Announcements

1. **Redo 4th exam – due 4/24/05**
2. **Extra credit session?**

   Monday evening?

   Tuesday evening?

   Wednesday evening?
3. **Today's topic quantum mechanics – particle-like behavior of electromagnetic radiation and wave-like behavior of particles**

   Scanning tunneling microscopes

Quantum Physics

- Changes in physical laws for very small distances, short times
- Electromagnetic waves have some particle-like properties

  discrete energies – photoelectron effect

  transfer momentum in collisions – Compton effect

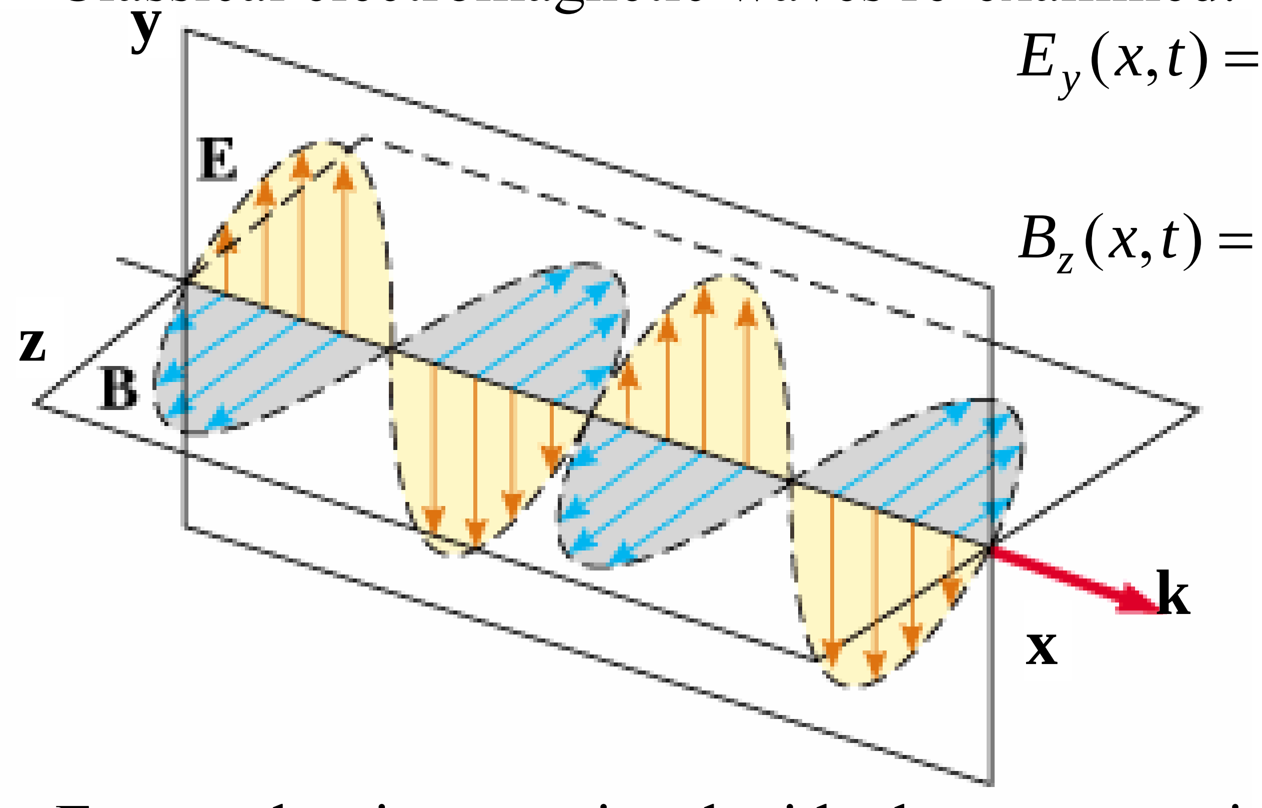
- Particles have some wave-like properties:

  de Broglie wavelength

  interference effects – Davisson-Germer experiment

  Bohr model of the atom

Classical electromagnetic waves re-examined:



$$E_y(x,t) = E_{\max} \sin\left(\frac{2\pi x}{\lambda} - 2\pi f t\right)$$

$$B_z(x,t) = \frac{E_{\max}}{c} \sin\left(\frac{2\pi x}{\lambda} - 2\pi f t\right)$$

Energy density associated with electromagnetic wave:

$$u = \tfrac{1}{2}\varepsilon_0 E^2 + \tfrac{1}{2} B^2/\mu_0 = \tfrac{1}{2}\left(\varepsilon_0 E_{\max}^2 + B_{\max}^2/\mu_0\right)\sin^2\left(\frac{2\pi x}{\lambda} - 2\pi f t\right)$$

$$\langle u \rangle_{avg} = \tfrac{1}{2}\varepsilon_0 E_{\max}^2$$

Poynting vector: $\langle \mathbf{S} \rangle_{avg} = \dfrac{\hat{\mathbf{x}}}{2\mu_0 c} E_{\max}^2$

Quantum theory of electromagnetic waves -- apparent at low intensities

$$\langle u \rangle_{avg} = hf\left(n + \tfrac{1}{2}\right) \qquad n = 0,1,2,3\cdots$$

h = 6.6261 x 10^{-34} J s

= 4.1323 x 10^{-15} eV s

$$\langle \mathbf{S} \rangle_{avg} = \hat{\mathbf{x}} h \frac{f}{c}\left(n + \tfrac{1}{2}\right) = \hat{\mathbf{x}} \frac{h}{\lambda}\left(n + \tfrac{1}{2}\right)$$

