Announcements

1. First hour exam – Friday, February 4, 2005 – covering Chapters 22-28. (Not including 28.8.)

   May bring 1 8½” x 11” sheet of paper to the exam (to be turned in with your exam papers).

   The exam will be proctored by Professor G. Holzwarth.

   Practice exam available on website.

2. Today’s topics –

   DC circuits including $\mathcal{E}$, R, and I (will consider the effects of C and Q next week)

   Kirchhoff’s analysis method

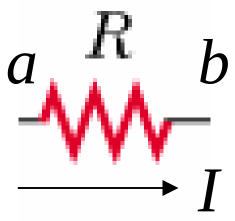
Analysis of DC circuits:

Elements:

wire

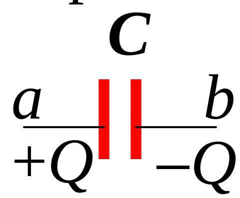
$V_{ab} \approx 0$

resistor



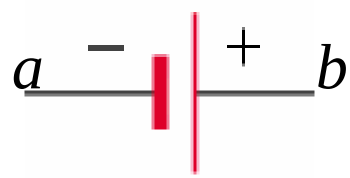
$V_{ab} = -IR$

capacitor



$V_{ab} = -Q/C$

emf

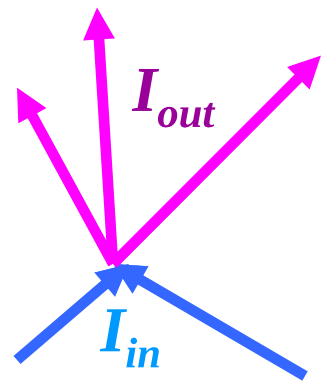


$\mathrm{V}_{\mathrm{ab}} = \mathcal{E}$

The principles:

Kirchhoff's rules

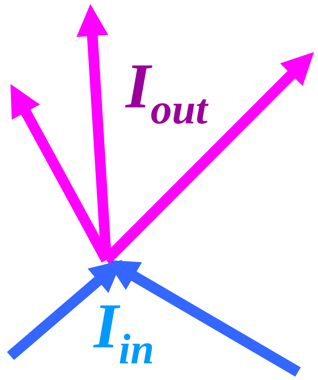
At any wire junction: $\sum I_{in} = \sum I_{out}$



For any closed wire loop: $\sum \Delta V = 0$

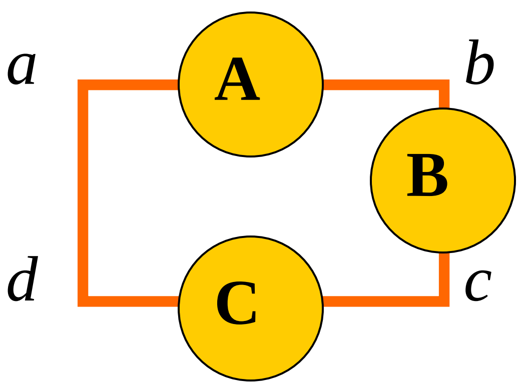
Why?

Junction rule: $\sum I_{in} = \sum I_{out}$



Consequence of conservation of charge; assumes no leakage, sparking, etc.

