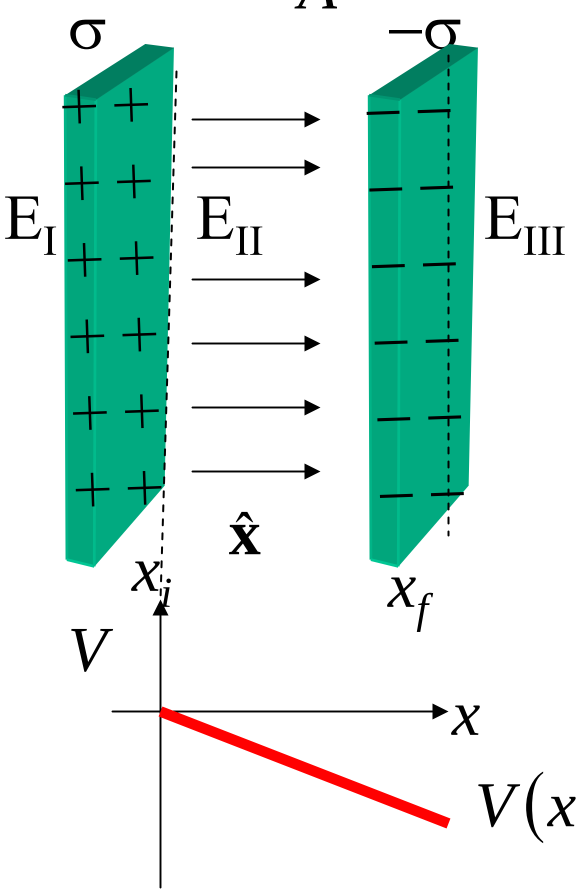
Announcements

1. Problem solving session tomorrow (Tuesday) 6 PM in Olin 101
2. Practice problems available -- http://www.wfu.edu/~natalie/s05phy114/extrapractice/
3. Today's topic – capacitance and dielectrics
   a. Parallel plate capacitors – relationship between charge and voltage
   b. Dielectric properties of materials and how that relates to capacitors
   c. Capacitors as components of a circuit

Electrostatic field and voltage between two charged plates

$$\sigma \equiv \frac{Q}{A}$$

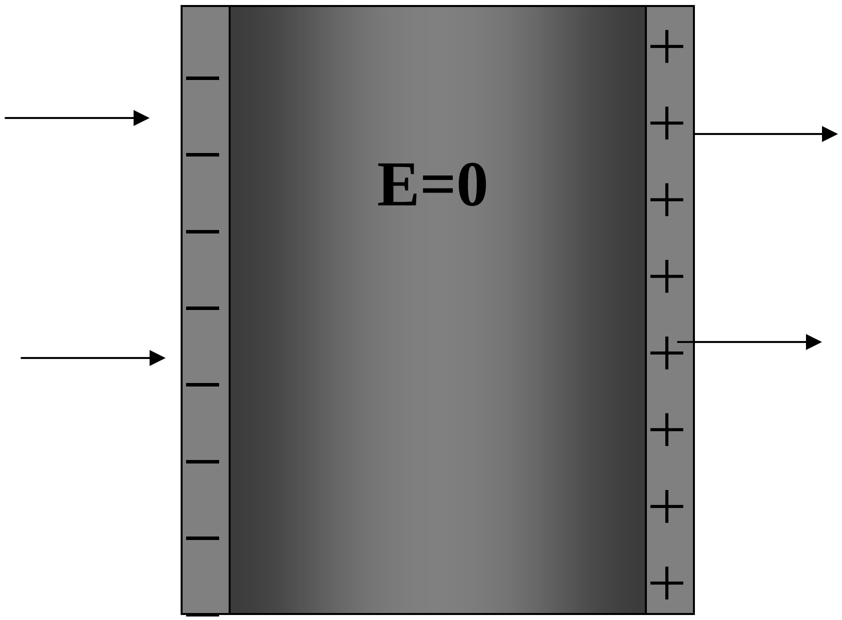


Last week we showed that the configuration of two charged sheets shown on the left correspond to the electrostatic fields $E_I = E_{III} = 0$ and $\mathbf{E}_{II} = \frac{\sigma}{\varepsilon_0}\hat{\mathbf{x}}$.

