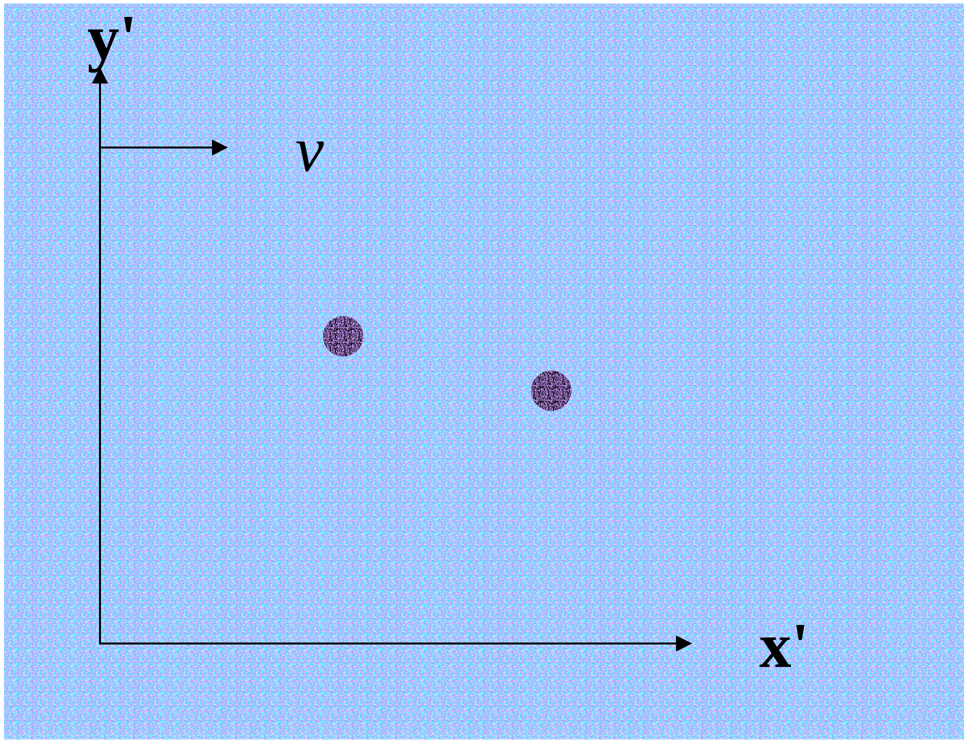
- The speed of light in vacuum c = 299792458 m/s is measured to be the same in all inertial reference frames.

The effects:

- c is the limiting speed in vacuum.

- The Lorentz transformation describes position and time relationships between frames of reference

- New formulations of momentum, and energy.



Lorentz transformation: $x' = \gamma(x - vt)$ $\quad$ $x = \gamma(x' + vt')$

$$\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad y' = y \qquad y = y'$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \qquad t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$

Review of the Lorentz transformations

$$x' = \gamma(x - vt)$$
$$y' = y$$
$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$
$$x = \gamma(x' + vt')$$
$$y = y'$$
$$t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$

where: $\gamma \equiv \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$ $\quad 1 \le \gamma \le \infty$

For example:

If $v = 0.5c, \gamma = 1.1547$

$v = 0.994c, \gamma = 9.14243$

$v = 0.99994c, \gamma = 91.2885$

Note that:

$$\Delta x = \gamma \Delta x' \text{ if } \Delta t' = 0$$
$$\Delta x' = \gamma \Delta x \text{ if } \Delta t = 0$$
$$\Delta t = \gamma \Delta t' \text{ if } \Delta x' = 0$$
$$\Delta t' = \gamma \Delta t \text{ if } \Delta x = 0$$