➔one "photon" has a quantum of energy *hf*

momentum h/λ

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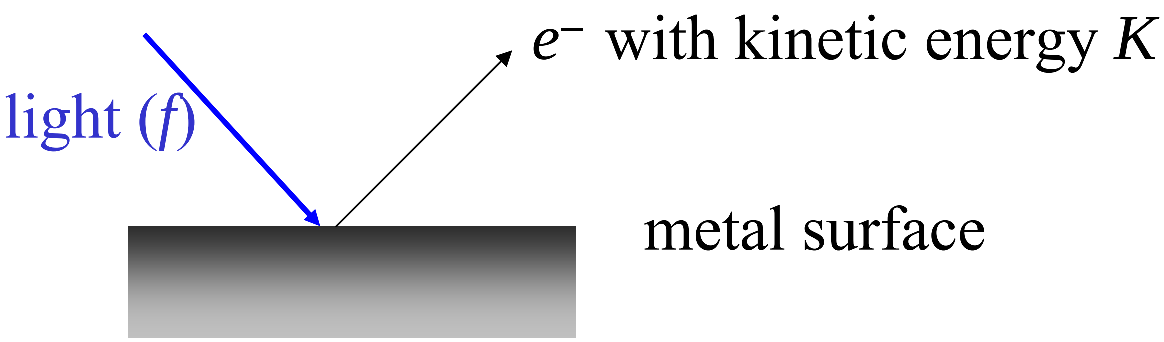
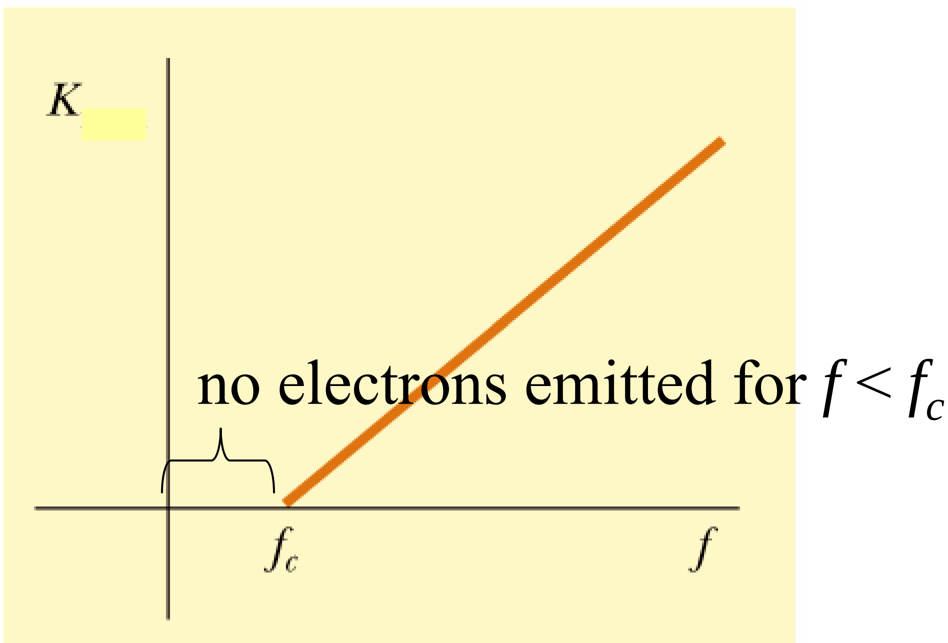
momentum h/λ

Experimental evidence:

1. Analysis by Max Planck of radiation distribution from thermal source (such as a glowing tungsten wire). (Requires assumptions from statistical formulation of thermal physics.)
2. Albert Einstein’s analysis of the photoelectric effect
3. Arthur H. Compton’s analysis of the scattering of light by an electron.

Photoelectron effect:

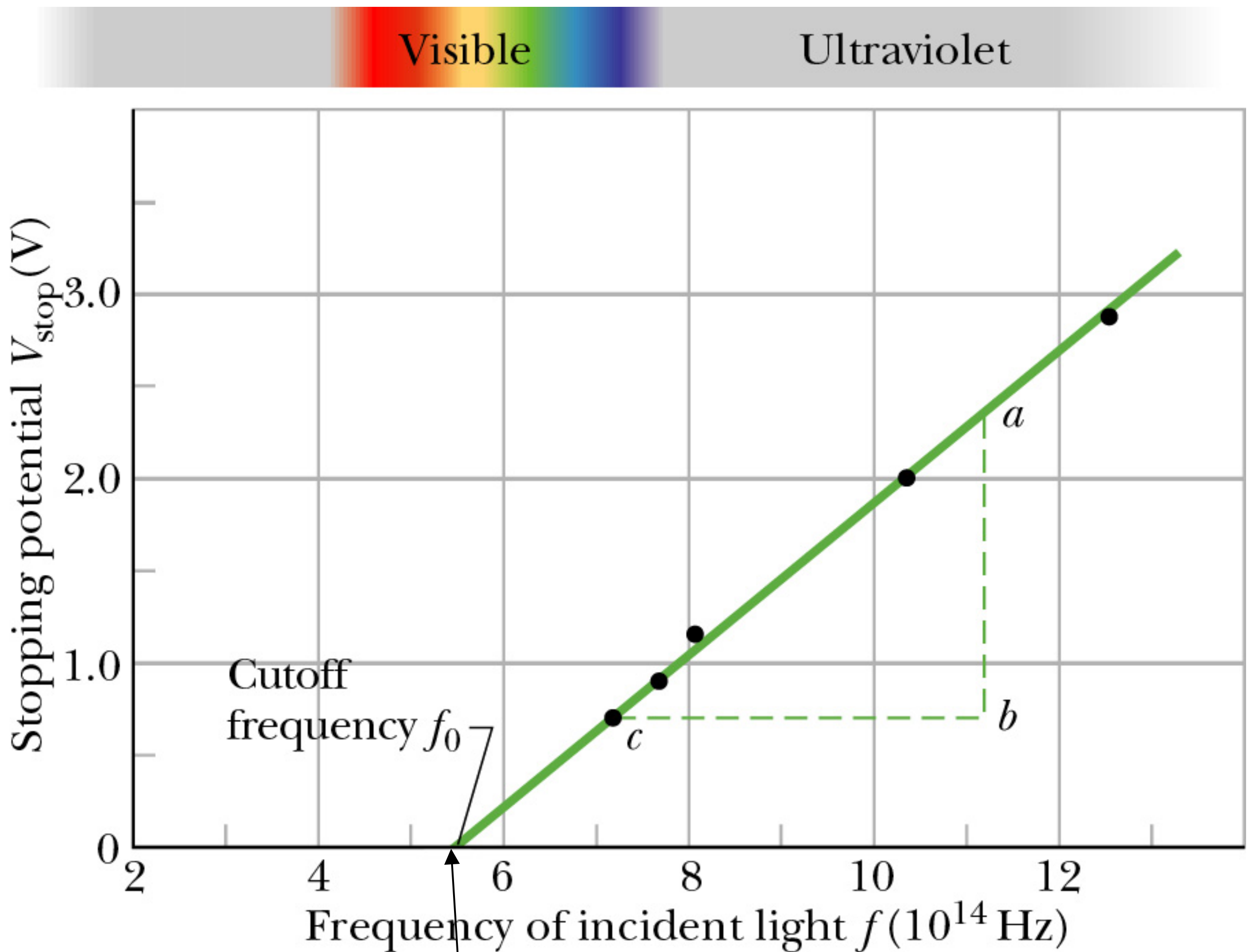




Einstein's idea based on the fact that each surface has a "work function" ϕ

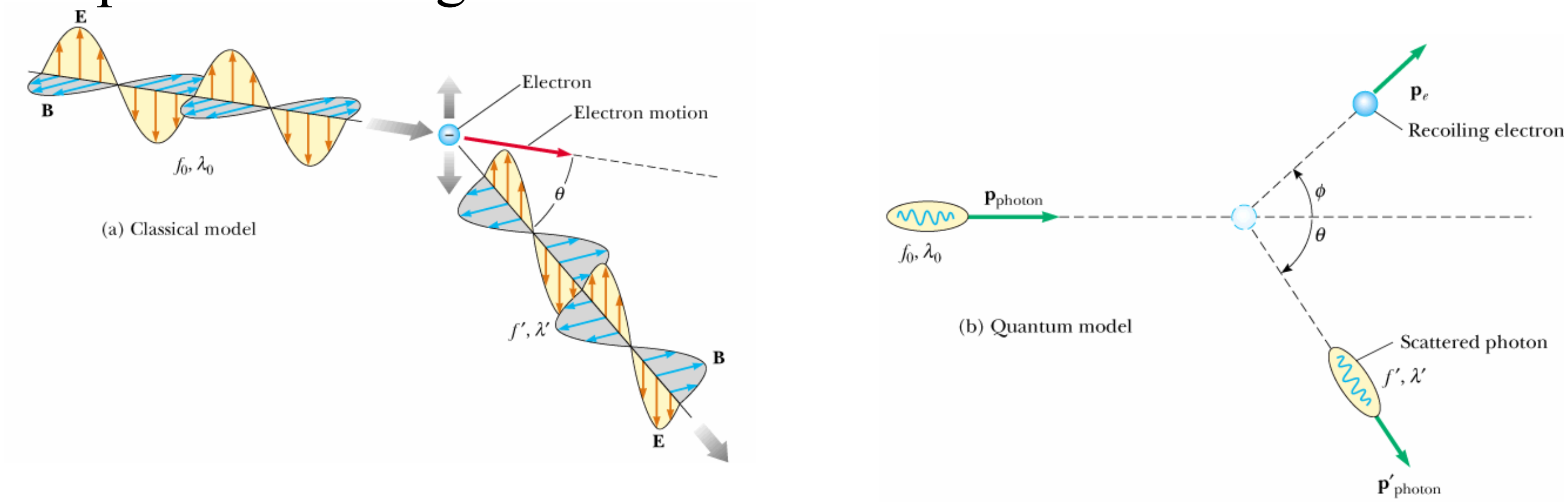
$$\boxed{hf_c = \phi}$$

$K = hf - \phi$
for $f > f_c$



$hf \cong 4.14 \cdot 10^{-15}\ 5.5 \cdot 10^{14}\ eV = 2.3\ eV$

Compton scattering



By analyzing the process with the assumption that the "photon" has an energy $E=hf$ and a momentum of magnitude $p=hf/c$, Compton was able to explain his data.

Note: Since the "photon" is traveling at the speed of light, we must treat this problem relativistically.

$$\text{Recall:} \quad E^2 = p^2c^2 + m^2c^4$$

$$\text{For photon:} \quad \Rightarrow E = pc = hf = h\frac{c}{\lambda}$$

Analysis of Compton effect continued:

Conservation of energy: $$h\frac{c}{\lambda_0} = h\frac{c}{\lambda'} + \left(\gamma mc^2 - mc^2\right)$$

