Announcements

1. **Fourth hour exam scheduled for Friday 4/15/05**

   The following students have arranged to take the exam on the alternate date: Lesly Bankson, Daniel Blackburn, Tripp Cockerham, Drew Harston, Sylvia Holcombe, Eric Jewett, Laura Millns, Rachel Morgan, Yee Yee Pu, Chris Vellano

   Prepare equation sheet
   Review homework and class examples
   Formulate and find answers to your questions
   Previous exam

2. Problem solving session at 6 PM in Olin 101?????

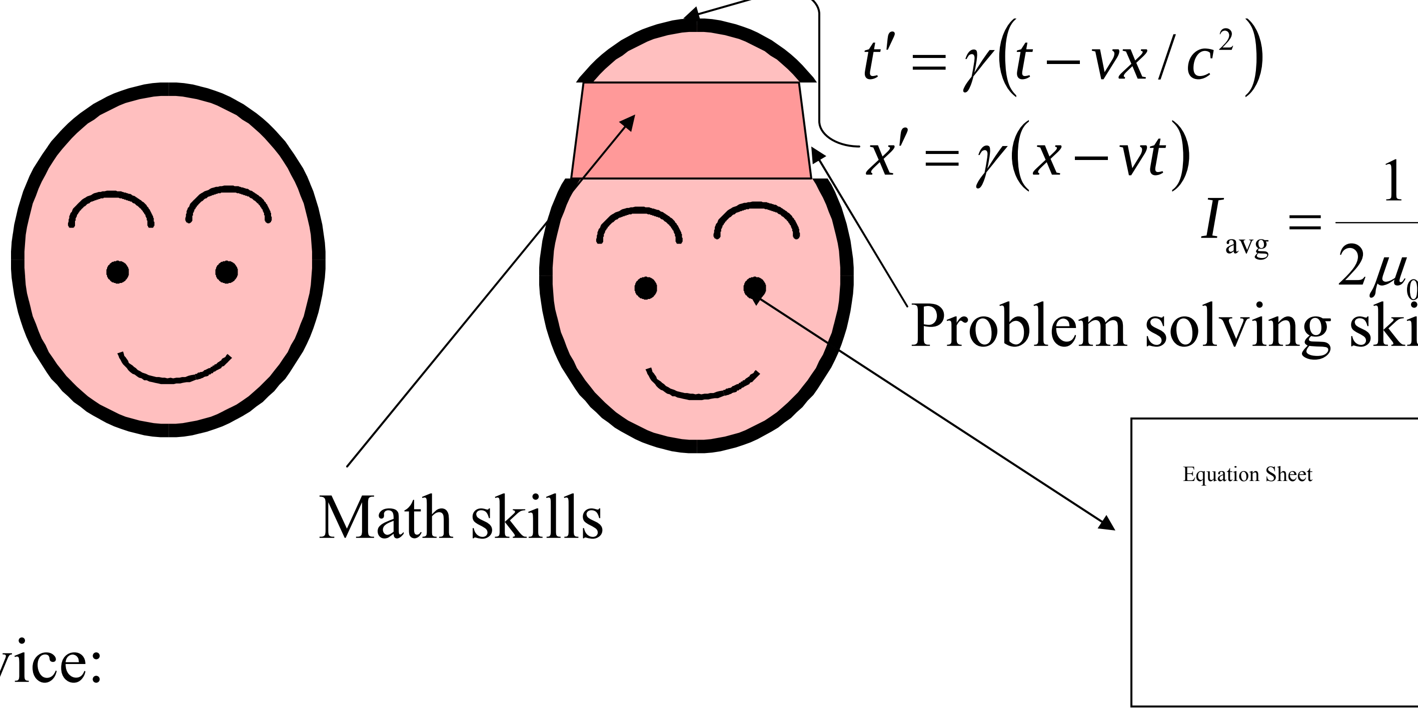
3. Topic for today -- Review of Chapters 35-38
   - Geometrical optics
   - Interference and diffraction
   - Special theory relativity

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f} \qquad m=-\frac{i}{p}$$

$$t'=\gamma\left(t-vx/c^2\right)$$

$$x'=\gamma\left(x-vt\right)$$

$$I_{\text{avg}}=\frac{1}{2\mu_0 c}\left|E_{\max}\right|^2$$



Advice:

1. Keep basic concepts and equations at the top of your head.
2. Practice problem solving and math skills
3. Develop an equation sheet that you can consult.

Problem solving steps

1. Visualize problem – labeling variables
2. Determine which basic physical principle(s) apply
3. Write down the appropriate equations using the variables defined in step 1.
4. Check whether you have the correct amount of information to solve the problem (same number of knowns and unknowns).
5. Solve the equations.
6. Check whether your answer makes sense (units, order of magnitude, etc.).

Geometrical optics

•Basic physics reflection (mirrors) and refraction (lenses) of EM radiation

•Ray diagram traces the propagation vectors of EM radiation

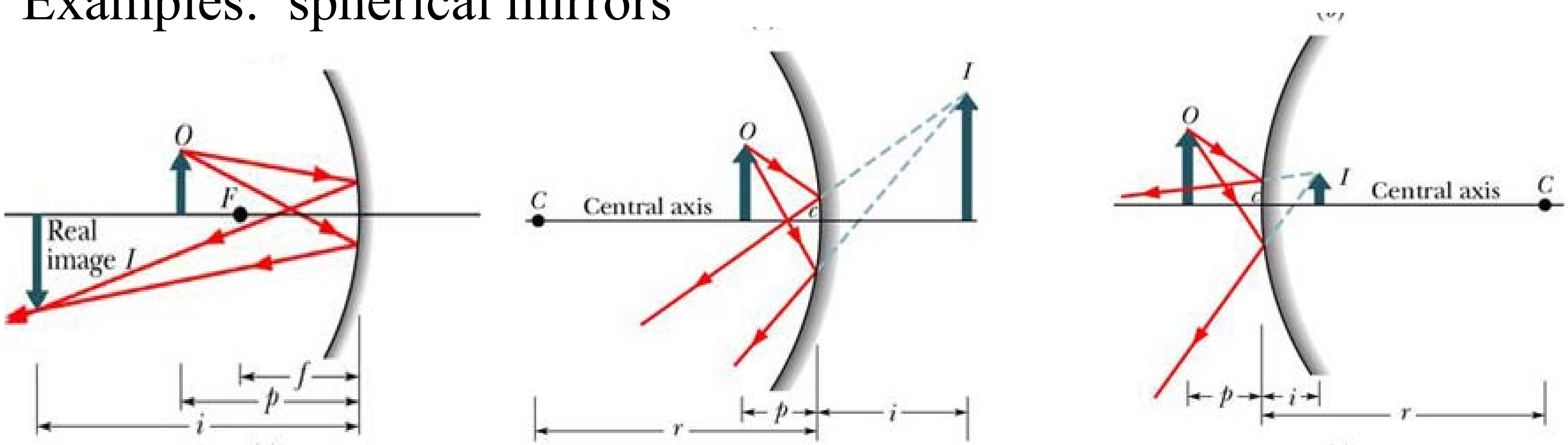
•Focused images (real and virtual) using mirrors and lenses

Mirror and thin lens equation

$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f} \qquad m=\frac{h_i}{h_p}=-\frac{i}{p}$$

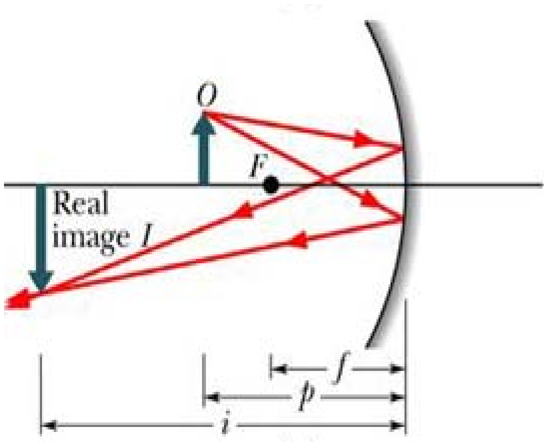
Note: sign conventions are important for this equation.