$$V(x) = -\int_{x_i}^{x} E\,dx = -\frac{Q}{A\varepsilon_0}(x - x_i)$$

Some special properties of metals:

- Within a conducting material in equilibrium, the electrostatic field vanishes
- Excess or asymmetric charges reside only at the surfaces of conductors at equilibrium



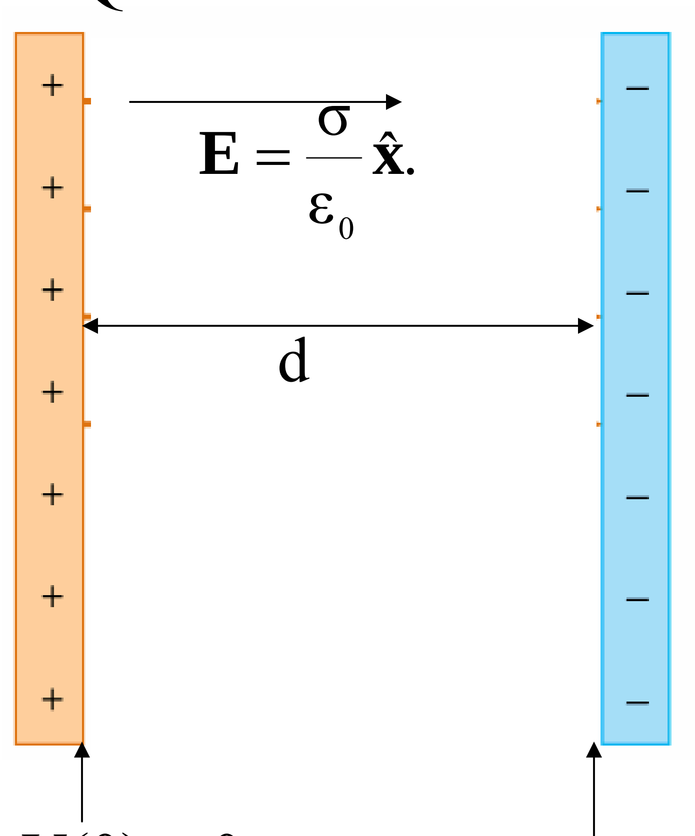
Note that if $\mathbf{E} = 0$

$\Rightarrow V = \text{(constant)}$

Thus, the interior of a metal is a region of *constant* electrostatic potential *V*. (This remains approximately true in the presence of currents.)

Electrostatic field and voltage between two charged plates

$\sigma = Q/A$ $\qquad\qquad$ $-\sigma$



$V(0) = 0$

$V(d) = -\sigma d/\varepsilon_0$

$$|\Delta V| = \frac{\sigma d}{\varepsilon_0} = \frac{Qd}{A\varepsilon_0} \equiv \frac{Q}{C}$$

$$\text{where, } C \equiv \frac{A\varepsilon_0}{d}.$$

unit of capacitance:

1 F = 1 “Farad” (named for Michael Faraday)

= 1 Coulomb/Volt

Relationship between voltage and charge:

$$|\Delta V| = \frac{Q}{C}$$ ➔ general relationship for many geometries

$$\text{for parallel plate configuration: } C \equiv \frac{A\varepsilon_0}{d}.$$

$$\text{Example: } A = 0.02\ \text{m}^2, d = 0.003\ \text{m}$$

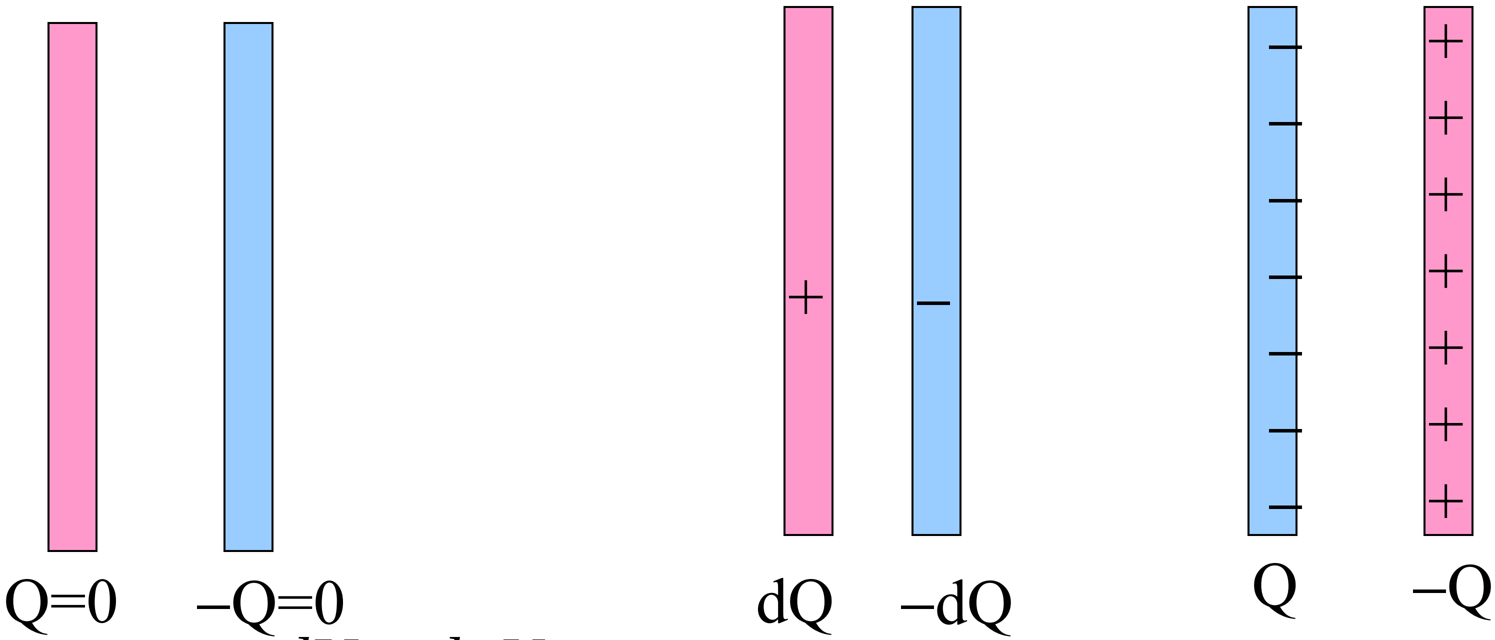
$$C \equiv \frac{A\varepsilon_0}{d} = \frac{0.02\ \text{m}^2 \cdot 8.85\times10^{-12}\ \text{F/m}}{0.003\ \text{m}} = 5.9\times10^{-11}\ \text{F}.$$

➔ For $|\Delta V| = 100$ V, $Q = C|\Delta V| = 5.9 \times 10^{-9}$ Coulombs

A capacitor can be used to store charge and energy.

Energy stored in a capacitor.