Conservation of momentum: $$\frac{h}{\lambda_0} = \frac{h}{\lambda'}\cos\theta + \gamma mv\cos\varphi$$

$$0 = \frac{h}{\lambda'}\sin\theta - \gamma mv\sin\varphi$$

Result: $$\lambda' - \lambda_0 = \frac{h}{mc}\left(1 - \cos\theta\right)$$

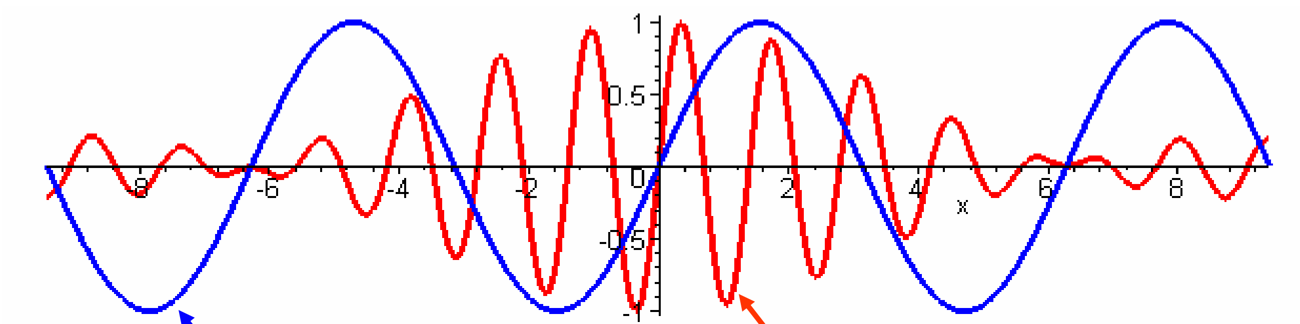
Compelling evidence of discrete "quanta" of light:

$E_{photon} = hf$ $\qquad$ $p_{photon} = h/\lambda$

Mathematical representation of particle and wave behaviors.

Consider a superposition of periodic waves at $t=0$:

$$E(x,t) = \sum_i E_{\max} \sin(k_i x)$$

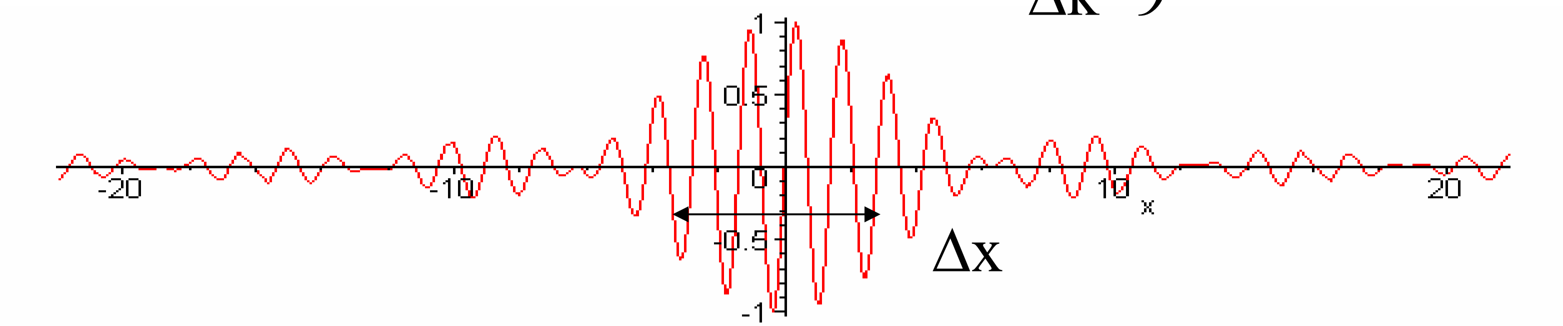


single wave
(one value of k)

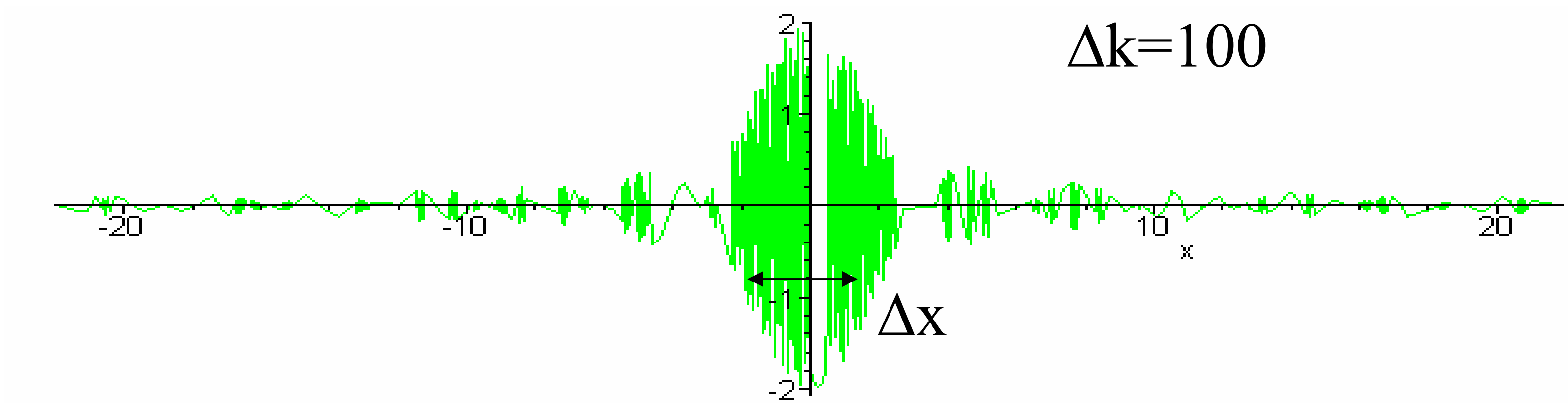
superposed wave
(many values of k)

More details about superposition:

$\Delta k=9$



$\Delta k=100$



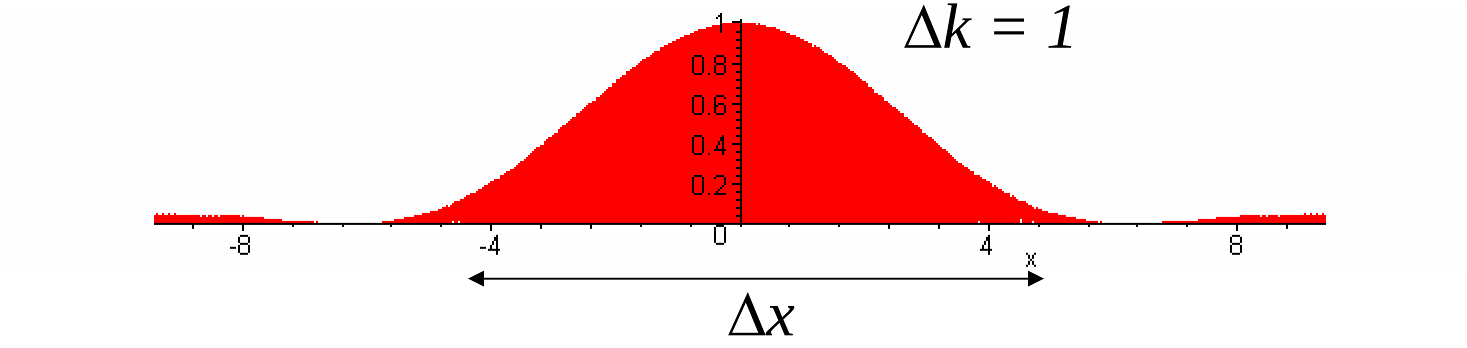
For intensities, we find $\Delta x \, \Delta k \approx$ (constant)

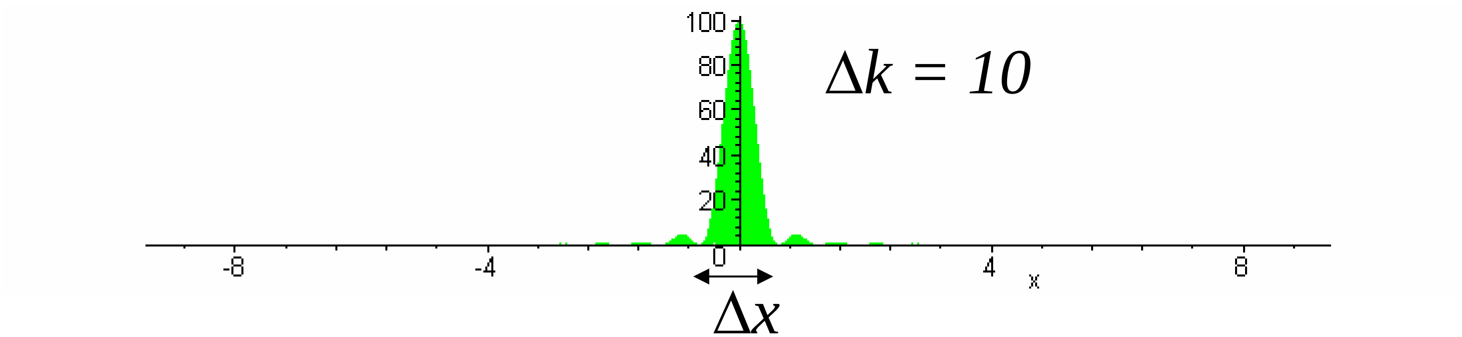
Δx smaller ➔ more particle like

Δk smaller ➔ more wave like

$$[E(x,0)]^2 = \left(\sum_i E_{\max} \sin(k_i x)\right)^2$$

$$\Delta x \, \Delta k \approx 2\pi$$





Δx smaller ➔ more particle like

Δk smaller ➔ more wave like

$\Delta x\ \Delta k \approx 2\pi$ ➔ Heisenberg's uncertainty principle

De Broglie's particle moment – wavelength relation:

$$p = \frac{h}{\lambda} = \frac{h/2\pi}{\lambda/2\pi} = \hbar k$$

Heisenberg's hypotheses: $\Delta x \Delta p \geq \frac{\hbar}{2}$

$$\Delta t \Delta E \geq \frac{\hbar}{2}$$

Wave equations

Electromagnetic waves:

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} = c^2 \frac{\partial^2 \mathbf{E}}{\partial x^2}$$

Matter waves: (Schrödinger equation)

$$-i\hbar \frac{\partial}{\partial t}\Psi(x,t) = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right]\Psi(x,t)$$

Electromagnetic waves	Matter waves
Vector – **E** or **B** fields Second order *t* dependence Examples: $E_y(x,t) = E_{\max}\sin(kx-\omega t)$ $B_z(x,t) = \frac{E_{\max}}{c}\sin(kx-\omega t)$	Scalar – probability amplitude First order *t* dependence Examples: $\Psi(x,t) = \Psi_0 \sin(kx) e^{-iEt/\hbar}$ "bound" states

What is the meaning of the matter wave function $\Psi(x,t)$?

- $\Psi(x,t)$ is not directly measurable
- $|\Psi(x,t)|^2$ is measurable – represents the density of particles at position x at time t.
- For a single particle system, $\int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = 1$
- For many systems of interest, the wave function can be written in the form $\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$

$$|\Psi(x,t)|^2 = |\psi(x)|^2$$

Wave-like properties of particles

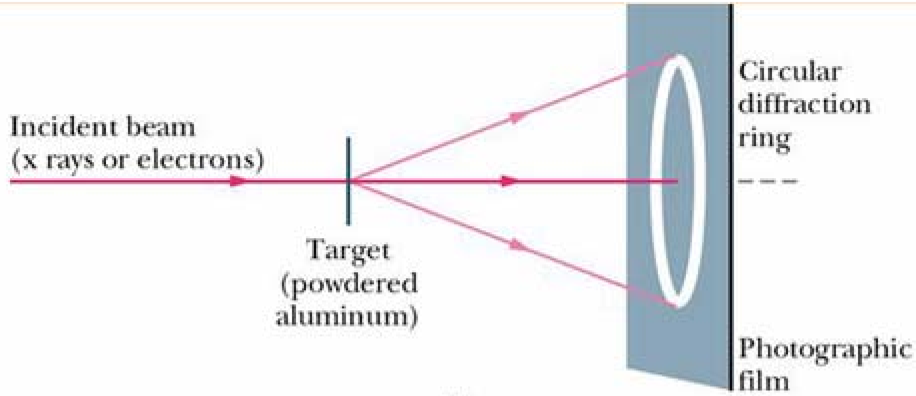
Louis de Broglie suggested that a wavelength could be associated with a particle's momentum