Examples: spherical mirrors



Sign conventions

Mirrors:



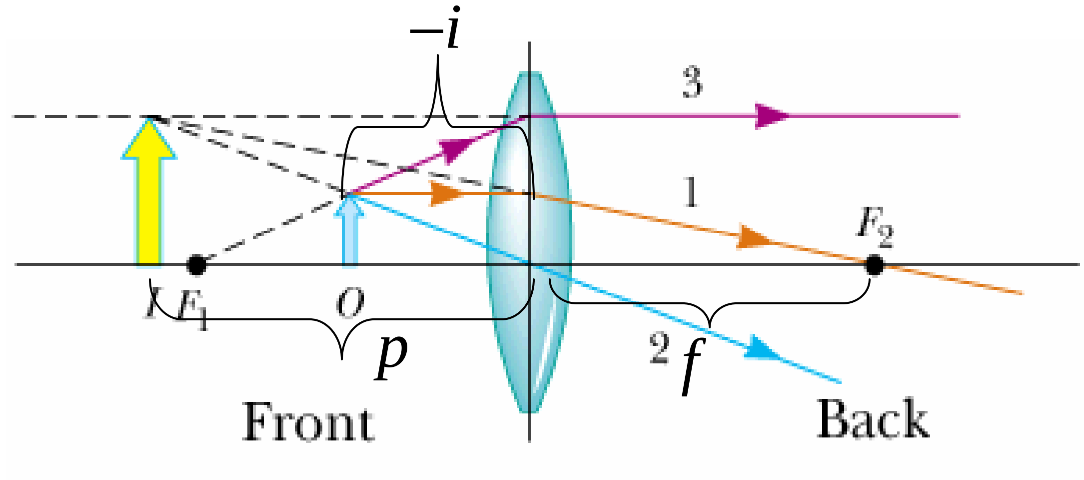
$p,i > 0$ real image

$p,i < 0$ virtual image

$f>0$ for concave

$f<0$ for convex

Lenses



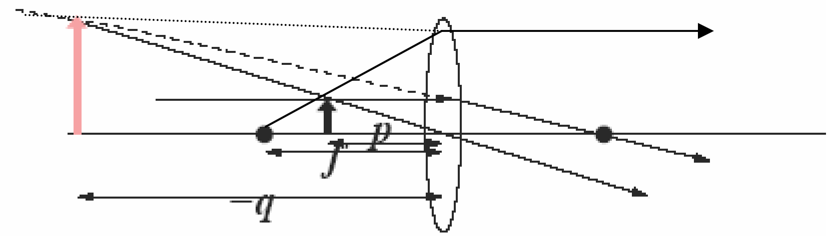
$p > 0 \;\; i<0$ virtual image

$p<0 \;\; i>0$ real image

$f>0$ for converging

$f<0$ for diverging

3. The figure above shows Sherlock Holmes looking through a converging lens at a piece of evidence. Assume that the focal length of his lens is $f = 10$ cm. If he adjusts the distance p of the lens relative to the object appropriately, he is able to see the image magnified by 3 times its original size. In the space below, draw the ray diagram for this case, and determine the object and image distances p and q. Indicate whether the image is real or virtual.



3.

The ray diagram is shown above. The image is virtual

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}.$$

$$M = \frac{-q}{p} \quad \Rightarrow q = -Mp.$$

$$\frac{1}{p}\left(1 - \frac{1}{M}\right) = \frac{1}{f}.$$

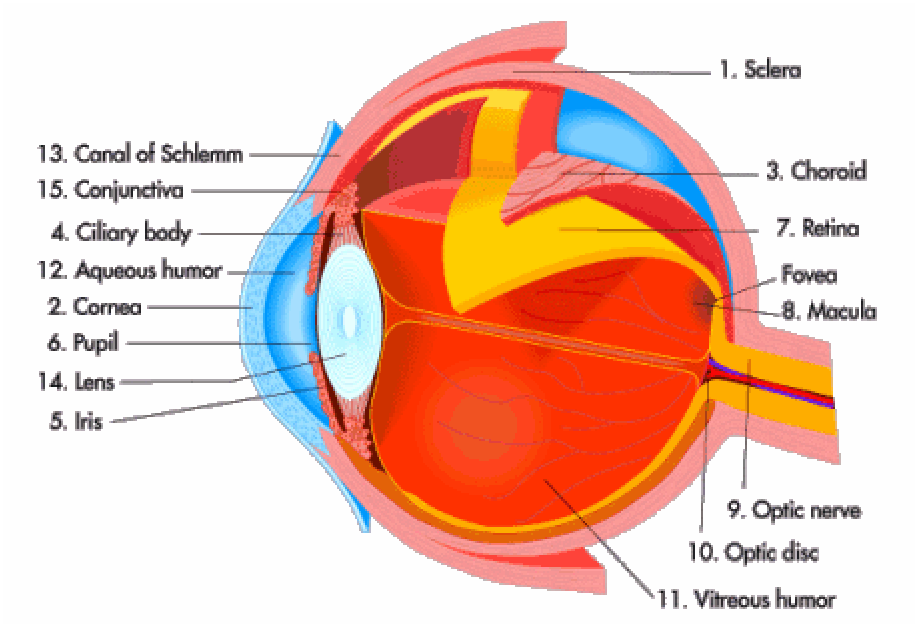
Solving this expression for p:

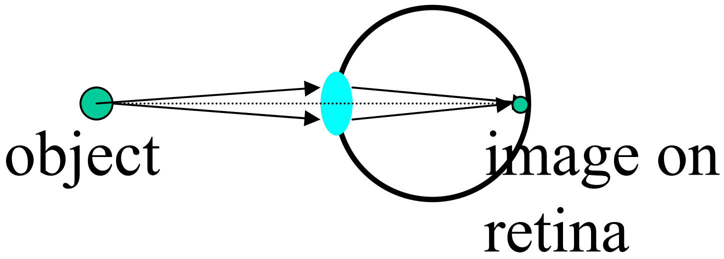
$$p = \frac{2}{3}f = 6.67cm.$$

$$q = -3p = -20cm.$$

View of the eye from

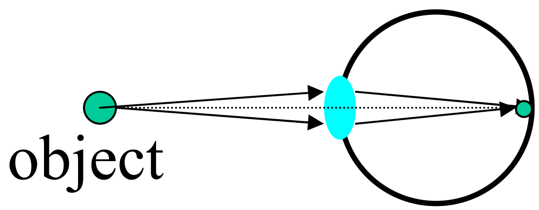
http://science.howstuffworks.com/eye1.htm





Vision problems and corrective lenses

Ideal vision:



Near sighted vision – problem with "Far point"

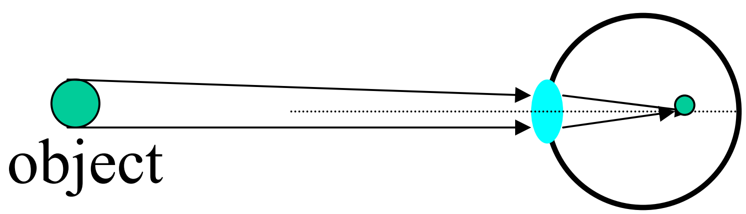


image in front of retina ➔ need diverging corrective lens

Far sighted vision – problem with "Near point"

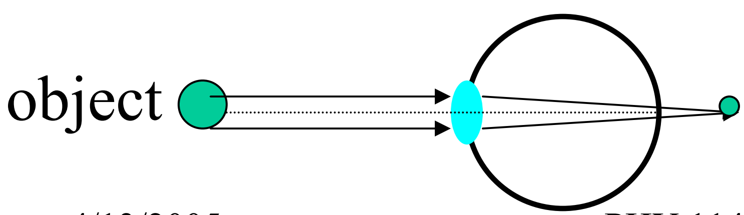
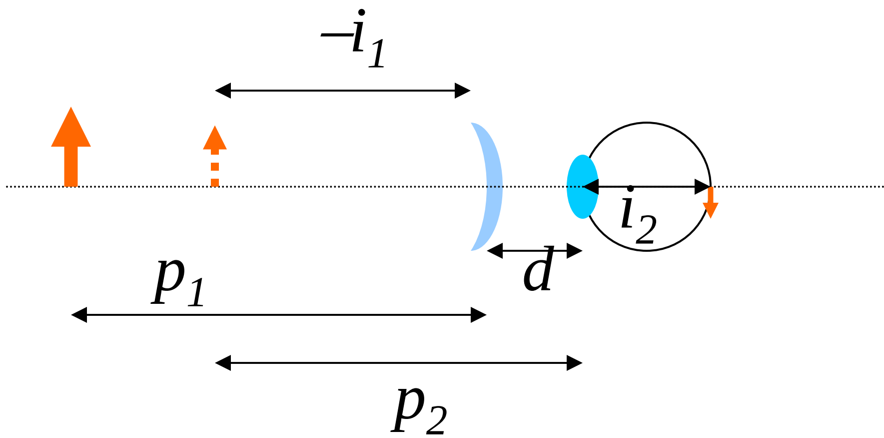


image in back of retina ➔ need converging corrective lens

Refraction from two or more lenses

→ Successive use of lens equation



$$\frac{1}{p_1}+\frac{1}{i_1}=\frac{1}{f_1}$$

$$p_2=-i_1+d$$

$$\frac{1}{p_2}+\frac{1}{i_2}=\frac{1}{f_2}$$

Example:

$p_1=50\ \text{cm};\quad f_1=-25\ \text{cm};\quad f_2=1.4\ \text{cm};\ d=1.0\ \text{cm}$

We find:

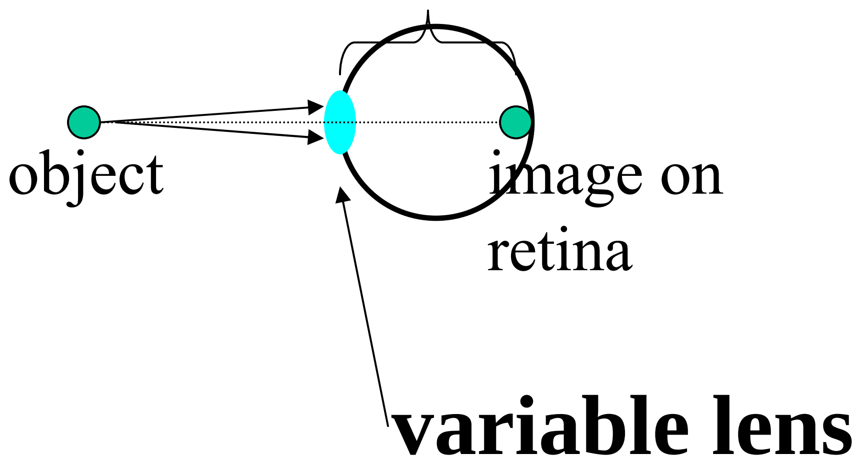
$i_1=-16.667\ \text{cm};\quad i_2=1.52\ \text{cm}$