Review of Lorentz transformation of velocities

$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$

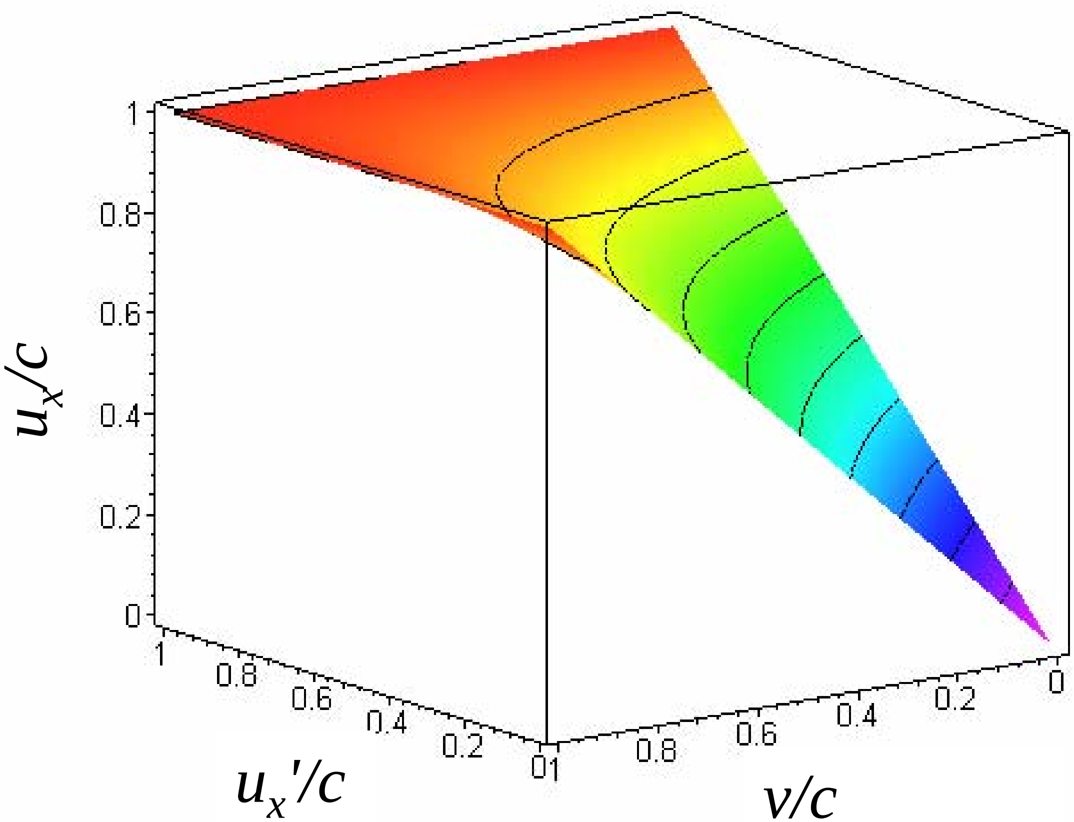
$$y' = y \qquad y = y'$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \qquad t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$

$$u'_x = \frac{dx'}{dt'} = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} \qquad u_x = \frac{dx}{dt} = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}}$$

$$u'_y = \frac{dy'}{dt'} = \frac{u_y}{\gamma\left(1 - \frac{u_x v}{c^2}\right)} \qquad u_y = \frac{dy}{dt} = \frac{u'_y}{\gamma\left(1 + \frac{u'_x v}{c^2}\right)}$$

Behavior of transformation for $$u_x = \frac{dx}{dt} = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}}$$



Lorentz transformations for electromagnetic waves

$$E = E_{\max}\sin(kx - \omega t) = E_{\max}\sin(k'x' - \omega' t')$$

$$x' = \gamma(x - vt) \qquad k' = \gamma\left(k - \frac{v\omega}{c^2}\right)$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \qquad \omega' = \gamma\left(\omega - \frac{v}{c}k\right)$$

$$|k'| = \frac{\omega'}{c} \qquad \text{if } k \parallel v, \Rightarrow \frac{\omega'}{c} = \gamma\frac{\omega}{c}\left(1 - \frac{v}{c}\right)$$

$$\omega' = \omega\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}} \qquad \text{or } f' = f\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$

Doppler effect for electromagnetic waves

$$f' = f\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$

$c ==$ velocity of light

$- \Rightarrow$ away

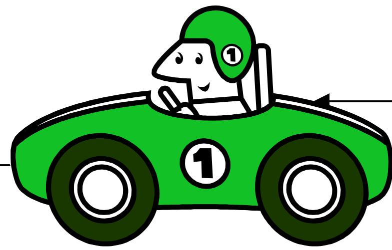
$+ \Rightarrow$ towards

Doppler effect for sound waves :

$$f' = f\,\frac{1 \pm v_D/s \;\;\begin{matrix}\text{toward}\\ \text{away}\end{matrix}\;\; \text{(detector)}}{1 \mp v_S/s \;\;\begin{matrix}\text{toward}\\ \text{away}\end{matrix}\;\; \text{(source)}}$$