Electrical work without motion--



$$dU = dq\,V$$

$$U = \int_0^U dU = \int_0^Q dq\,V = \int_0^Q dq\,\frac{q}{C} \qquad \Rightarrow U(Q) = \frac{Q^2}{2C}$$

Summary:

Voltage difference in a capacitor: $|\Delta V| = \frac{Q}{C}$

Energy stored in a charged capacitor: $U(Q) = \frac{Q^2}{2C} = \frac{1}{2}C|\Delta V|^2$

Example: $A = 0.02\ \mathrm{m}^2,\ d = 0.003\ \mathrm{m}$

$$C \equiv \frac{A\varepsilon_0}{d} = \frac{0.02\ \mathrm{m}^2 \cdot 8.85\times10^{-12}\ \mathrm{F/m}}{0.003\ \mathrm{m}} = 5.9\times10^{-11}\ \mathrm{F}.$$

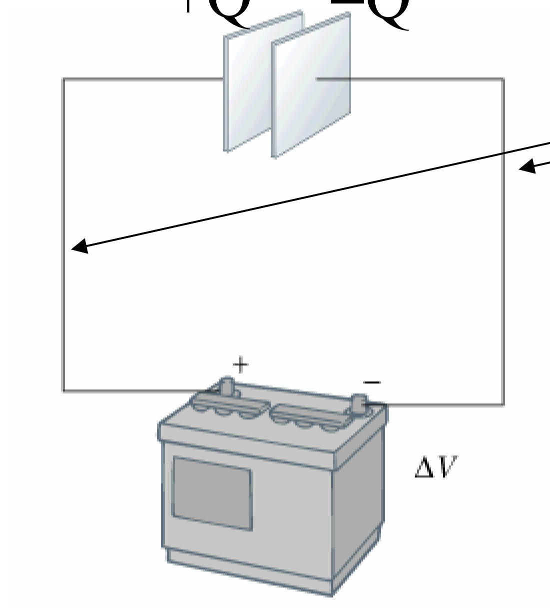
➔ For $|\Delta V| = 100$ V, $Q = C|\Delta V| = 5.9 \times 10^{-9}$ Coulombs

$$U(Q) = \frac{Q^2}{2C} = \frac{1}{2}C|\Delta V|^2 = \frac{1}{2}\cdot 5.9\times10^{-11}\ \mathrm{F}\cdot(100\ \mathrm{V})^2 = 2.95\times10^{-7}\ \mathrm{J}$$

Capacitors in a circuit:

$Q = \Delta V\, C$

+Q –Q



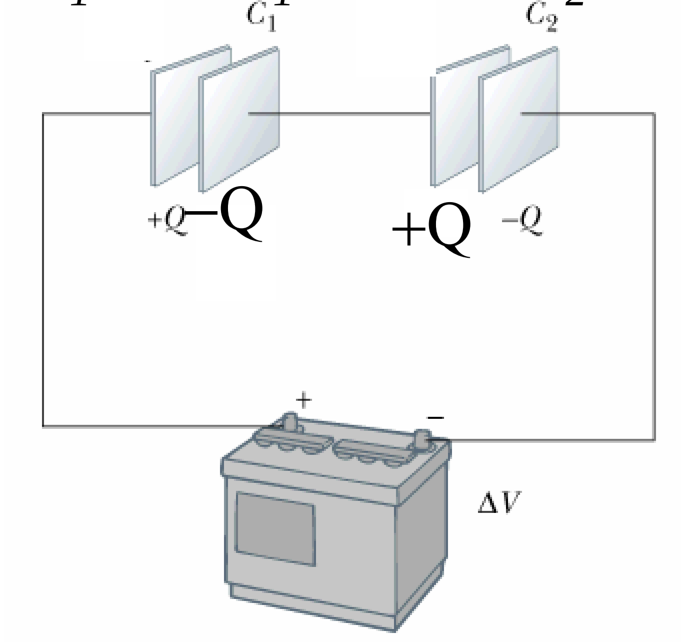
conductor

battery – source of charge (thanks to chemistry)

Two capacitors in a circuit –

Consider the following configuration:

$\Delta V_1 = Q/C_1$ $\Delta V_2 = Q/C_2$



$$\Delta V = \Delta V_1 + \Delta V_2$$

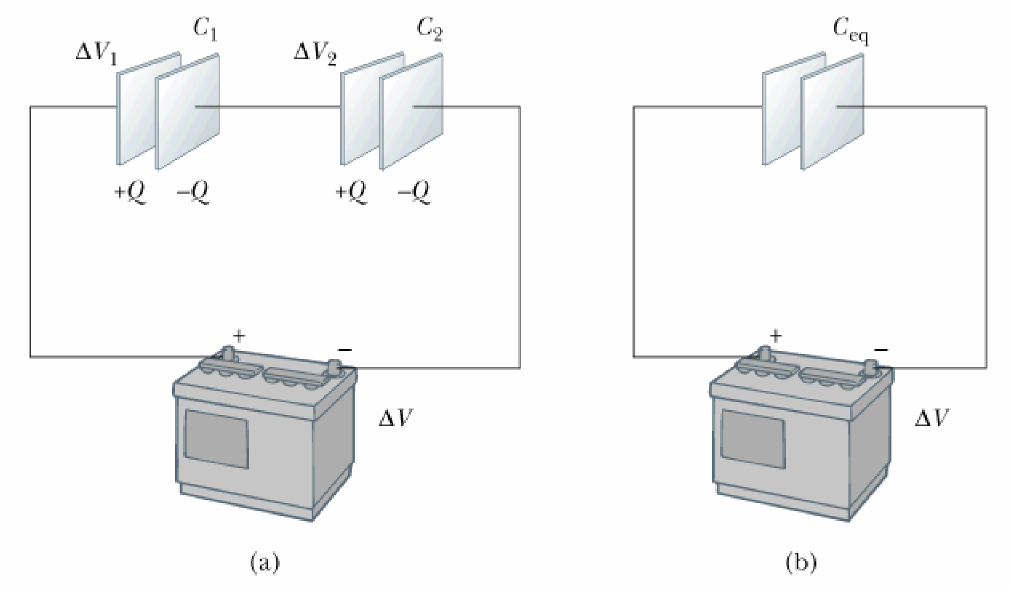
$$= \frac{Q}{C_1} + \frac{Q}{C_2} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

Capacitors connected in *series* are equivalent to C_{eq}:

$$\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i}$$

Peer instruction question

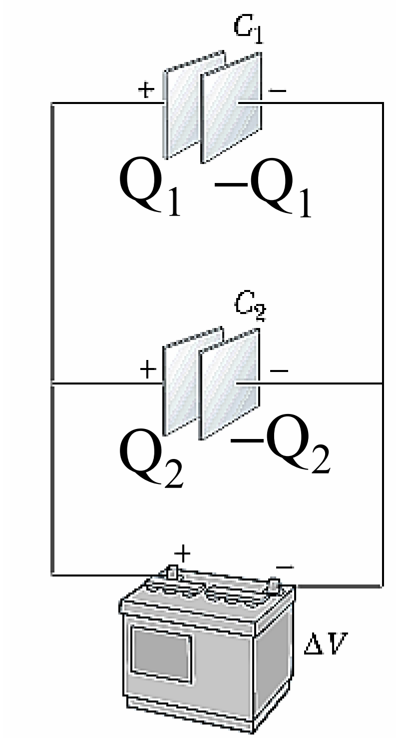
Consider a circuit with two capacitors $C_1 = C_2 = 2$ pF as shown. If these were replace by a single capacitance C_{eq}, what would be its value?



(A) 1 pF   (B) 2 pF   (C) 3 pF   (D) 4 pF

Two capacitors in a circuit –

Consider the following configuration:



$$\Delta V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$Q = Q_1 + Q_2 = C_1 \Delta V + C_2 \Delta V$$