Without the diverging lens:

$i_2=1.44\,\text{cm}$ (short of retina)

More details about eye:

$i \approx 2.5\text{cm}$



$$\frac{1}{p}+\frac{1}{i}=\frac{1}{f}$$

$$\frac{1}{25}+\frac{1}{2.5}=\frac{1}{f}$$

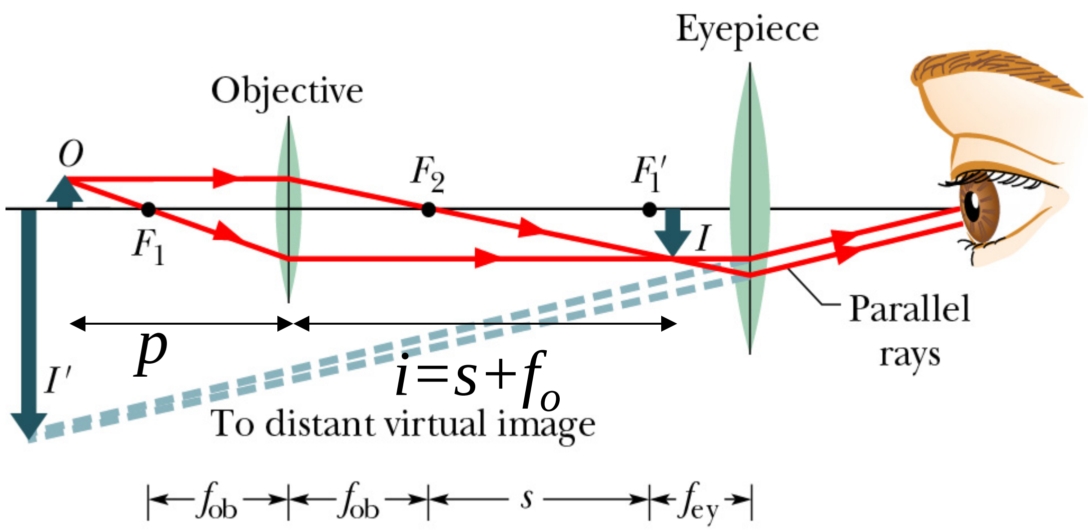
$$f=\frac{1}{\frac{1}{25}+\frac{1}{2.5}}=2.27\,\text{cm}$$

"near point" ≡ closest point that the eye can focus

25 cm standard value

7-200 cm depending on person

Physics of the microscope

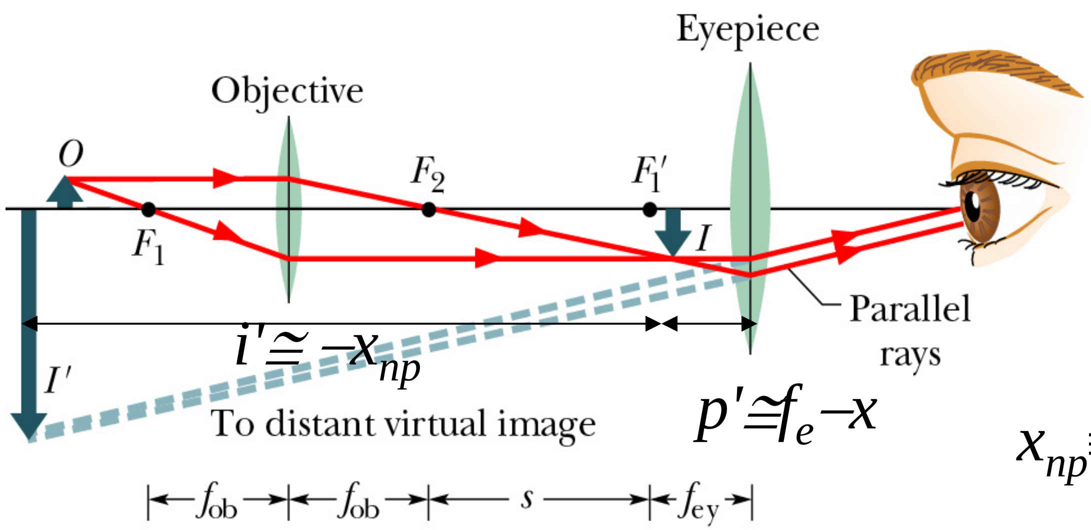


Objective:

$$\frac{1}{p}+\frac{1}{s+f_o}=\frac{1}{f_o} \qquad p=\frac{f_o(s+f_o)}{s}$$

$$m=\frac{-i}{p}=-\frac{s+f_o}{\frac{f_o(s+f_o)}{s}}=-\frac{s}{f_o}$$

Physics of the microscope

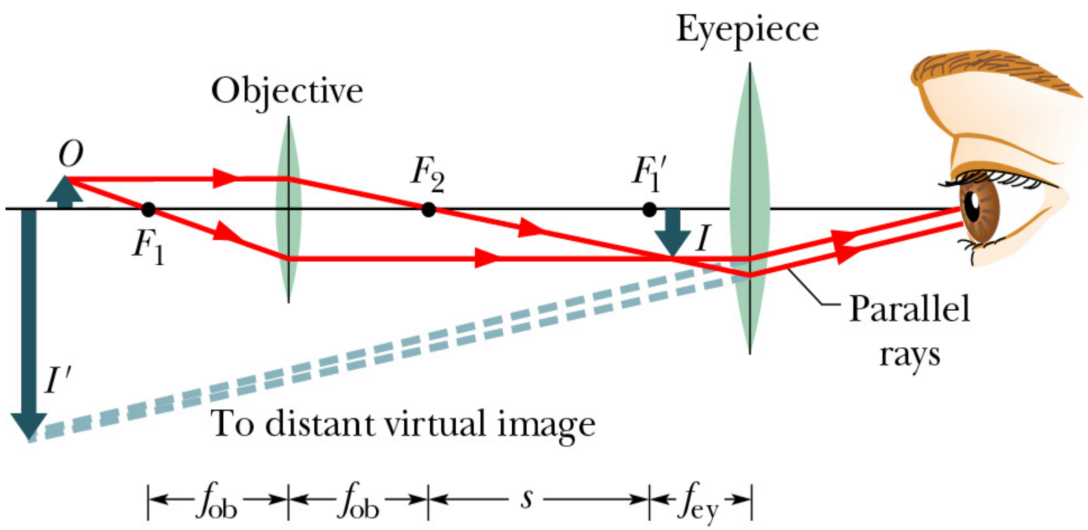


$x_{np} \equiv$ “near point” distance

Eyepiece: $$\frac{1}{f_e - x} + \frac{1}{-x_{np}} = \frac{1}{f_e} \qquad x = \frac{f_e^{\,2}}{x_{np} + f_e} \qquad \left(f_e << x_{np}\right)$$

$$m' = \frac{-i'}{p'} = \frac{x_{np}}{f_e - x} \approx \frac{x_{np}}{f_e}$$

Physics of the microscope



Net magnification :

$$M = mm' = \frac{-s}{f_o}\frac{x_{np}}{f_e}$$

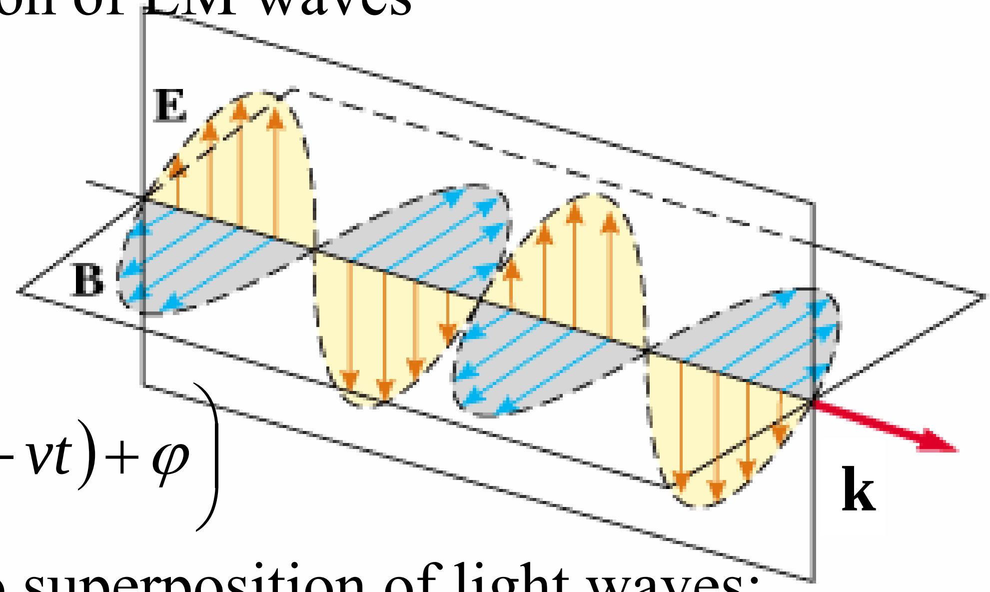
Example : $f_o = 6\text{mm}$; $f_e = 20\text{mm}$;

$$s = 150\text{mm}$$

$$M = -\frac{150}{6}\frac{250}{20} = -312.5$$

Interference and diffraction of EM waves

Plane polarized electromagnetic wave at an instant of time:



$$E_y(x,t) = E_{\max} \sin\left(\frac{2\pi}{\lambda}(x - vt) + \varphi\right)$$

Interference effects due to superposition of light waves:

$$E_{tot}(x,t) = E_1(x,t) + E_2(x,t)$$

$$= E_{\max} \sin\left(\frac{2\pi}{\lambda}(x - vt)\right) + E_{\max} \sin\left(\frac{2\pi}{\lambda}(x - vt) + \varphi\right)$$

$$= 2E_{\max} \sin\left(\frac{2\pi}{\lambda}(x - vt) + \frac{1}{2}\varphi\right) \cos\left(\frac{\varphi}{2}\right)$$

$$I = |\mathbf{S}|_{av} = \frac{4E_{\max}^2}{2\mu_0 c} \cos^2\left(\frac{\varphi}{2}\right) \equiv I_{\max} \cos^2\left(\frac{\varphi}{2}\right)$$

