Announcements

1. **Comments about Exam 4 and final exam format**
2. **Extra credit sessions:**

   Monday evening at 8 PM – 4 students so far

   Tuesday evening at 8 PM – 3 students so far

   Wednesday evening at 8 PM – 0 students so far

3. **Today's topic: (Chapters 39 & 40 of your text)**

   Continued discussion of quantum mechanics

   Lasers

4. **Wednesday – general review**

Quantum physics –

- Electromagnetic waves sometimes behave like particles

➔one "photon" has a quantum of energy E=hf

momentum p=$h/\lambda=hf/c$

- Particles sometimes behave like waves

➔"wavelength" of particle related to momentum:

$$\lambda=h/p$$

➔ quantum particles can "tunnel" to places classically "forbidden"

➔ Stationary quantum states have quantized energies

Wave equations

Electromagnetic waves:

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} = c^2 \frac{\partial^2 \mathbf{E}}{\partial x^2}$$

Matter waves: (Schrödinger equation)

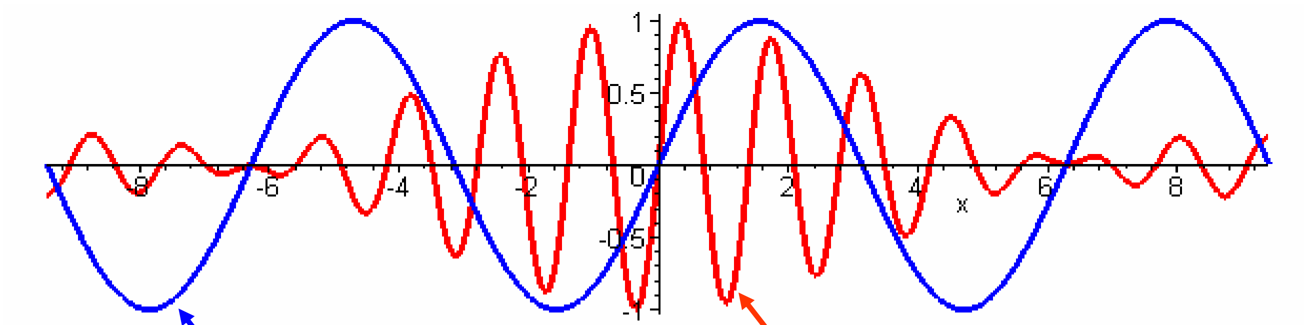
$$-i\hbar \frac{\partial}{\partial t}\Psi(x,t) = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right]\Psi(x,t)$$

Particle properties	Wave properties
Position as a function of time is known -- **r**(t)	Phonomenon is spread out over many positions at an instant of time.
Particle is spatially confined when E≤U(r).	Finite probability that particle can be found in regions other than E≤U(r).
Particles are independent.	Interference effects.

Mathematical representation of particle and wave behaviors.

Consider a superposition of periodic waves at *t=0:*

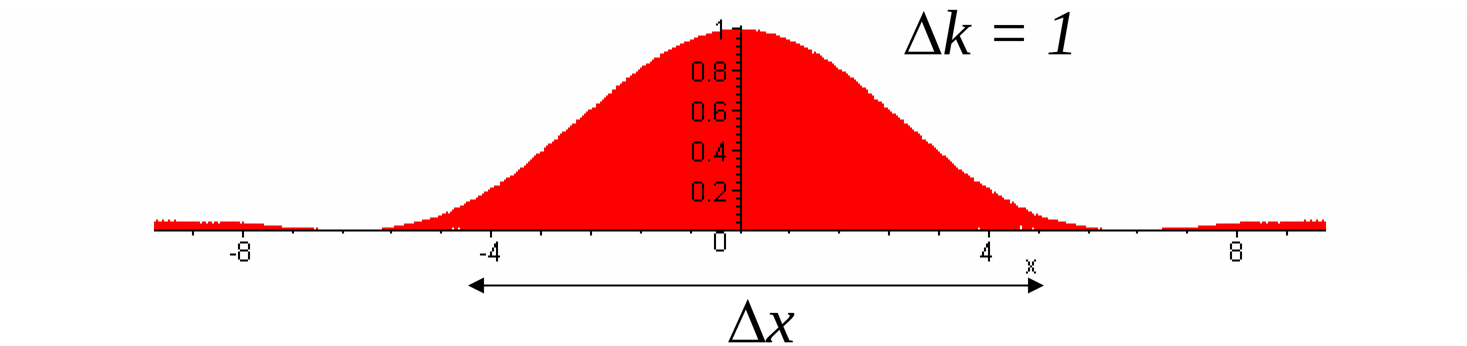
$$E(x,t) = \sum_i E_{\max} \sin(k_i x)$$

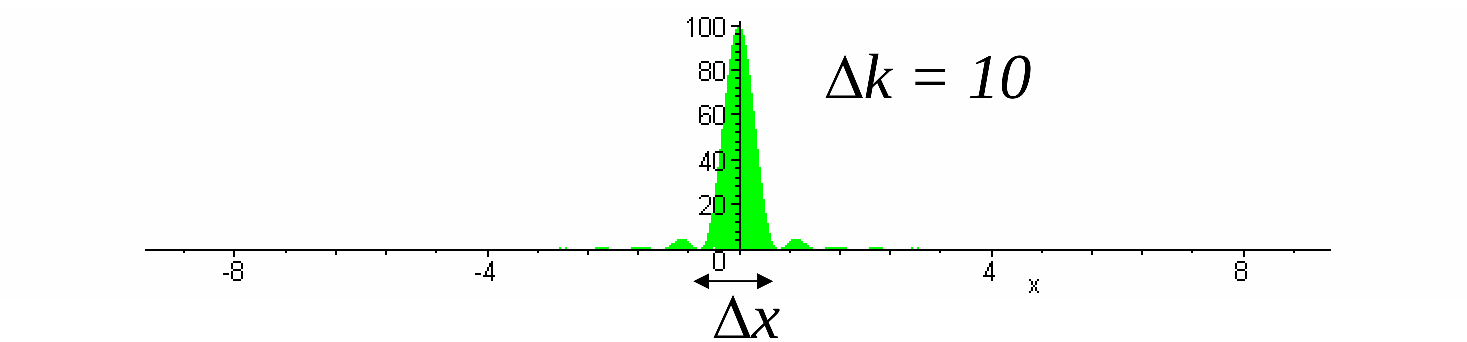


single wave
(one value of *k*)

superposed wave
(many values of *k*)

$$[E(x,0)]^2 = \left(\sum_i E_{\max} \sin(k_i x) \right)^2 \qquad \Delta x \, \Delta k \approx 2\pi$$