Loop rule: $\sum \Delta V = 0$

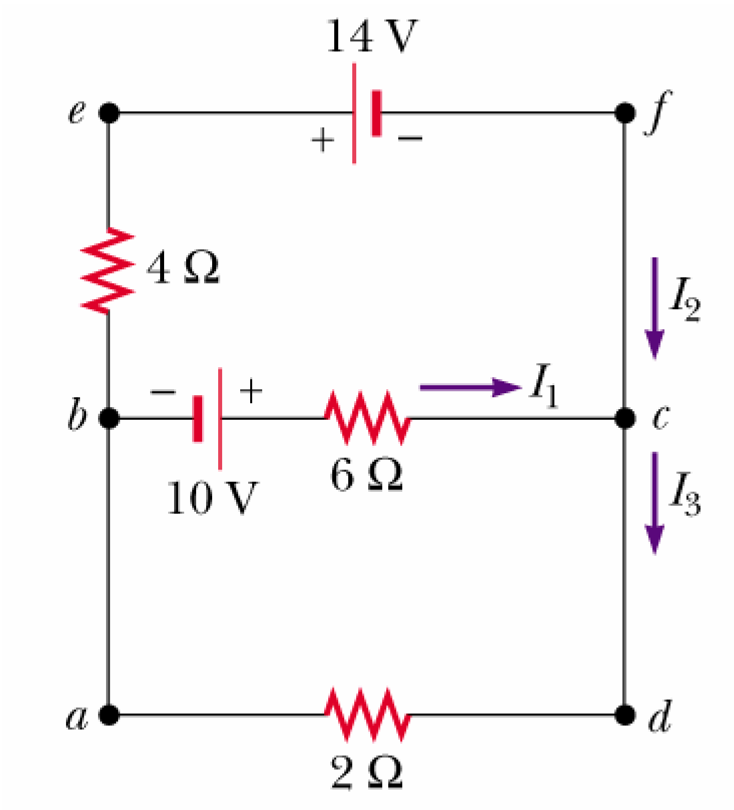


Consequence of electrostatic potential being derived from a conservative electric field

$$\sum \Delta V = (V_b - V_a) + (V_c - V_b) + (V_d - V_c) + (V_a - V_d) = 0$$

Peer instruction question

Which of the loops of the following circuit follow the "loop rule"?



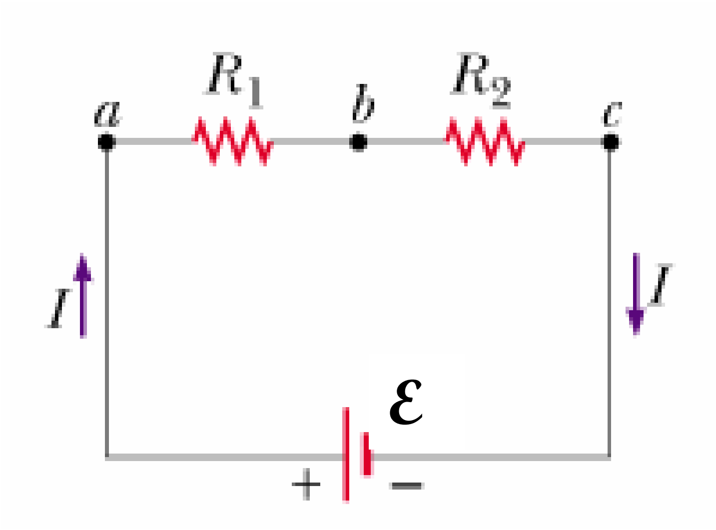
(A) abcda

(B) befcb

(C) abefcda

(D) All of these.

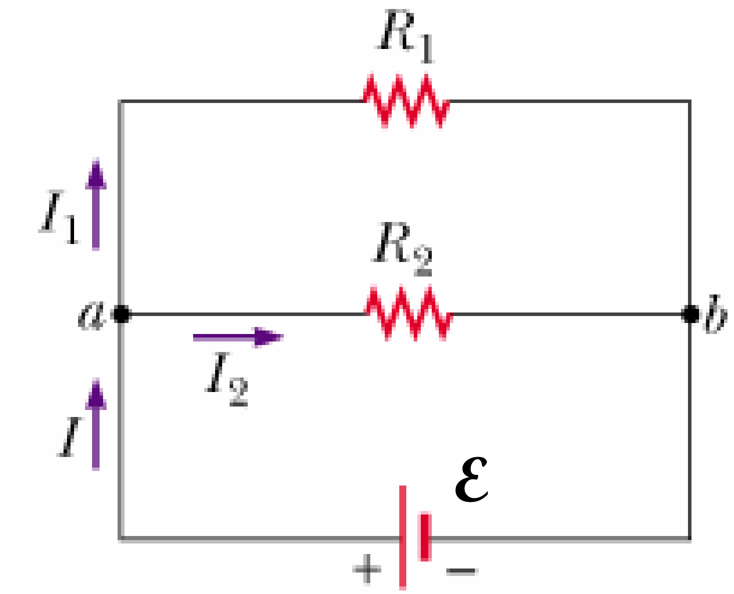
Example: resistors in series:



$$R_{eq} = \sum_i R_i$$

$$-IR_1 - IR_2 + \mathcal{E} = 0 \;\Rightarrow\; \mathcal{E} = I\,\underbrace{(R_1 + R_2)}_{R_{eq}}$$