Interference and diffraction of EM waves

Basic physics: EM waves from different sources or from different parts of the same source **add**

Trigonometric identities:

$$\sin(\alpha)+\sin(\beta)=2\sin\left(\tfrac{1}{2}(\alpha+\beta)\right)\cos\left(\tfrac{1}{2}(\alpha-\beta)\right)$$

$$\sin(\alpha)-\sin(\beta)=2\cos\left(\tfrac{1}{2}(\alpha+\beta)\right)\sin\left(\tfrac{1}{2}(\alpha-\beta)\right)$$

$$\cos(\alpha)+\cos(\beta)=2\cos\left(\tfrac{1}{2}(\alpha+\beta)\right)\cos\left(\tfrac{1}{2}(\alpha-\beta)\right)$$

$$\cos(\alpha)-\cos(\beta)=-2\sin\left(\tfrac{1}{2}(\alpha+\beta)\right)\sin\left(\tfrac{1}{2}(\alpha-\beta)\right)$$

average phase interference phase

Some details:

$$E_y^{tot}(x,t) = E_{y,A}(x,t) + E_{y,B}(x,t)$$

$$E_y(x,t) = E_{\max}\sin\left(\frac{2\pi}{\lambda}(x-vt)\right) + E_{\max}\sin\left(\frac{2\pi}{\lambda}(x-vt)+\varphi\right)$$

$$= 2E_{\max}\sin\left(\frac{2\pi}{\lambda}(x-vt)+\frac{1}{2}\varphi\right)\cos\left(\frac{\varphi}{2}\right)$$

Note that this result follows from the trigonometric identity:

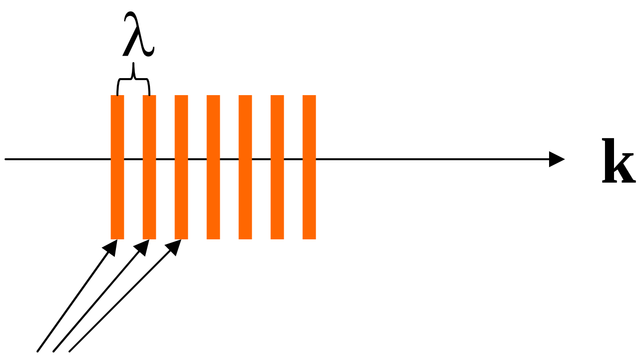
$$\sin(A)+\sin(B) = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

Intensity:

$$I_A = \frac{1}{2c\mu_0}\left|E_{\max}\right|^2 = I_B$$

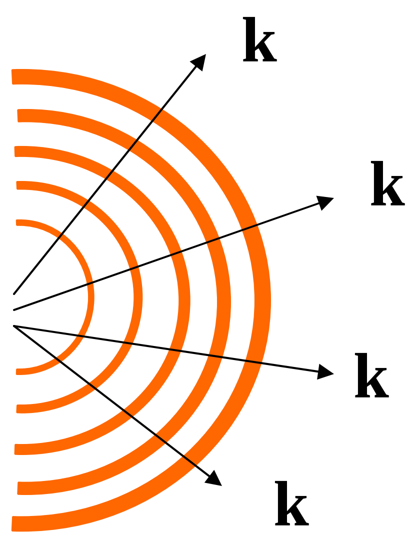
$$I^{tot} = \frac{4}{2c\mu_0}\left|E_{\max}\cos\left(\frac{\varphi}{2}\right)\right|^2 = 4I_A\cos^2\left(\frac{\varphi}{2}\right)$$

Top view of plane EM wave

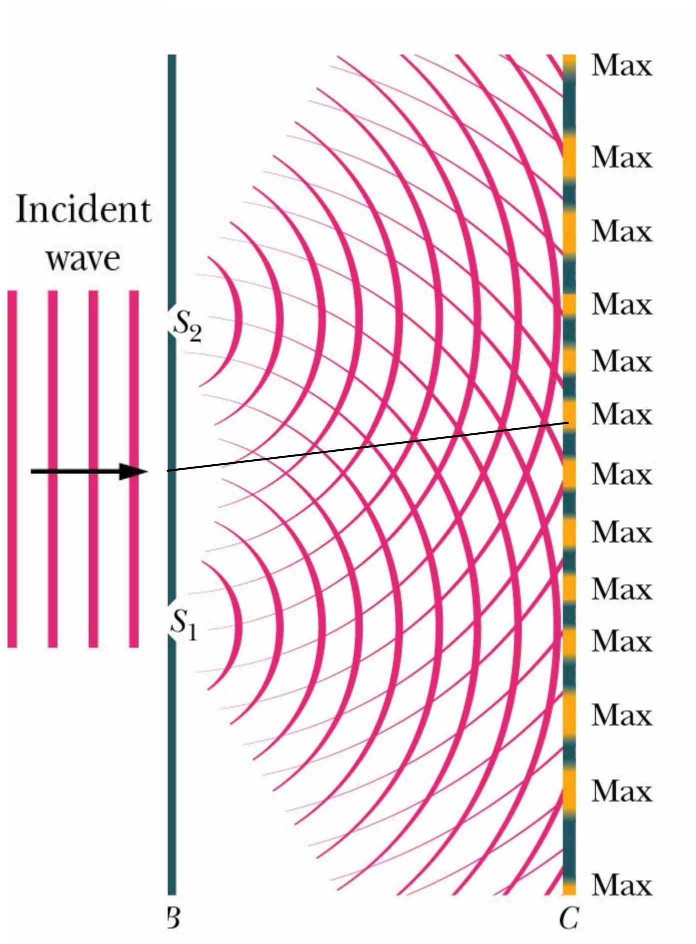


peaks of E field (perpendicular to screen)

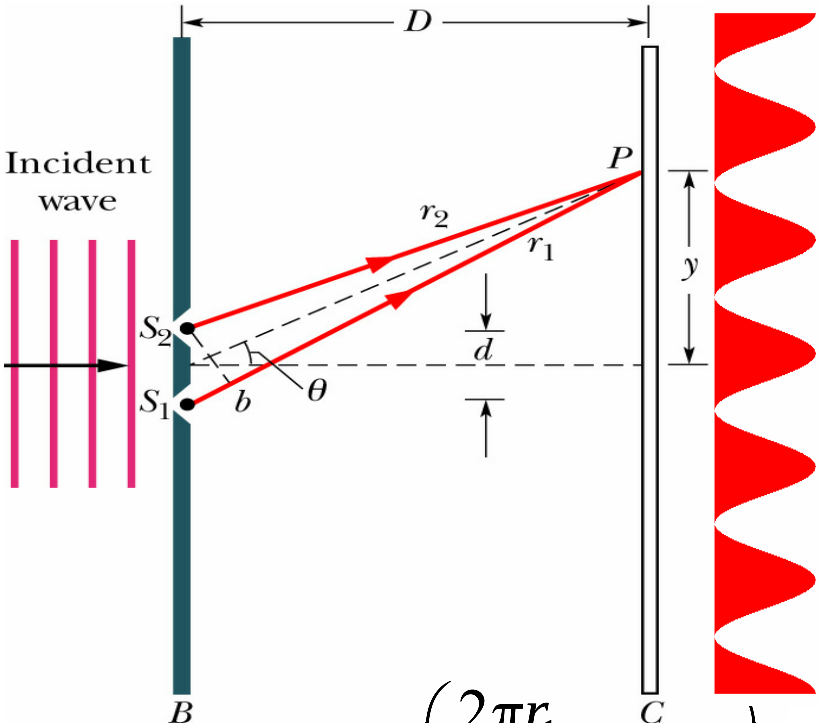
Top view of spherical EM wave

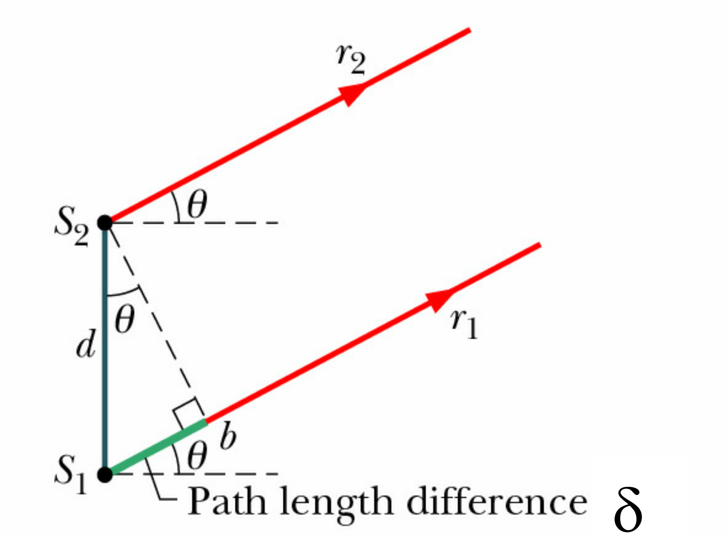


Interference of spherical waves in phase:



Diffraction pattern from a plane wave incident on a double slit:





$$r_1 - r_2 = \delta = d \sin\theta$$

$$E(P,t) = E_{\max}\sin\left(\frac{2\pi r_1}{\lambda} - 2\pi f t\right) + E_{\max}\sin\left(\frac{2\pi r_2}{\lambda} - 2\pi f t\right)$$

$$= 2E_{\max}\sin\left(\frac{\pi(r_1 + r_2)}{\lambda} - 2\pi f t\right)\cos\left(\frac{\pi(r_1 - r_2)}{\lambda}\right)$$

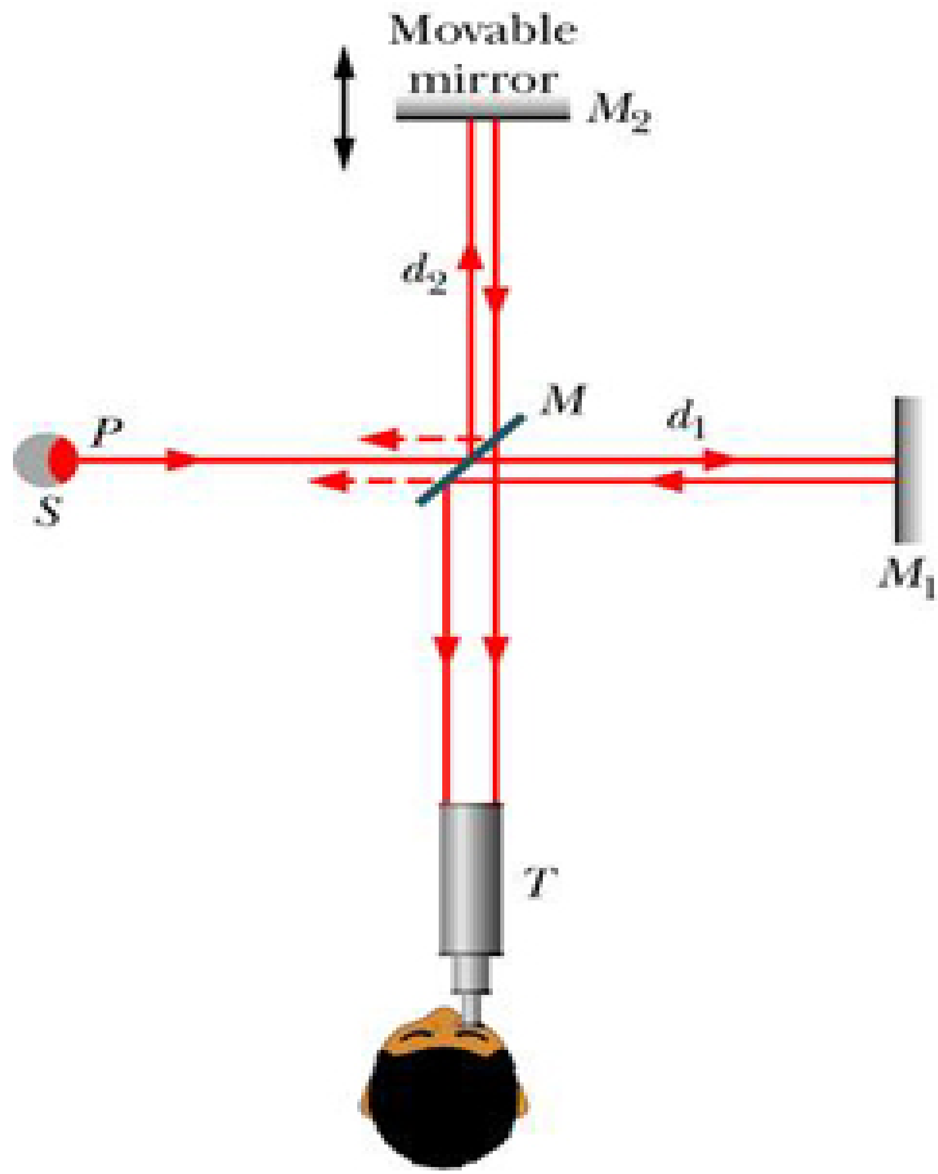
➔ intensity maxima occur for $\frac{\pi(r_1 - r_2)}{\lambda} = m\pi \quad \Rightarrow d\sin\theta = m\lambda$

Michelson interferometer

$$\varphi = \frac{2\pi}{\lambda}(d_1 - d_2)$$

Intensity :

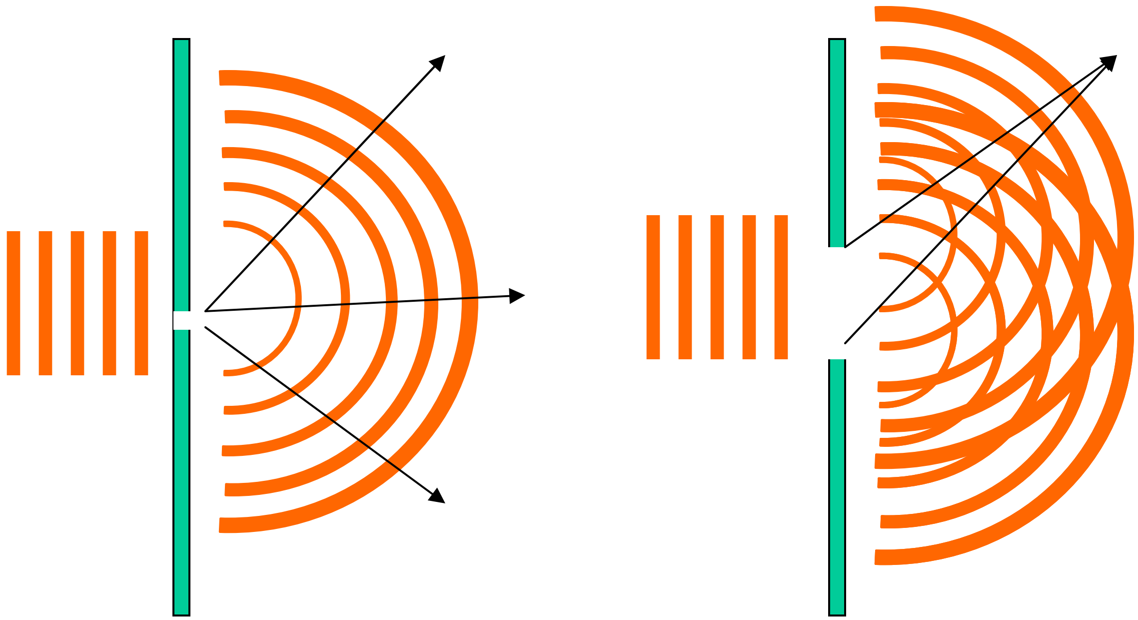
$$I = I_{max} \sin^2\left(\frac{\varphi}{2}\right)$$