Δx smaller ➔ more particle like

Δk smaller ➔ more wave like

$\Delta x\, \Delta k \approx 2\pi$ ➔ Heisenberg's uncertainty principle

De Broglie's particle moment – wavelength relation:

$$p = \frac{h}{\lambda} = \frac{h/2\pi}{\lambda/2\pi} = \hbar k$$

Heisenberg's hypotheses: $\Delta x \Delta p \geq \frac{\hbar}{2}$

$$\Delta t \Delta E \geq \frac{\hbar}{2}$$

$h = 6.6\times10^{-34}$ Js $= 4.14\times10^{-15}$ eVs

Electromagnetic waves	Matter waves
Vector – **E** or **B** fields Second order *t* dependence Examples: $E_y(x,t) = E_{\max}\sin(kx - \omega t)$ $B_z(x,t) = \frac{E_{\max}}{c}\sin(kx - \omega t)$	Scalar – probability amplitude First order *t* dependence Examples: $\Psi(x,t) = \Psi_0 \sin(kx) e^{-iEt/\hbar}$ "bound" states

What is the meaning of the matter wave function $\Psi(x,t)$?

- $\Psi(x,t)$ is not directly measurable
- $|\Psi(x,t)|^2$ is measurable – represents the density of particles at position x at time t.
- For a single particle system, $\int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = 1$
- For many systems of interest, the wave function can be written in the form $\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$

$$|\Psi(x,t)|^2 = |\psi(x)|^2$$

Wave-like properties of particles

Louis de Broglie suggested that a wavelength could be associated with a particle's momentum

$$p = \frac{h}{\lambda}$$

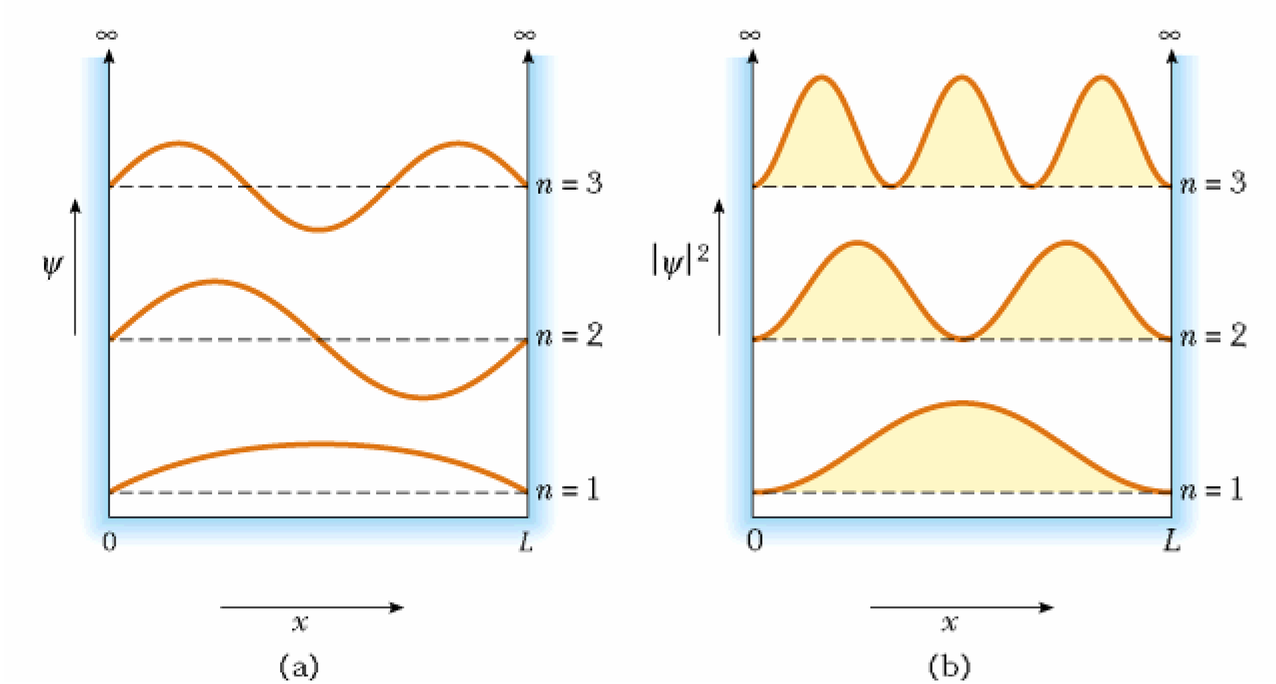
"Wave" equation for particles – Schrödinger equation

$$\left[-\frac{h^2}{(2\pi)^2 m}\frac{\partial^2}{\partial x^2} + V(x)\right]\Psi(x,t) = -i\,\frac{h}{2\pi}\frac{\partial}{\partial t}\Psi(x,t)$$

$$\text{Stationary-state wavefunctions:} \quad \Psi(\mathbf{r},t) = \psi(\mathbf{r})e^{-iEt/\hbar}$$

$$\left[-\frac{h^2}{(2\pi)^2 m}\frac{\partial^2}{\partial x^2} + V(x)\right]\Psi(x,t) = E\Psi(x,t)$$

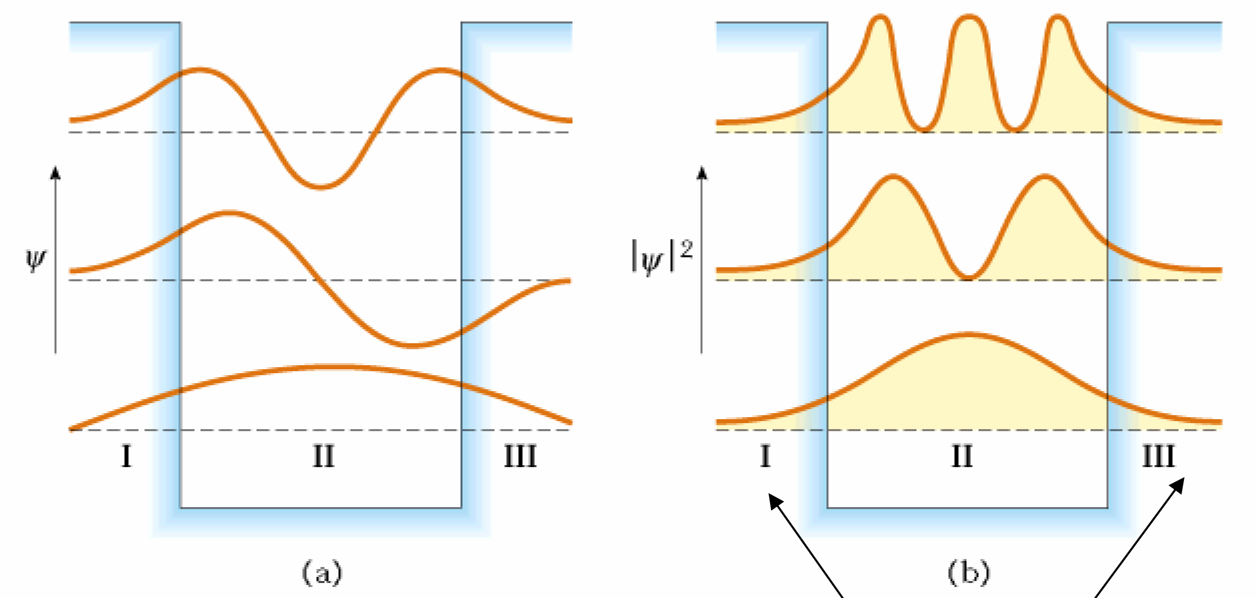
Electrons in an infinite box:



$$E\,\psi(x) = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\right]\psi(x) \quad \text{for } 0 \le x \le L$$

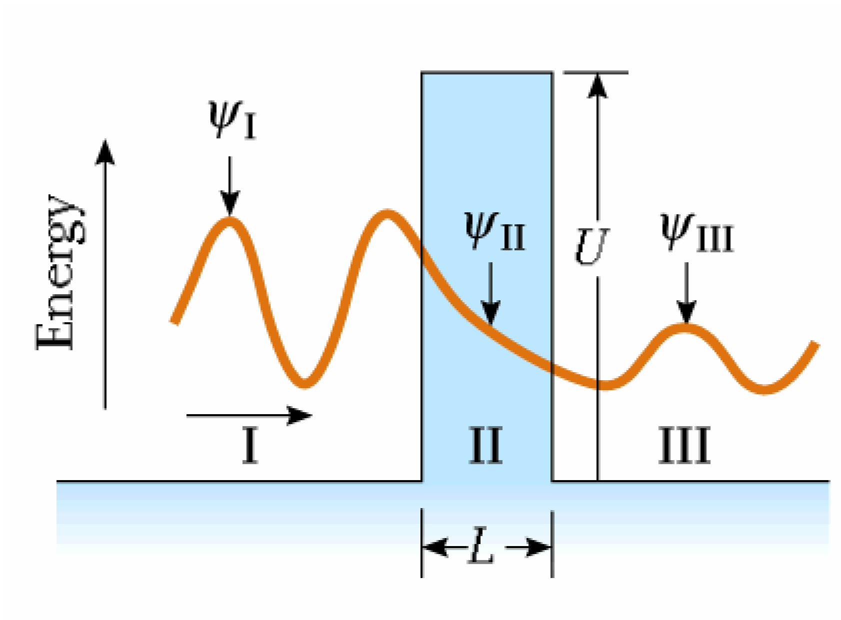
$$\psi(x) = \psi_0 \sin\left(\frac{n\pi x}{L}\right) \qquad n = 1,2,3\cdots \qquad E_n = \frac{\hbar^2\pi^2 n^2}{2m}$$

Electrons in a finite box:



finite probability of electron existing outside of classical region

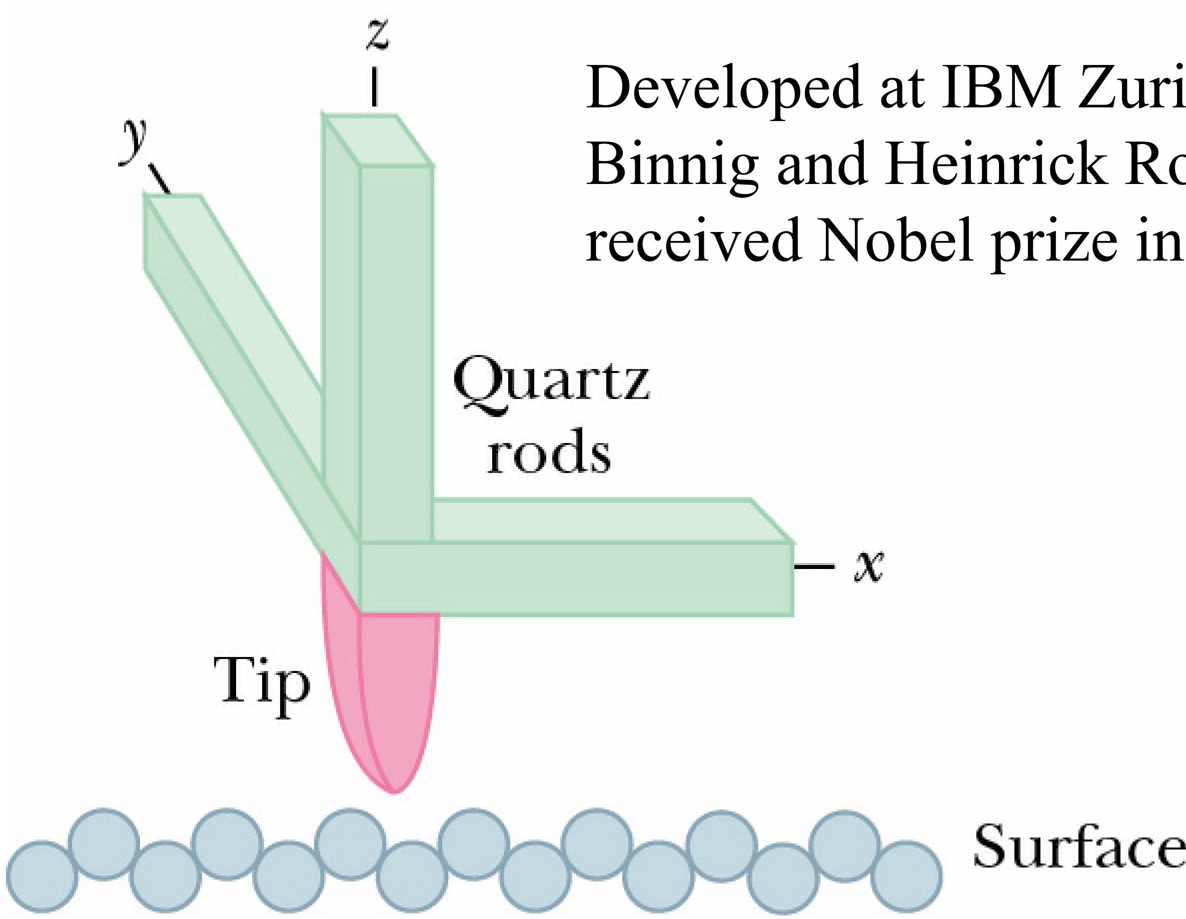
Tunneling through a barrier



surface region tip

vacuum

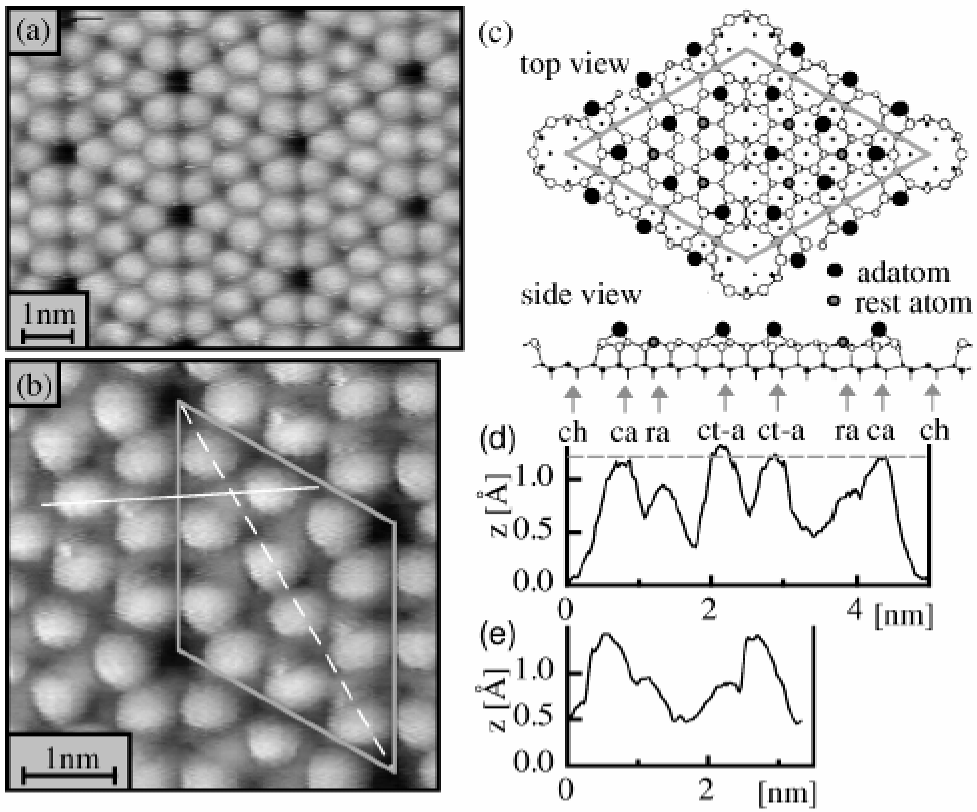
How a scanning tunneling microscope works:



Developed at IBM Zurich by Gerd Binnig and Heinrick Rohrer who received Nobel prize in 1986.

Visualization of $|\psi(x)|^2$

A surface if a nearly perfect Si crystal



Physical Review Letters -- March 20, 2000 -- Volume 84, Issue 12, pp. 2642-2645

The physics of atoms –

Features are described by solutions to the matter wave equation – Schrödinger equation:

$$-i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t)=\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial \mathbf{r}^2}+V(\mathbf{r})\right]\Psi(\mathbf{r},t)$$

"reduced" mass of electron and proton

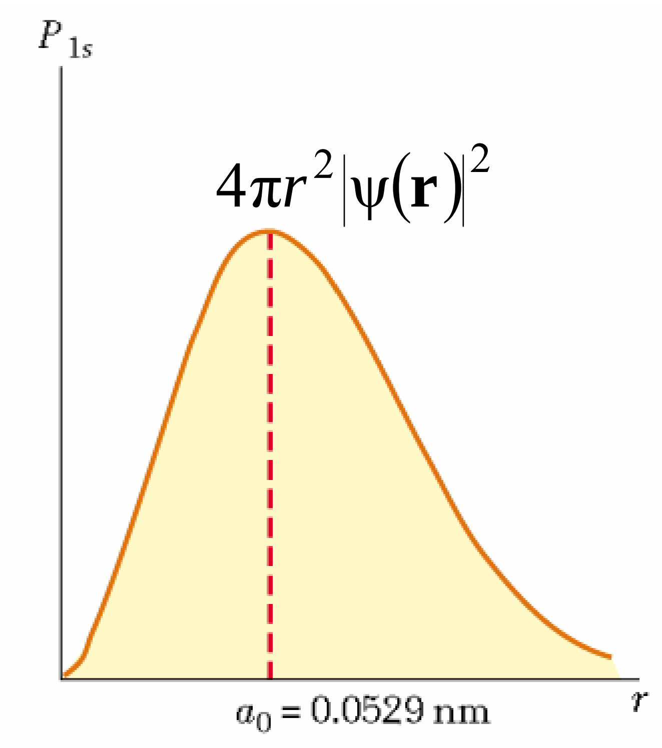
$$-\frac{Ze^2}{4\pi\varepsilon_0 r}$$

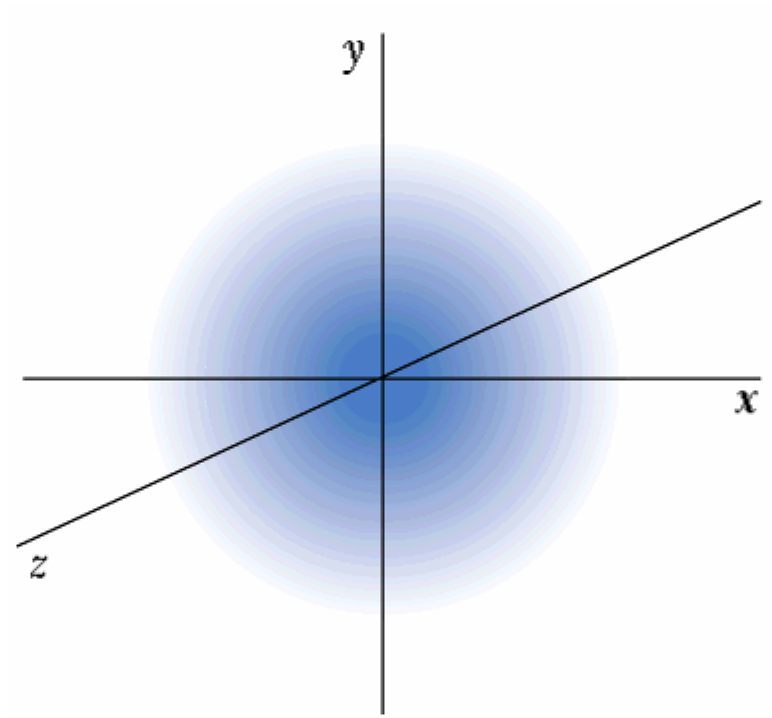
$$\text{Stationary-state wavefunctions:}\quad \Psi(\mathbf{r},t)=\psi(\mathbf{r})e^{-iEt/\hbar}$$

$$\text{Solutions:}\quad E_n=-\frac{Z^2e^2}{8\pi\varepsilon_0 a_0}\frac{1}{n^2}=-13.6\frac{Z^2}{n^2}\text{ eV}$$

$$a_0=\frac{4\pi\varepsilon_0\hbar^2}{me^2}=0.0529\text{ nm}$$

Form of probability density for ground state (n = 1)