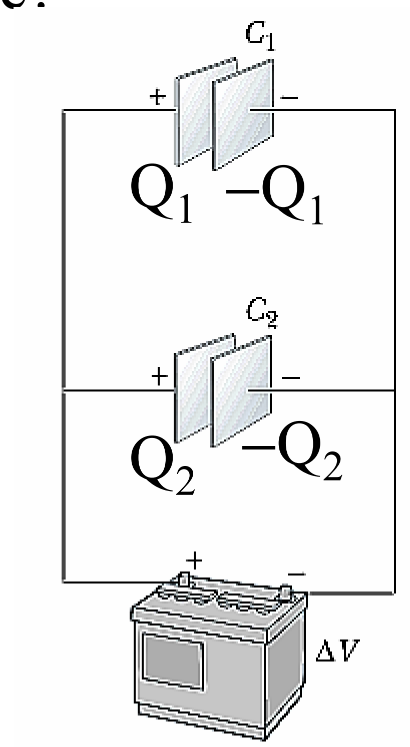
$$= (C_1 + C_2)\Delta V$$

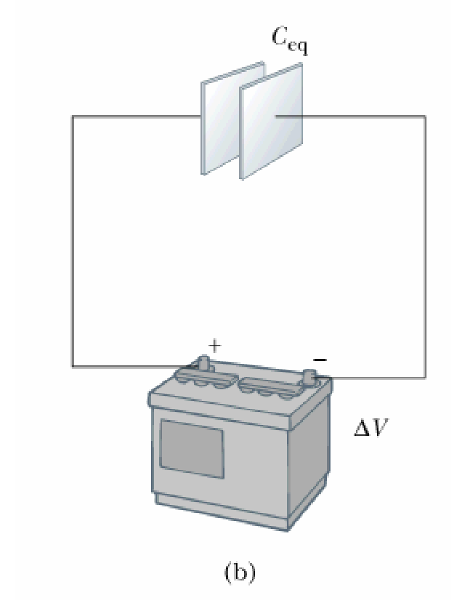
Capacitors connected in *parallel* are equivalent to C_{eq}:

$$C_{eq} = \sum_i C_i$$

Peer instruction question

Consider a circuit with two capacitors $C_1 = C_2 = 2$ pF as shown. If these were replace by a single capacitance C_{eq}, what would be its value?