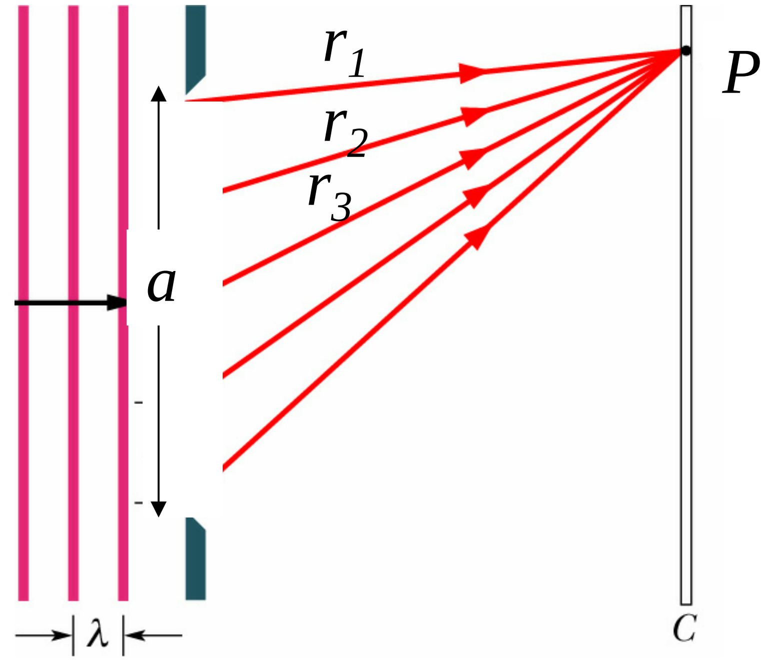
More details --

Actual interference effects within a single finite size slit

thin slit thicker slit



Mathematical description of single slit diffraction



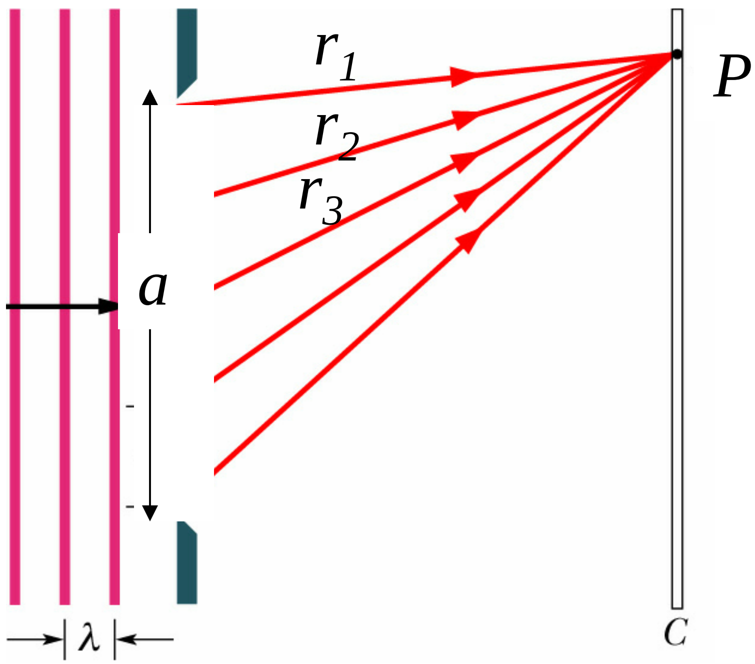
$$E(P,t) = \sum_i E_{\max} \sin\left(\frac{2\pi r_i}{\lambda} - 2\pi f t\right)$$

$$\approx (\text{Constant}) E_{\max} \int_{-a/2}^{a/2} \sin\left(\frac{2\pi\left(r_{av} - x\sin\theta\right)}{\lambda} - 2\pi f t\right) dx$$

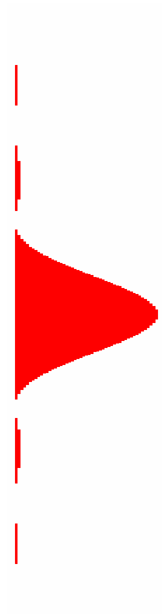
$$= 2(\text{Constant}) E_{\max} \sin\left(\frac{2\pi r_{av}}{\lambda} - 2\pi f t\right) \left\{ \frac{\sin\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\left(\frac{\pi a \sin\theta}{\lambda}\right)} \right\}$$

Single slit intensity pattern:

$$\langle I \rangle_{av} = I_{\max} \left\{ \frac{\sin\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\left(\frac{\pi a \sin\theta}{\lambda}\right)} \right\}^2$$

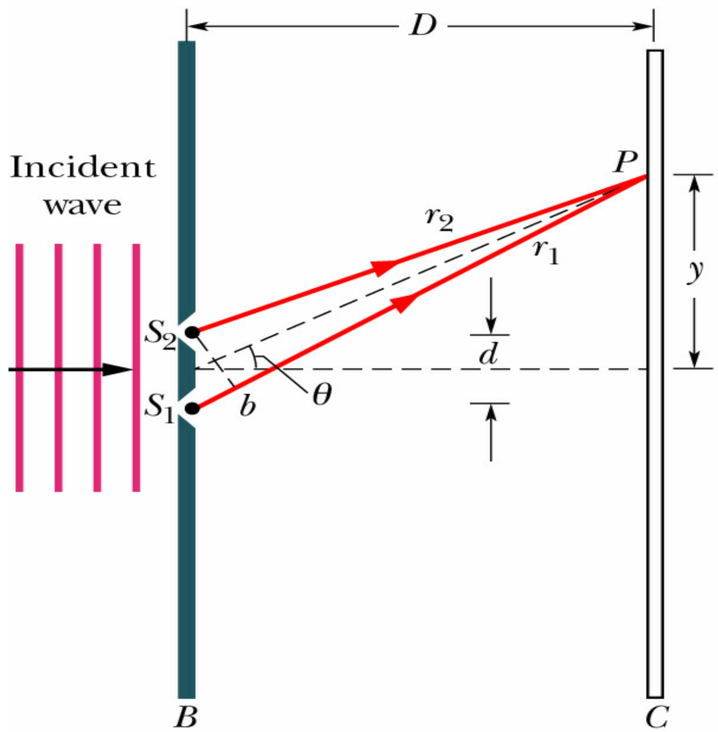


small *a*

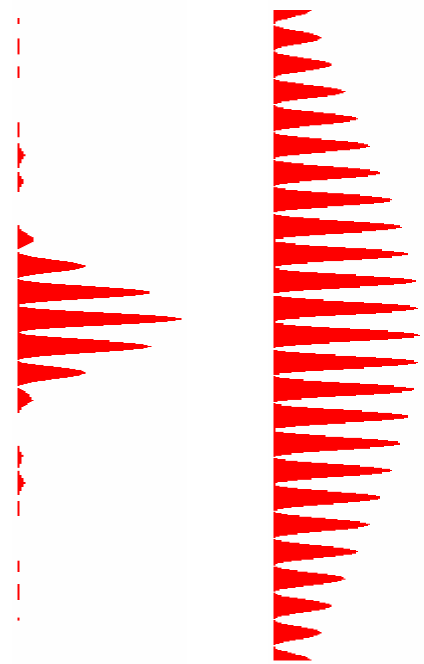


smaller *a*

Effect of slit size on double slit pattern



$a = d/4$ $a = d/20$



$$I = I_{\max}\left[\cos\left(\frac{\pi d \sin\theta}{\lambda}\right)\right]^2 \left[\frac{\sin\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\frac{\pi a \sin\theta}{\lambda}}\right]^2$$

1. [HRW6 37.P.028.] For $d = 6a$ in Fig. 37-38, how many bright interference fringes lie in the central diffraction envelope?

$$I = I_{max}\left[\cos\left(\frac{\pi d \sin\theta}{\lambda}\right)\right]^2 \left[\frac{\sin\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\frac{\pi a \sin\theta}{\lambda}}\right]^2$$

$$-\pi < \frac{\pi a \sin\theta}{\lambda} < \pi$$

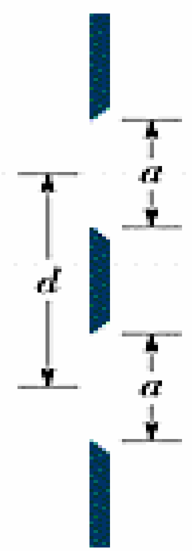
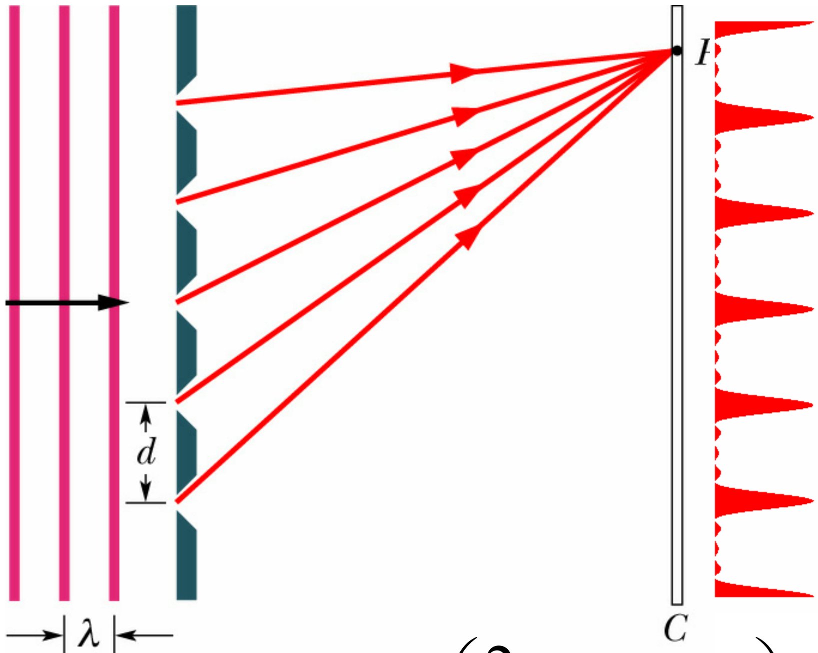
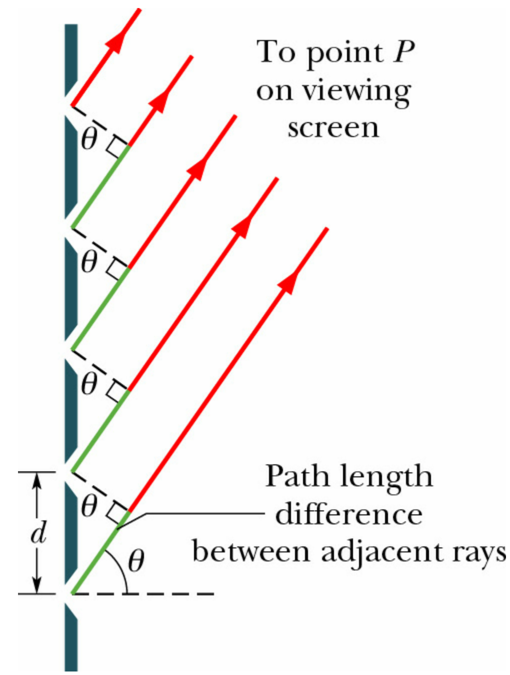


Figure 37-38.