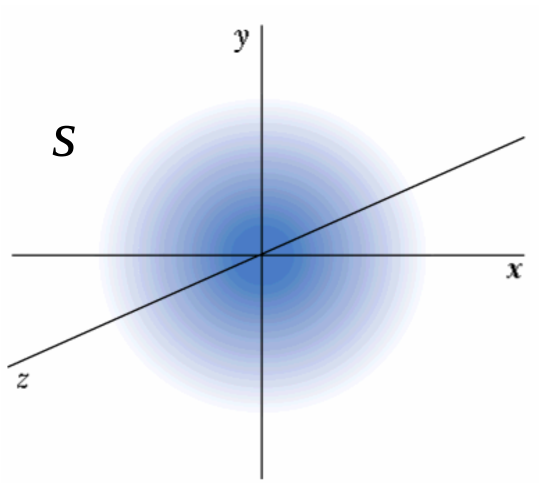
Angular degrees of freedom

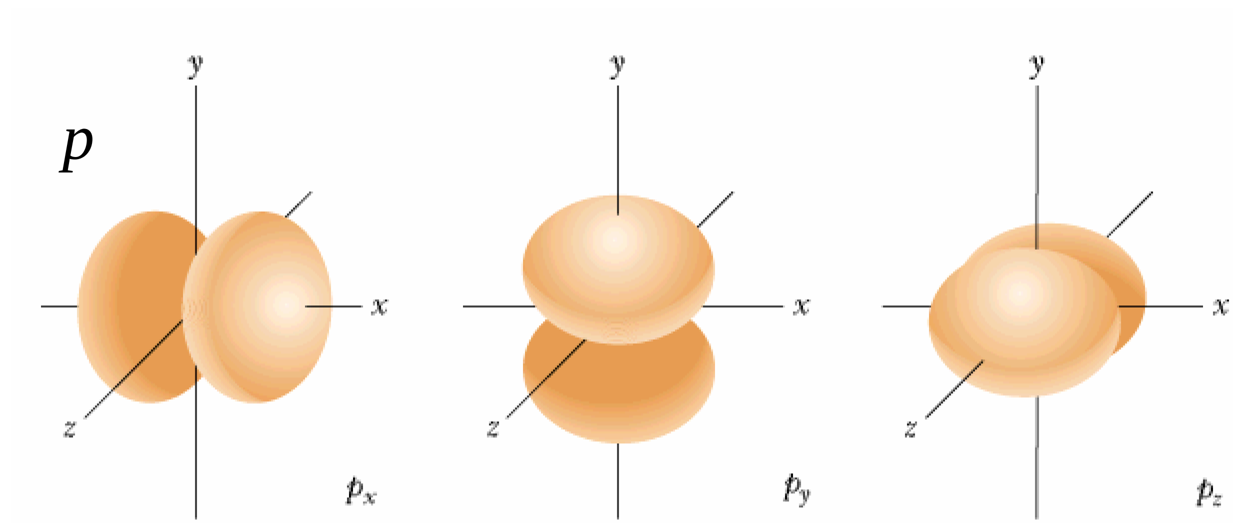
-- since the force between the electron and nucleus depends only on distance and not on angle, angular momentum $\mathbf{L} \equiv \mathbf{r} \times \mathbf{p}$ is conserved. Quantum numbers associated with angular momentum:

$$\mathbf{L}^2 = \hbar^2 \ell(\ell+1) \qquad \ell = 0,1,2,....(n-1)$$

$$L_z = \hbar m \qquad -\ell \le m \le \ell \quad \text{total of } 2\ell+1 \text{ states}$$

Notation: $\ell = 0 \Rightarrow s, \quad 1 \Rightarrow p, \quad 2 \Rightarrow d$





Summary of results for H-atom:

$$E_n = -13.6 \frac{Z^2}{n^2} \text{ eV}$$

Balmer series

spectra

degeneracy associated with each n: $2n^2$

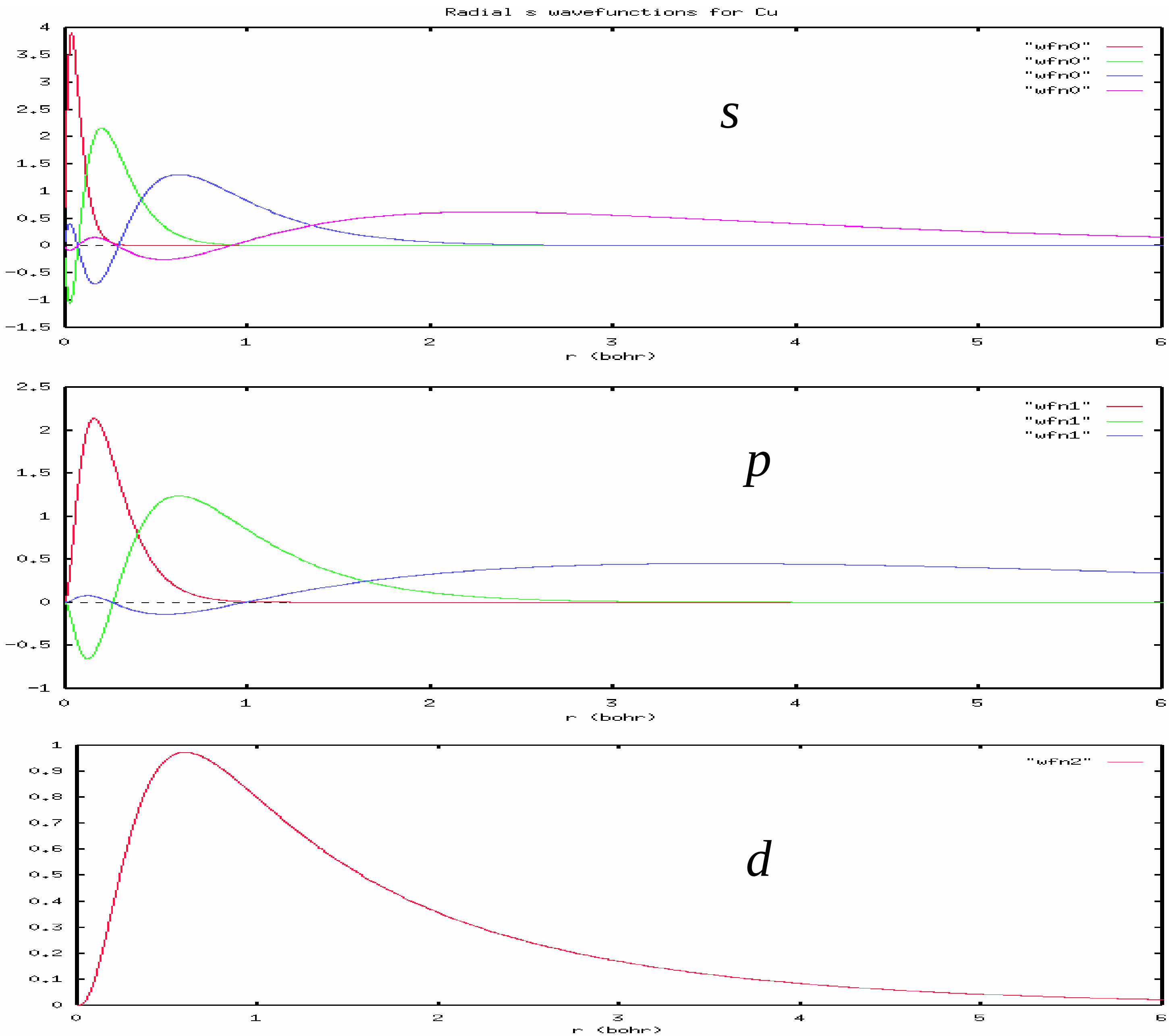
Atomic states of atoms throughout periodic table:

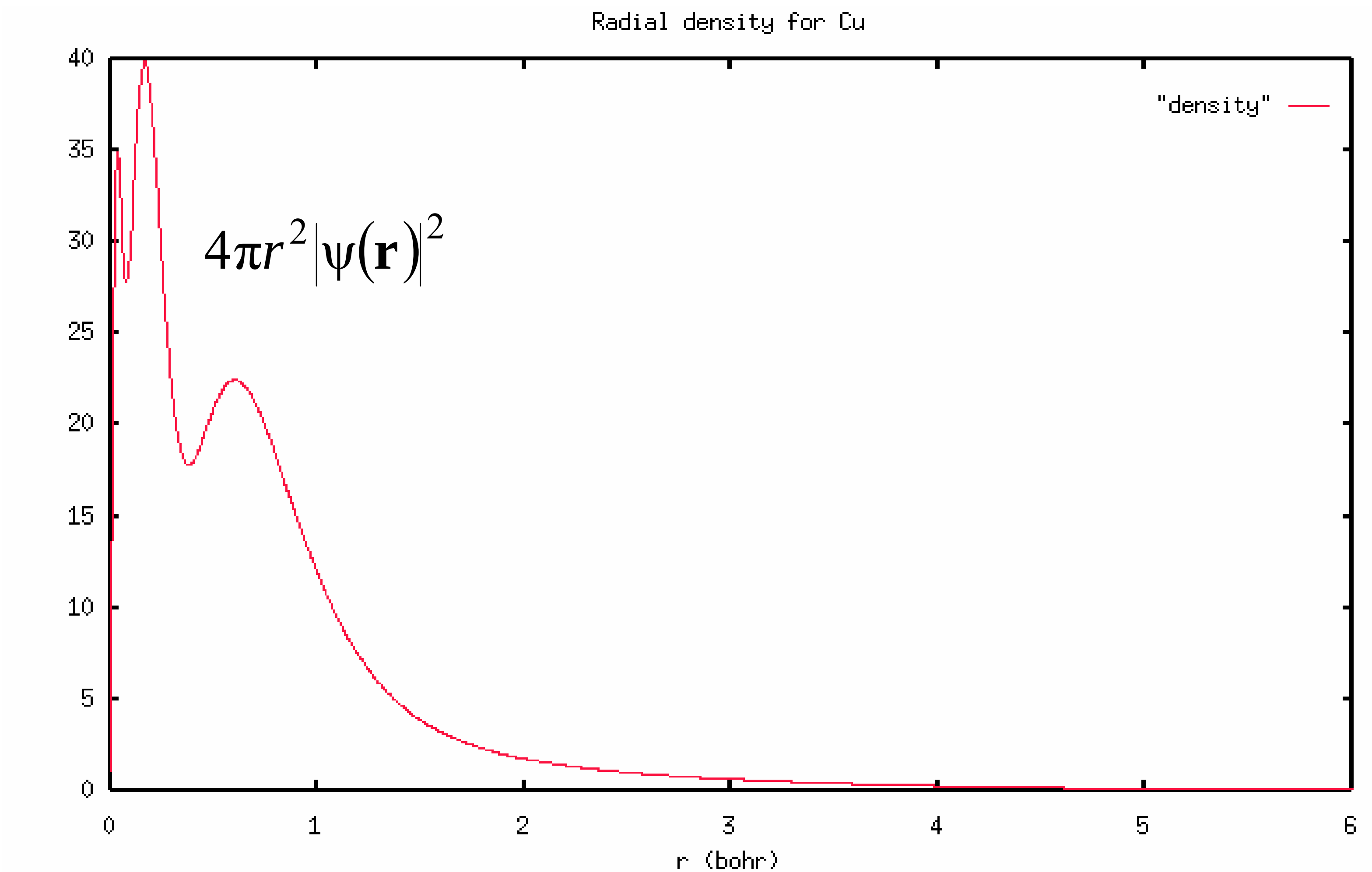
1 H																	2 He
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 As	36 Kr
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba	57 La	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	89 Ac															

$$E\psi(\mathbf{r}) = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial \mathbf{r}^2} + V(\mathbf{r}) \right]\psi(\mathbf{r})$$

effective potential for an electron in atom

Example: Cu (Z=29) $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^1$




Radial density for Cu
"density"
$4\pi r^2 |\psi(\mathbf{r})|^2$
40
35
30
25
20
15
10
5
0
0
1
2
3
4
5
6
r (bohr)