$s ==$ velocity of sound

Incident radar



v

$$f' = f_0 \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$

f_0

Reflected radar

$$f'' = f' \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$

$$f'' = f_0 \frac{1-\frac{v}{c}}{1+\frac{v}{c}} \approx f_0\left(1-2\frac{v}{c}\right)$$

Example: $\quad v = 30m/s \qquad \frac{v}{c} = \frac{30}{3\times10^{8}} = 1\times10^{-7}$

$$f'' \approx f_0\left(1-2\frac{v}{c}\right); \qquad f'' - f_0 \; = 1\times10^{9}\,Hz\left(-2\cdot1\times10^{-7}\right) = -200Hz$$

Other results from the Special Theory of Relativity

➔Notice new physics at velocities *v* comparable to *c* (speed of electromagnetic waves in a vacuum). Note: Maxwell's equations are already consistent with notions of relativity.

➔New energy – momentum relationships within a single reference frame:

➔New zero of energy: If a particle has mass *m* and has zero velocity, its "rest mass energy" is mc^2. We can define a new "total" energy (not including potential energy) as

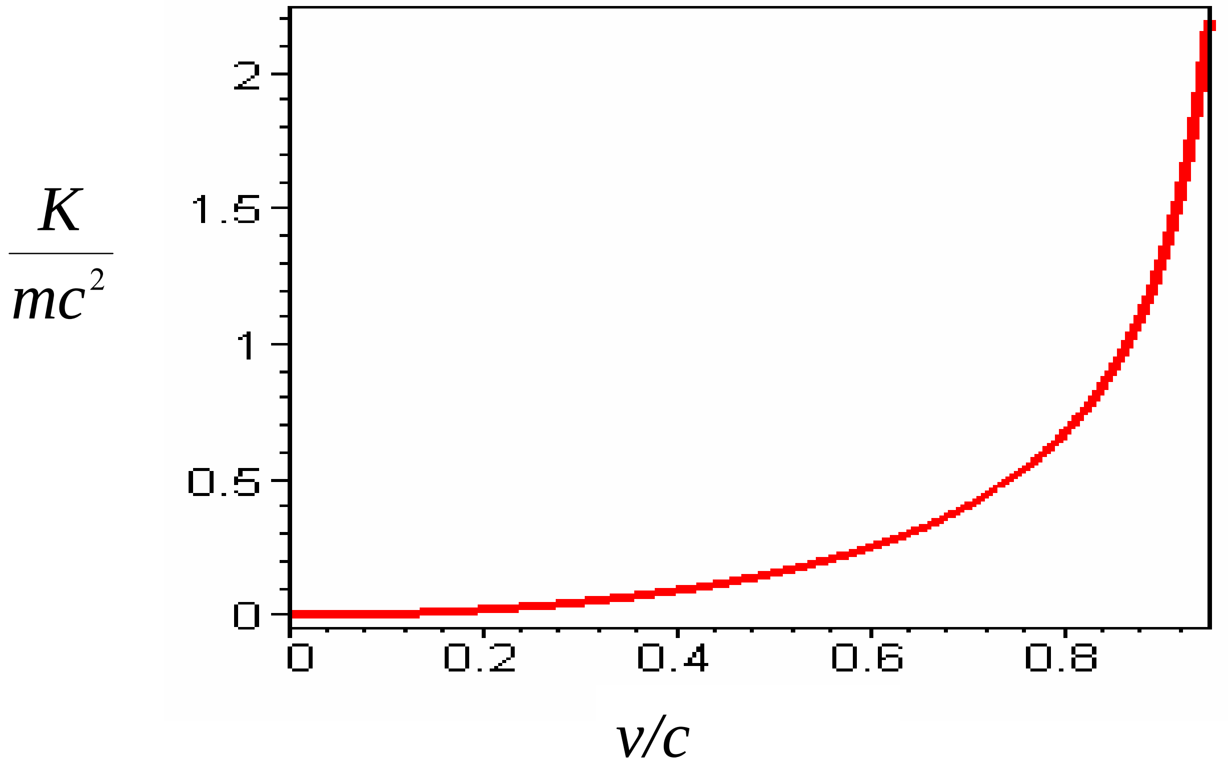
$$E = K + mc^2 = \gamma mc^2, \qquad \text{where } \gamma \equiv 1 \Big/ \sqrt{1-\frac{v^2}{c^2}}$$