Diffraction pattern for N slits – diffraction grating





$$E(P,t) = \sum_i E_{\max} \sin\left(\frac{2\pi r_i}{\lambda} - 2\pi f t\right)$$

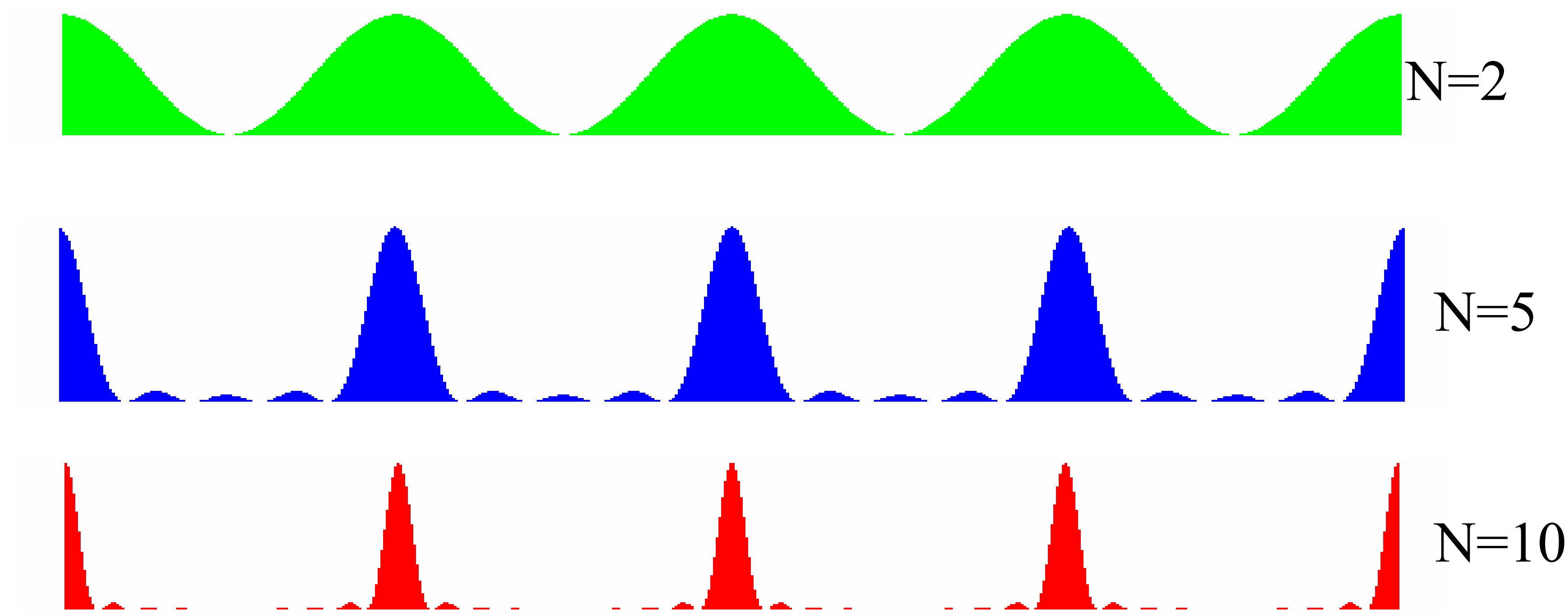
$$= E_{\max} \sin\left(\frac{2\pi r_{av}}{\lambda} - 2\pi f t\right) \frac{\sin\left(\frac{N\pi d \sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi d \sin\theta}{\lambda}\right)}$$

Intensity maxima at

$d \sin\theta = m\lambda$

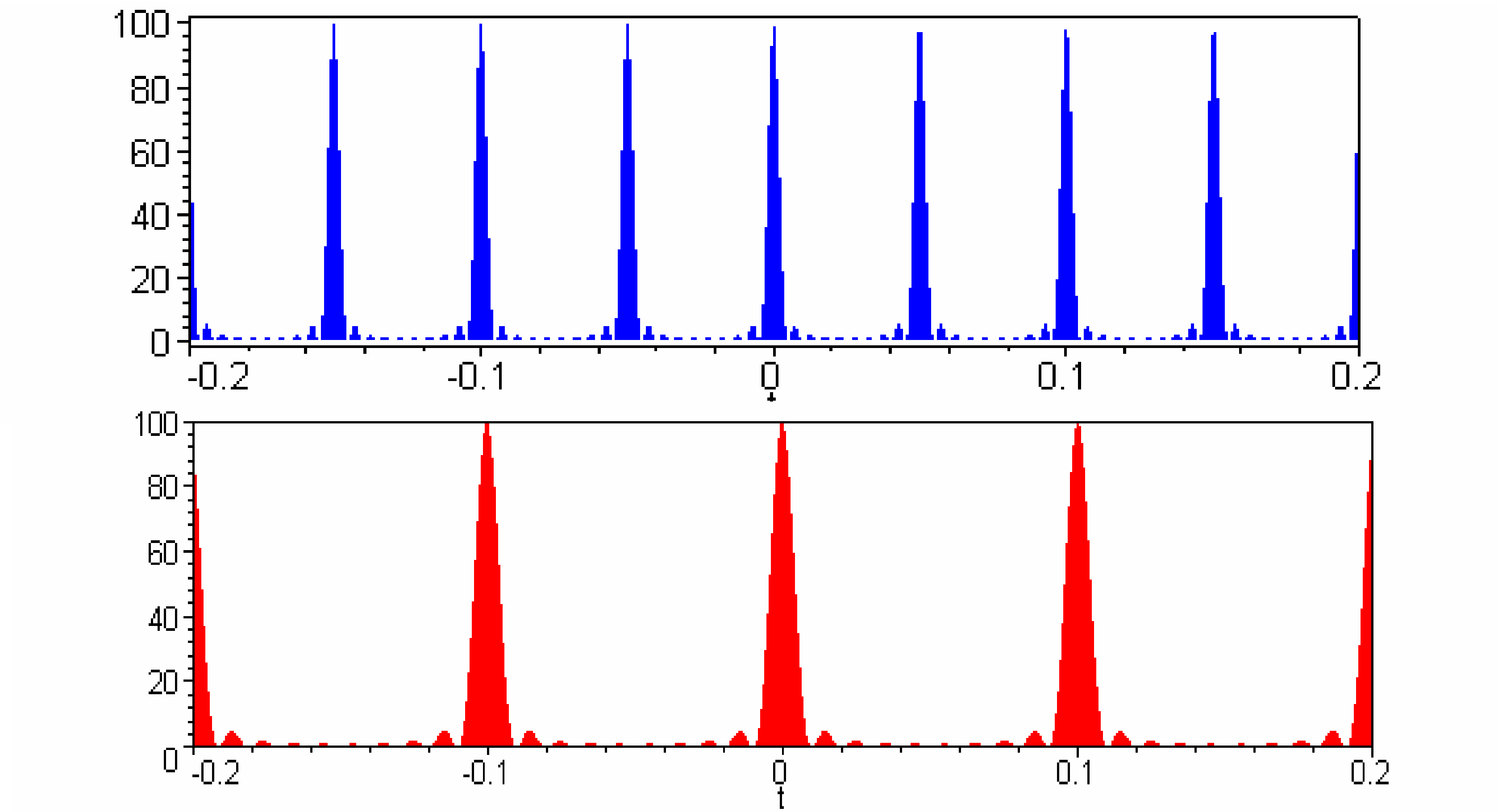
Intensity pattern from multiple slit grating:

$$I = I_{max}\left[\frac{\sin\left(\frac{N\pi d \sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi d \sin\theta}{\lambda}\right)}\right]^2$$