$$p = \frac{h}{\lambda}$$

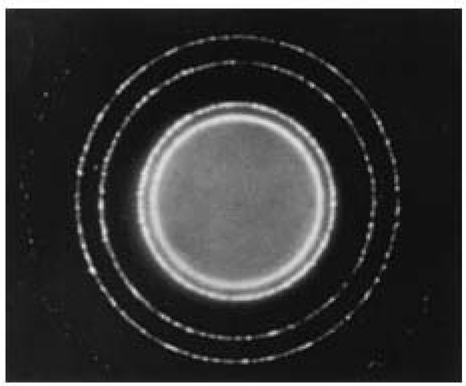
"Wave" equation for particles – Schrödinger equation

$$\left[-\frac{h^2}{(2\pi)^2 m}\frac{\partial^2}{\partial x^2} + V(x)\right]\Psi(x,t) = -i\frac{h}{2\pi}\frac{\partial}{\partial t}\Psi(x,t)$$

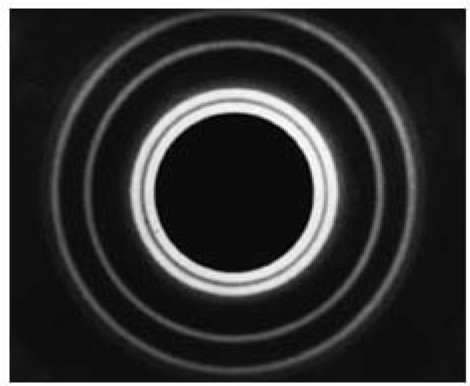
Experimental evidence of wave-like behavior of electrons – Davisson and Germer demonstration of electron diffraction



(a)



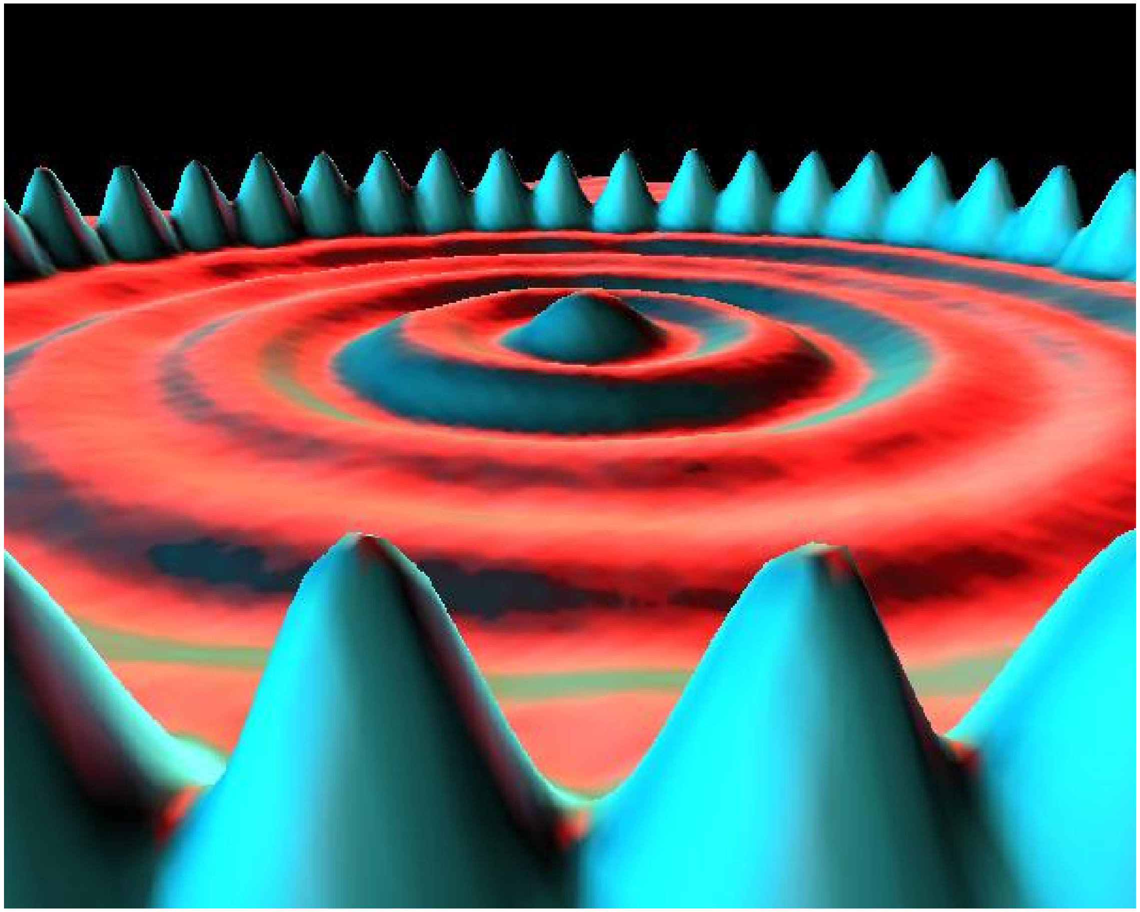
(b)



(c)

(b) Courtesy Riber Division of Instruments, Inc
(c) From PSSC film "Matter Waves", courtesy Education Development Center, Newton, Massachusetts