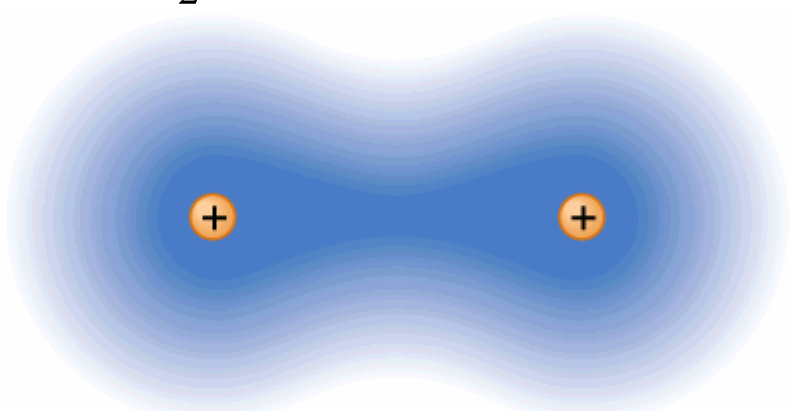
Physics of molecules and solids

$$E\psi(\mathbf{r}) = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial \mathbf{r}^2} + V(\mathbf{r}) \right]\psi(\mathbf{r})$$

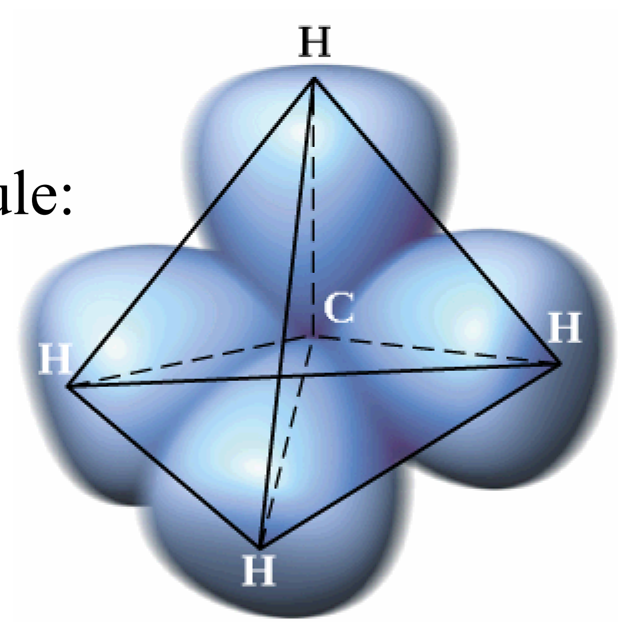
effective potential for electron in molecule or solid

Example: electron density associated with

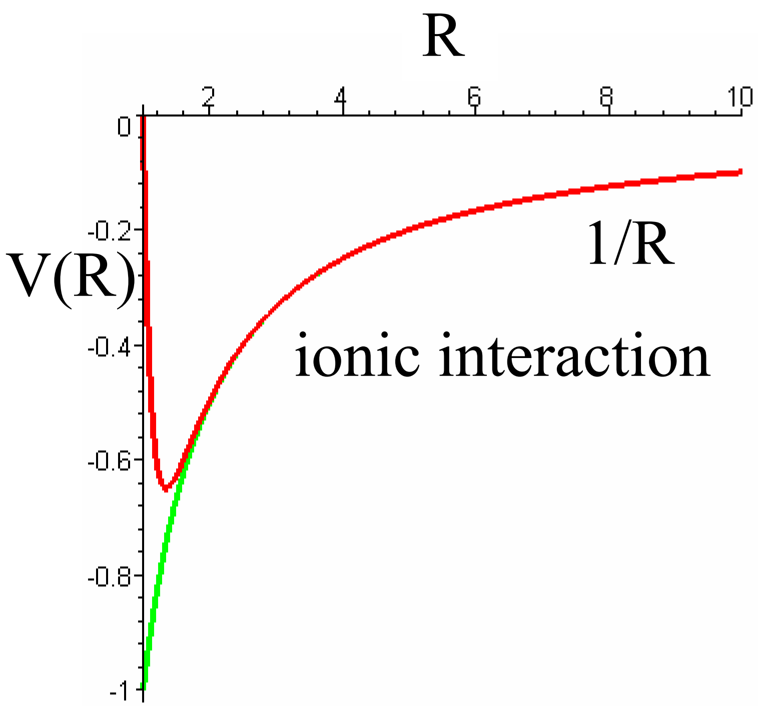
H_2 molecule:

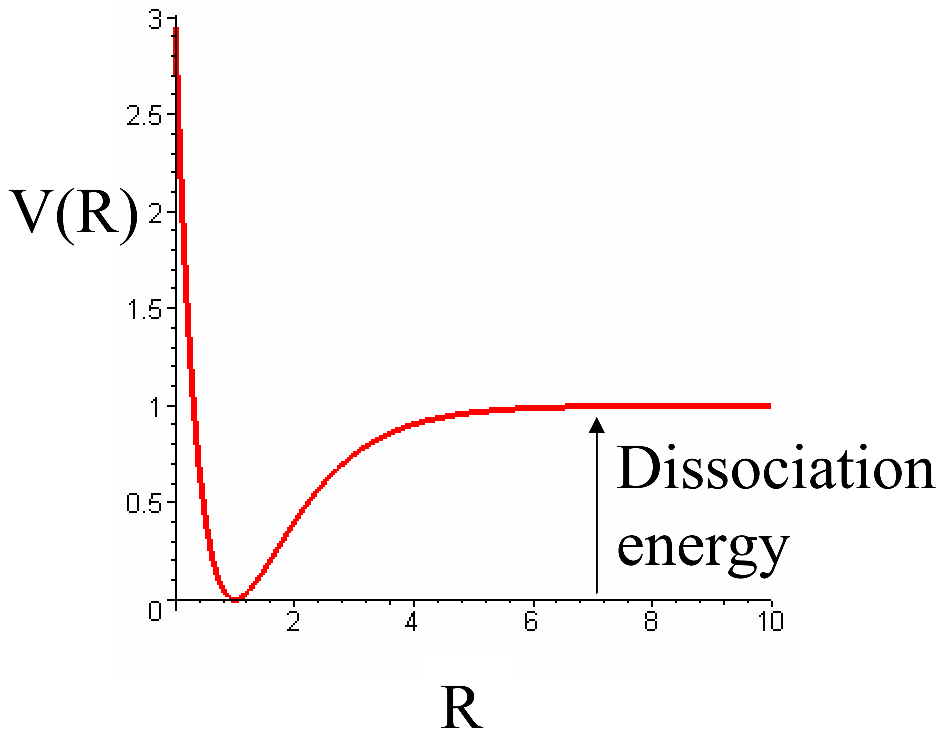


CH_4 molecule:



Molecular binding of nuclei due to electron "glue":



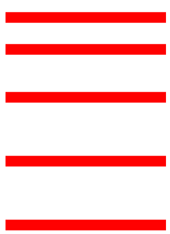


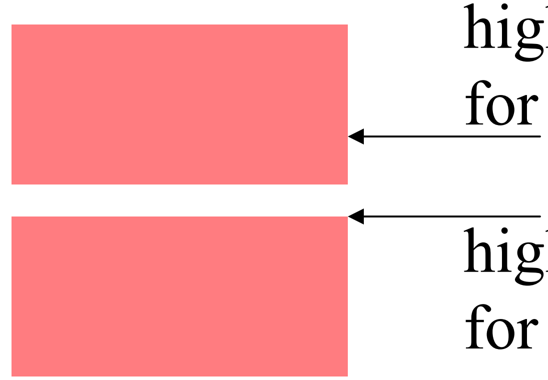
Physics of solids

$$E\psi(\mathbf{r}) = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial \mathbf{r}^2} + V(\mathbf{r}) \right]\psi(\mathbf{r})$$

effective potential for electron in molecule or solid

Energy spectrum of atom or molecule :



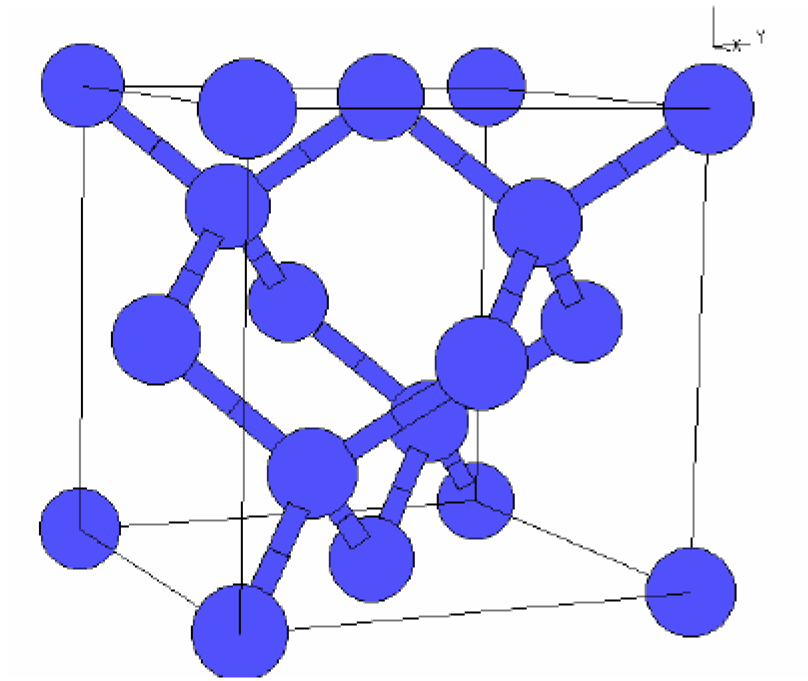


highest filled state for metal

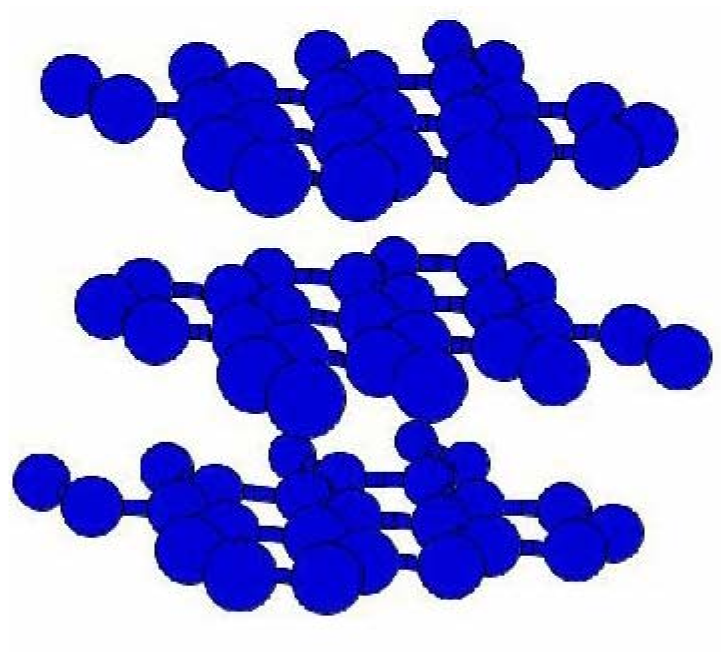
highest filled state for insulator

Example: 2 materials made of pure carbon:

diamond (insulator)



graphite (semi-metal)



Laser technology:

- System with ground and excited state at desired λ ($E_{ex}-E_g=hc/\lambda$).
- Standing EM wave
- Mechanism for "population inversion"

The Helium–Neon Gas Laser

Figure 41-21 shows a type of laser commonly found in student laboratories. It was developed in 1961 by Ali Javan and his coworkers. The glass discharge tube is filled with a 20:80 mixture of helium and neon gases, neon being the medium in which laser action occurs.

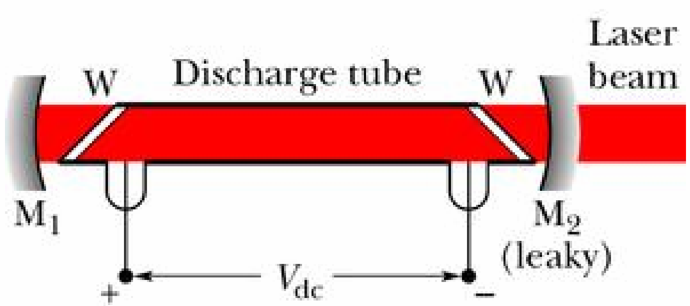


Fig. 41-21 The elements of a helium–neon gas laser. An applied potential V_{dc} sends electrons through a discharge tube containing a mixture of helium gas and neon gas. Electrons collide with helium atoms, which then collide with neon atoms, which emit light along the length of the tube. The light passes through transparent windows W and reflects back and forth through the tube from mirrors M_1 and M_2 to cause more neon atom emissions. Some of the light leaks through mirror M_2 to form the laser beam.

Figure 41-22 shows simplified energy-level diagrams for the two atoms. An electric current passed through the helium–neon gas mixture serves—through collisions between helium atoms and electrons of the current—to raise many helium atoms to state E_3, which is metastable.