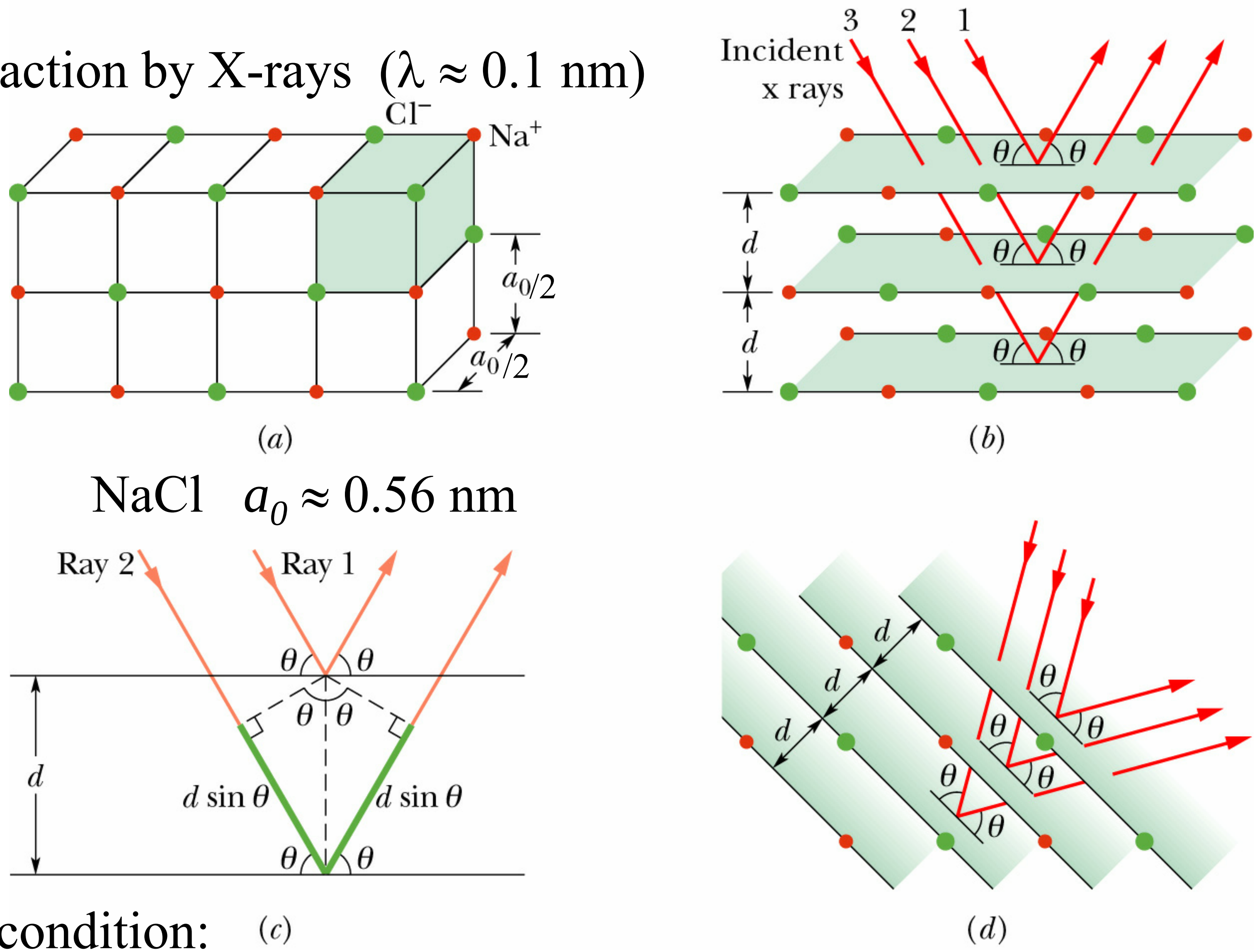


Effects of different wave lengths:

$$I = I_{\max}\left[\frac{\sin\left(\frac{N\pi d \sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi d \sin\theta}{\lambda}\right)}\right]^2$$



Diffraction by X-rays ($\lambda \approx 0.1$ nm)



NaCl $a_0 \approx 0.56$ nm

Bragg condition:

$$2d \sin \theta = m\lambda$$

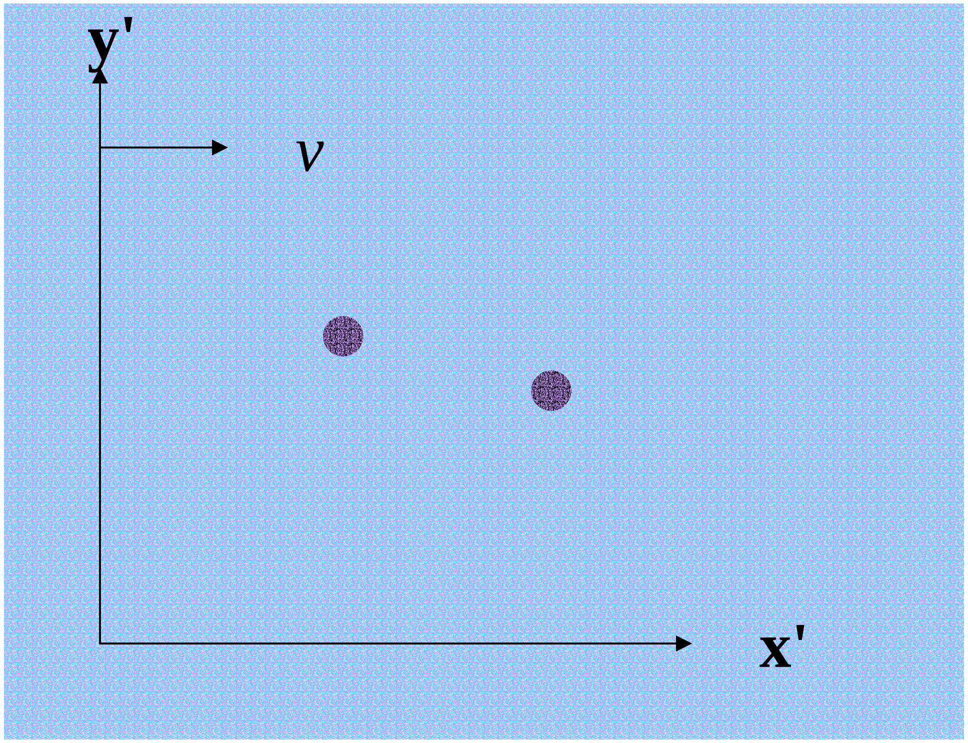
Some results from the special theory of relativity

Postulates:

- The fundamental laws of physics (Newton's laws, Maxwell's equations, etc.) are the same in all inertial reference frames. ("inertial reference frame" == reference frame moving at a constant velocity)
- The speed of light in vacuum c = 299792458 m/s is measured to be the same in all inertial reference frames.

The effects:

- c is the limiting speed in vacuum.
- The Lorentz transformation describes position and time relationships between frames of reference
- New formulations of momentum, and energy.



Lorentz transformation: $x' = \gamma(x - vt)$ $x = \gamma(x' + vt')$

$$\gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \qquad y' = y \qquad y = y'$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \qquad t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$

Example:

Consider some measurable process such as a decay of a cosmic ray particle $\mu \rightarrow e + \nu + \bar{\nu}$ which is known to the follow the relationship,

$N_\mu(t) = N_0 e^{-t/\tau}$ with τ=2.2μs.

In a classic experiment, Rossi and Hall (*Phys. Rev.* **59**,223 (1941)), measured μ particles traveling with v=0.994c on the top and bottom of a mountain with Δx = 2000 m .

Δt=2000m/0.994c=6.7μs ➔ expect $\dfrac{N_\mu(6.7\mu s)}{N_0} = e^{-6.7/2.2} = 0.048$

➔ found $\dfrac{N_\mu}{N_0} \approx 0.72 = e^{-0.7/2.2}$

$$\text{Infer}: \quad \underbrace{\Delta t_\mu}_{0.7\ \mu\text{s}} = \underbrace{\Delta t_{Earth}}_{6.7\ \mu\text{s}} / \gamma \qquad \gamma = \frac{1}{\sqrt{1-(0.994)^2}} = 9.1$$

Lorentz transformations for electromagnetic waves

$$E = E_{\max}\sin(kx - \omega t) = E_{\max}\sin(k'x' - \omega' t')$$

$$x' = \gamma(x - vt) \qquad k' = \gamma\left(k - \frac{v\omega}{c^2}\right)$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \qquad \omega' = \gamma\left(\omega - \frac{v}{c}k\right)$$

$$|k'| = \frac{\omega'}{c} \qquad \Rightarrow \frac{\omega'}{c} = \gamma\frac{\omega}{c}\left(1 - \frac{v}{c}\right)$$

$$\omega' = \omega\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}} \quad \text{or} \quad f' = f\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$

More general form:

$$f' = f\gamma\left(1 - \frac{v}{c}\cos\theta\right)$$

Other results from the Special Theory of Relativity

➔Notice new physics at velocities *v* comparable to *c* (speed of electromagnetic waves in a vacuum). Note: Maxwell's equations are already consistent with notions of relativity.

➔New energy – momentum relationships within a single reference frame:

➔New zero of energy: If a particle has mass *m* and has zero velocity, its "rest mass energy" is mc^2. We can define a new "total" energy (not including potential energy) as

$$E = K + mc^2 = \gamma mc^2, \qquad \text{where } \gamma \equiv 1 \Big/ \sqrt{1 - \frac{v^2}{c^2}}$$

$$\text{In this same scheme, momentum becomes: } \mathbf{p} = \gamma m \mathbf{u}$$

$$\Rightarrow E^2 = p^2c^2 + m^2c^4; \qquad p^2c^2 = K^2 + 2Kmc^2$$

Relativistic energies

$$E = K + mc^2 = \gamma mc^2, \qquad \text{where } \gamma \equiv 1 \Big/ \sqrt{1 - \frac{v^2}{c^2}}$$

Example (electron):

$$mc^2 = 9.11 \times 10^{-31} \cdot \left(3 \times 10^8\right)^2 = 8.199 \times 10^{-14} J \, \frac{1eV}{1.6 \times 10^{-19} J} = 0.511 MeV$$

Example (proton):

$$mc^2 = 1.67 \times 10^{-27} \cdot \left(3 \times 10^8\right)^2 = 1.503 \times 10^{-10} J \, \frac{1eV}{1.6 \times 10^{-19} J} = 938 MeV$$

Relativistic momentum: $\mathbf{p} = \gamma m\mathbf{u}$

Note: $p^2c^2 + m^2c^4 = E^2$

$$\gamma^2 m^2 u^2 c^2 + m^2 c^4 = \gamma^2 m^2 c^4 \left(\frac{u^2}{c^2} - \frac{1}{\gamma^2} \right) = \gamma^2 m^2 c^4 \left(\frac{u^2}{c^2} - \left(1 - \frac{u^2}{c^2} \right) \right)$$

$$= \gamma^2 m^2 c^4 = E^2$$

Note: if $p << mc$

$$E \approx mc^2 + \frac{p^2}{2m} = mc^2 + \frac{1}{2} mu^2$$