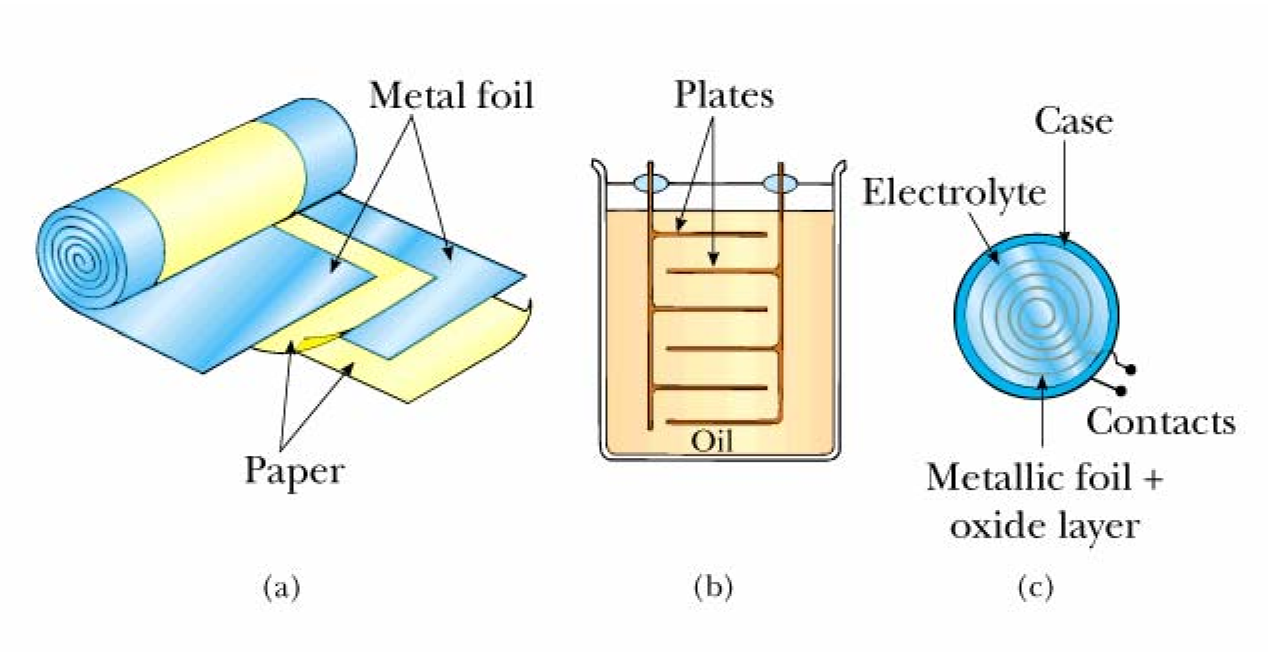
(A) 1 pF (B) 2 pF (C) 3 pF (D) 4 pF

Online Quiz for Lecture 5
Capacitance and dielectrics -- Jan. 21, 2005

1. Suppose that a parallel plate capacitor has a capacitance of $C=1x10^{-5}$ F and holds a voltage of 10 V. What is the charge on each of the plates?
(a)$1x10^{-6}$ C (b)$1x10^{-5}$ C (c)$1x10^{-4}$ C (d)$1x10^{-3}$ C
2. Two capacitors, each of capacitance C are connected in series. What is their equivalent capacitance?
(a) C/2 (b) C (c) 2C (d) 4C
3. Two capacitors, each of capacitance C are connected in parallel. What is their equivalent capacitance?
(a) C/2 (b) C (c) 2C (d) 4C

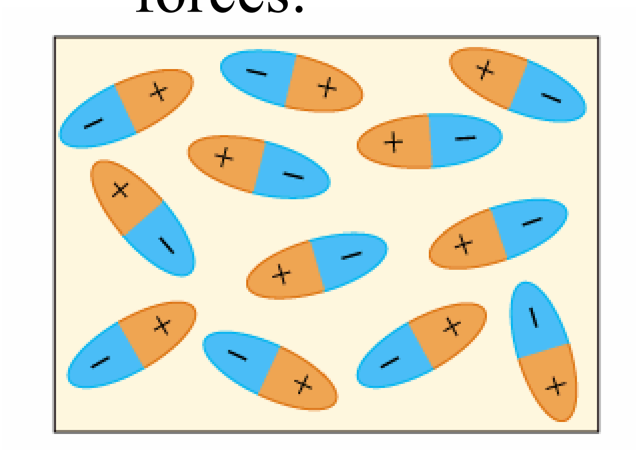
Practical capacitor design

- Compact geometries
- Dielectric media $\varepsilon_0 \rightarrow \varepsilon = \kappa\, \varepsilon_0$ $\qquad C_\kappa = \kappa\, C_0$