Visualization of $|\psi(x)|^2$

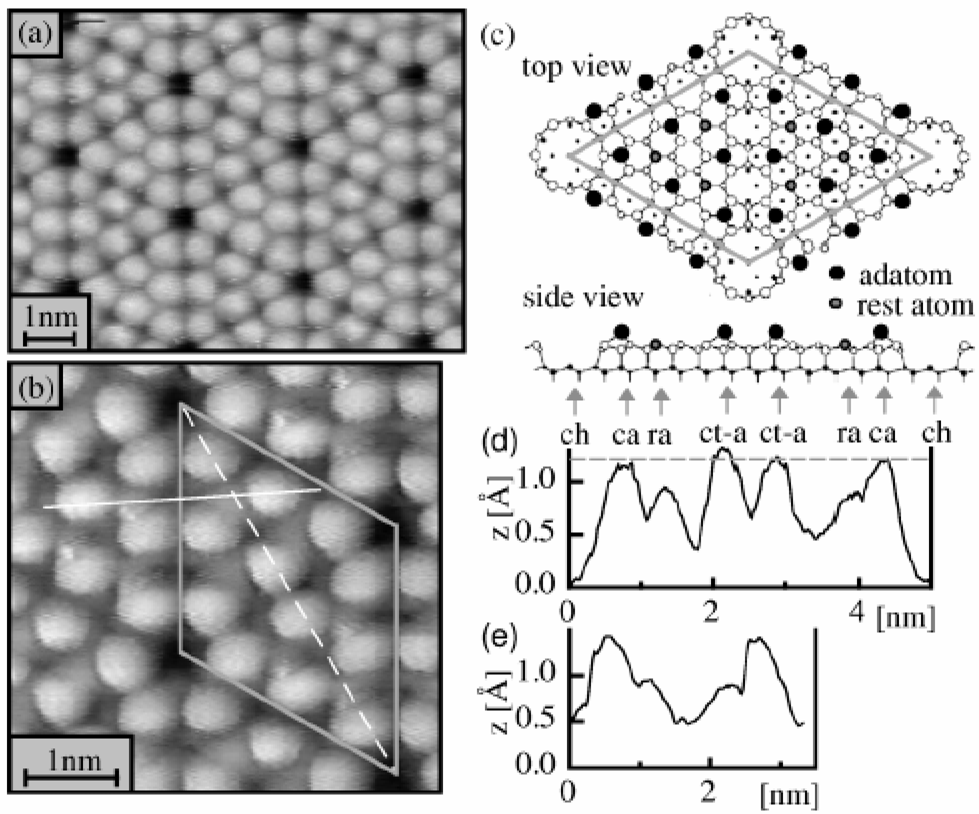


STM image of 48 iron atoms on a Cu surface.

From: M.F. Crommie, C.P. Lutz, D.M. Eigler. "Confinement of electrons to quantum corrals on a metal surface". Science **262**, 218-220 (1993).

Visualization of $|\psi(x)|^2$

A surface if a nearly perfect Si crystal



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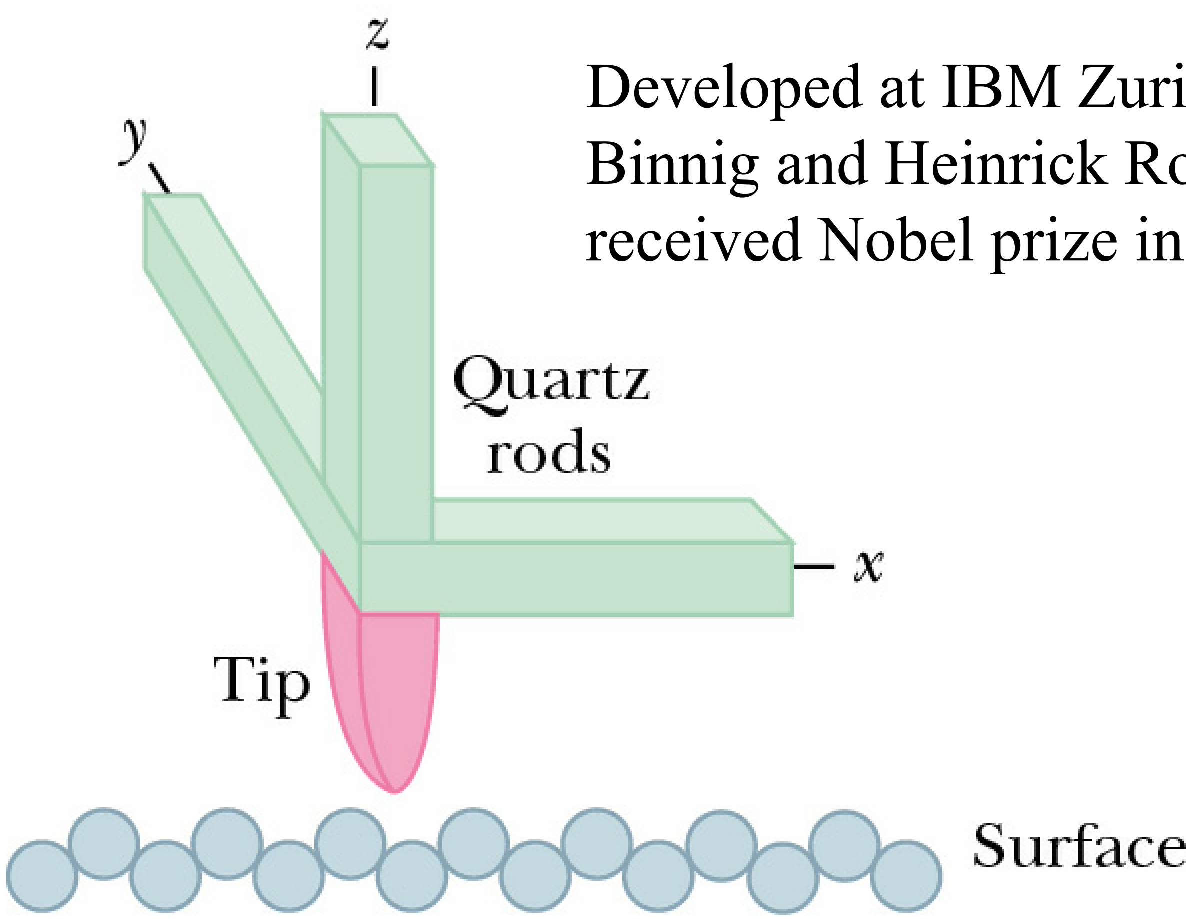
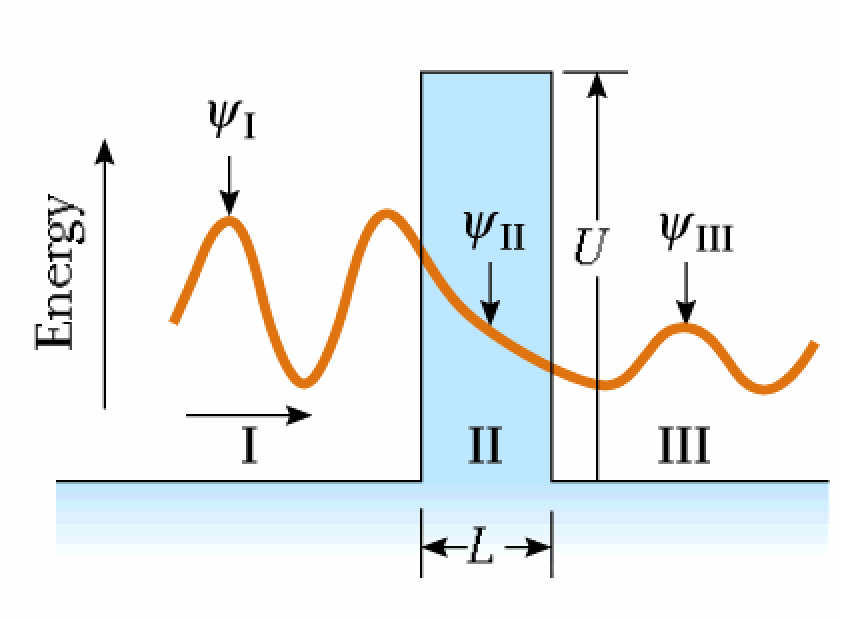
How a scanning tunneling microscope works:



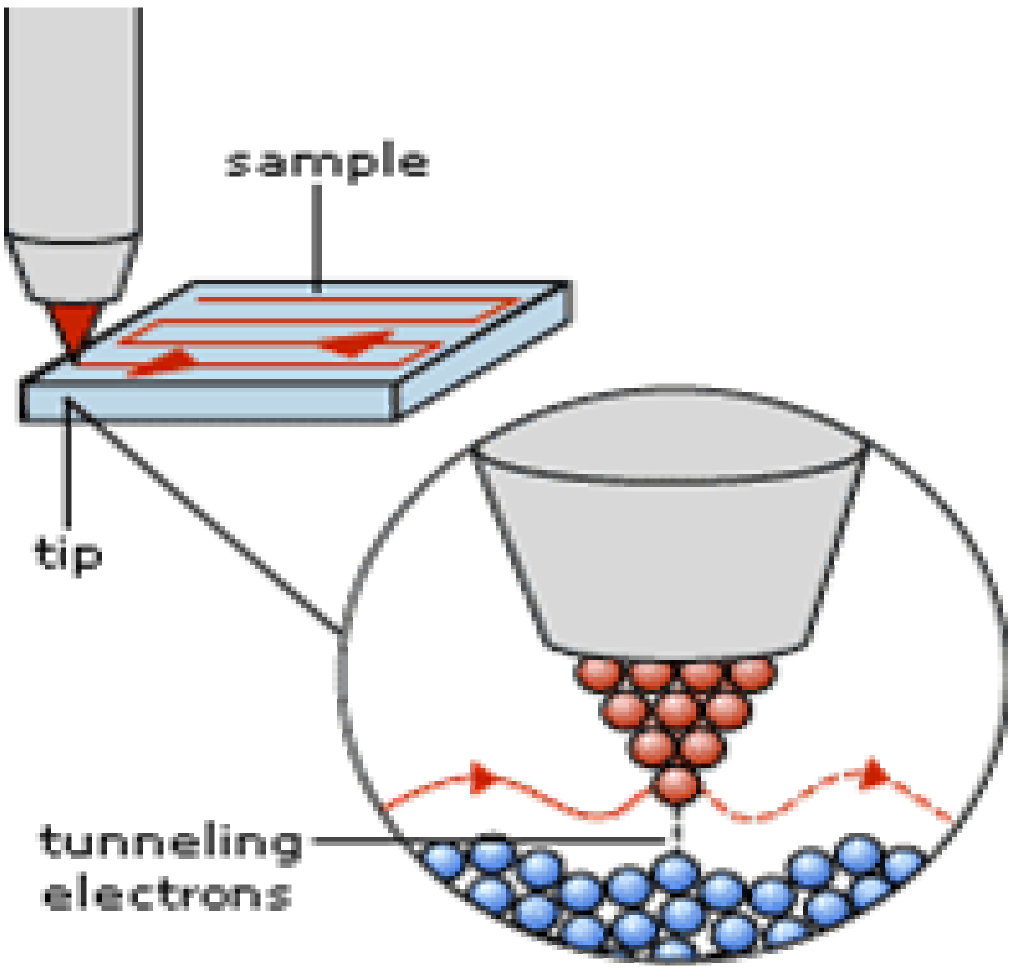
Developed at IBM Zurich by Gerd Binnig and Heinrick Rohrer who received Nobel prize in 1986.

Tunneling through a barrier



surface region tip

vacuum



http://nobelprize.org/physics/educational/microscopes/scanning/

Atomic force microscope – image of DNA taken by Professor Martin Guthold