$$\text{In this same scheme, momentum becomes: } \mathbf{p} = \gamma m\mathbf{u}$$

$$\Rightarrow E^2 = p^2c^2 + m^2c^4; \qquad p^2c^2 = K^2 + 2Kmc^2$$

Relativistic kinetic energy

$$K = E - mc^2 = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2$$



Relativistic energies

$$E = K + mc^2 = \gamma mc^2, \qquad \text{where } \gamma \equiv 1 \Big/ \sqrt{1 - \tfrac{v^2}{c^2}}$$

Example (electron):

$$mc^2 = 9.11\times10^{-31} \cdot \left(3\times10^8\right)^2 = 8.199\times10^{-14} J \, \frac{1eV}{1.6\times10^{-19} J} = 0.511 MeV$$

Example (proton):

$$mc^2 = 1.67\times10^{-27} \cdot \left(3\times10^8\right)^2 = 1.503\times10^{-10} J \, \frac{1eV}{1.6\times10^{-19} J} = 938 MeV$$

Energies for electrons

v/c	γ	E (MeV)	K (MeV)
0.0000	1.0000	0.5	0.0
0.5000	1.1547	0.6	0.1
0.9900	7.0888	3.6	3.1
0.9999	70.7124	36.1	35.6

Energies for protons

v/c	γ	E (MeV)	K (MeV)
0.0000	1.0000	938	0.0
0.5000	1.1547	1083	145
0.9900	7.0888	6649	5711
0.9999	70.7124	66328	65390

Online Quiz for Lecture 28
Diffraction patterns -- Apr. 11, 2005

The rest mass energy of an electron is 0.511 MeV.

1. Find the value of v/c which corresponds to an electron total energy of E=1 MeV
   A. 0.5111111
   B. 0.7342131
   C. 0.8595807
   D. 0.9999869
   E. 0.9999996
2. Find the value of v/c which corresponds to an electron total energy of E=100 MeV
   A. 0.5111111
   B. 0.7342131
   C. 0.8595807
   D. 0.9999869
   E. 0.9999996

Examples from nuclear physics

Consider the nucleus ^{238}U : M=238.051u

92 protons ➔ $92 \cdot 1.0073u$

146 neutrons ➔ $146 \cdot 1.0078u$

239.8104 u

"Binding energy" $\Delta Mc^2 = 1.759 \cdot 931.5$ MeV

$= 1600$ MeV

More examples from nuclear physics

Nuclear reaction $\quad {}^{238}U \rightarrow {}^{234}Th + {}^{4}He + K$

Mass change:

$$K = (M({}^{238}U) - M({}^{234}Th) - M({}^{4}He))c^2$$

$$= (238.051u - 234.043u - 4.003u)c^2$$

$$= 0.005u\ c^2 = 4\ \text{MeV}$$

Relativistic momentum: **p** = γm**u**

Note: $$p^2c^2 + m^2c^4 = E^2$$

$$\gamma^2 m^2 u^2 c^2 + m^2 c^4 = \gamma^2 m^2 c^4 \left(\frac{u^2}{c^2} - \frac{1}{\gamma^2} \right) = \gamma^2 m^2 c^4 \left(\frac{u^2}{c^2} - \left(1 - \frac{u^2}{c^2} \right) \right)$$

$$= \gamma^2 m^2 c^4 = E^2$$

Note: if $p << mc$

$$E \approx mc^2 + \frac{p^2}{2m} = mc^2 + \frac{1}{2}mu^2$$

Relationship between momentum and kinetic energy

Non-relativisitic: $K = \frac{p^2}{2m}$

Relativisitic: $K = \sqrt{p^2c^2 + m^2c^4} - mc^2$