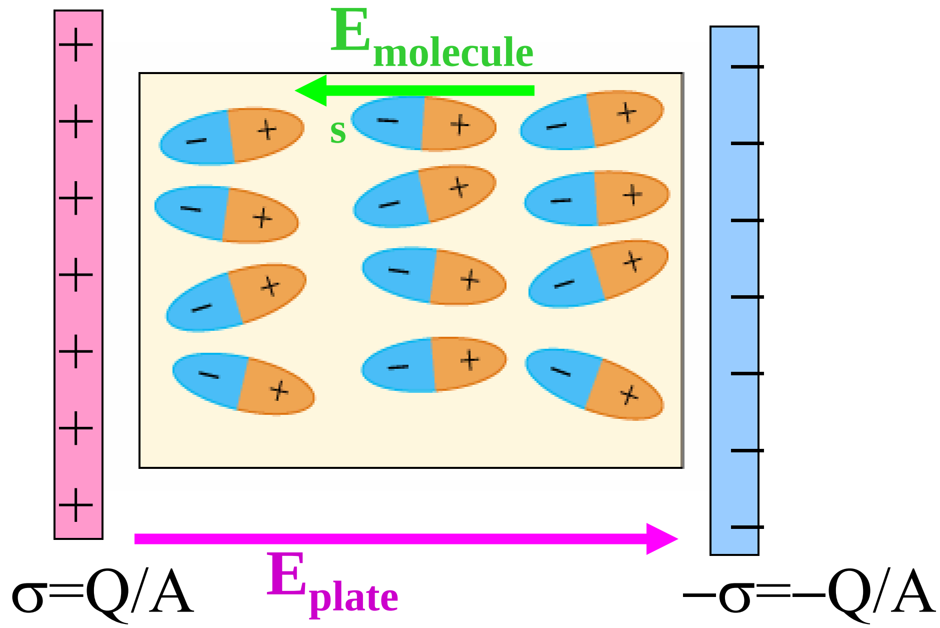


How dielectrics work:

Polar molecules in the absence of external forces:



Polar molecules in the aligned between two charged plates:



$$\mathbf{E}_{\mathbf{total}} = \mathbf{E}_{\mathbf{plate}} + \mathbf{E}_{\mathbf{molecules}} = \mathbf{E}_{\mathbf{plate}}/\kappa$$

$$V_{total} = V_{plate} + V_{molecules} = V_{plate}/\kappa$$

$$V_{total} = \frac{Qd}{A\kappa\varepsilon_0} \Rightarrow C(\kappa) = \frac{A\kappa\varepsilon_0}{d}$$

A few more details:

$\mathbf{E}_{\mathbf{molecules}} = -\chi\, \mathbf{E}_{\mathbf{plate}}$

$\Rightarrow \mathbf{E}_{\mathbf{total}} = (1 - \chi)\, \mathbf{E}_{\mathbf{plate}} = \mathbf{E}_{\mathbf{plate}} / \kappa$

$$\kappa = \frac{1}{1-\chi}$$

$$\text{For parallel plate capacitor:} \quad C = \kappa \frac{A\varepsilon_0}{d}$$

Some values of dielectric constants --

Material	κ
air	1
paper	3.7
water	80.0

1. [HRW6 26.P.003.] The capacitor in Fig. 26-24 has a capacitance of 21 μF and is initially uncharged. The battery provides a potential difference of 134 V. After switch S is closed, how much charge will pass through it?

[ ] C

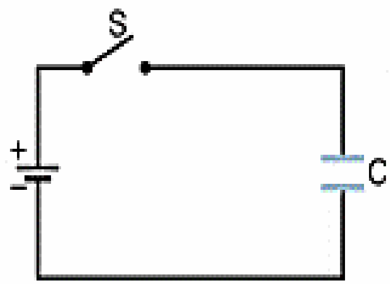


Figure 26-24.

4. [HRW6 26.P.030.] In Fig. 26-26, assume C_1 = 8 μF, C_2 = 3 μF, C_3 = 3 μF, and V = 50 V.

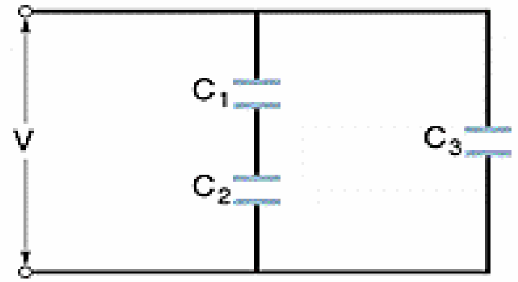


Figure 26-26.

Find,

(a) the charge on each capacitor,

[ ] μC (C_1)

[ ] μC (C_2)

[ ] μC (C_3)

(b) the potential difference on each capacitor,

[ ] V (C_1)

[ ] V (C_2)

[ ] V (C_3)

(c) the stored energy for each capacitor.

[ ] mJ (C_1)

[ ] mJ (C_2)

[ ] mJ (C_3)