Example: resistors in parallel



$$\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$$

$$\text{upper loop}: \; -I_1R_1 + I_2R_2 = 0$$

$$\text{lower loop}: \; -I_2R_2 + \mathcal{E} = 0$$

$$\text{junction}: \; I = I_1 + I_2 \quad \Rightarrow I = \mathcal{E}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{\mathcal{E}}{R_{eq}}$$

Shortcuts – using equivalent circuits

$$R_{eq} = \frac{1}{\frac{1}{7\Omega} + \frac{1}{R}}$$

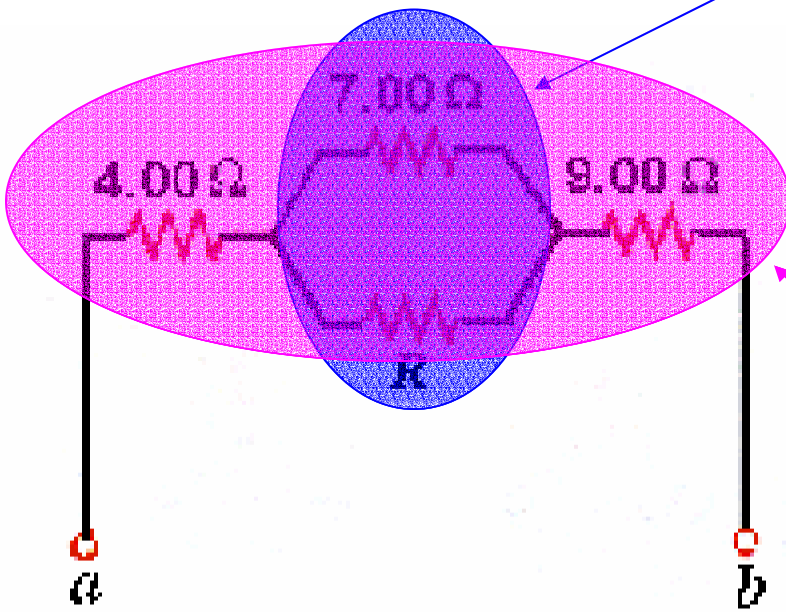
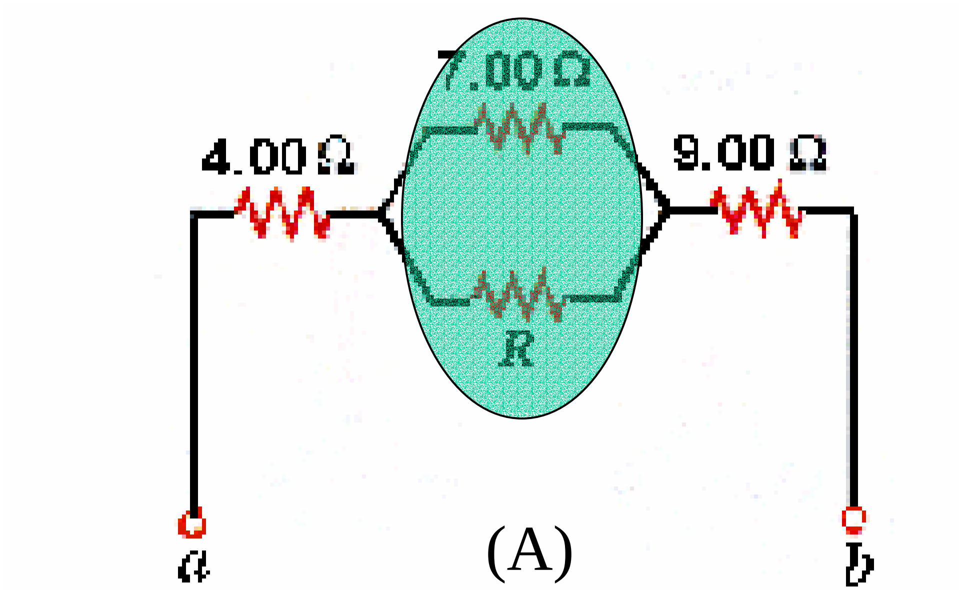


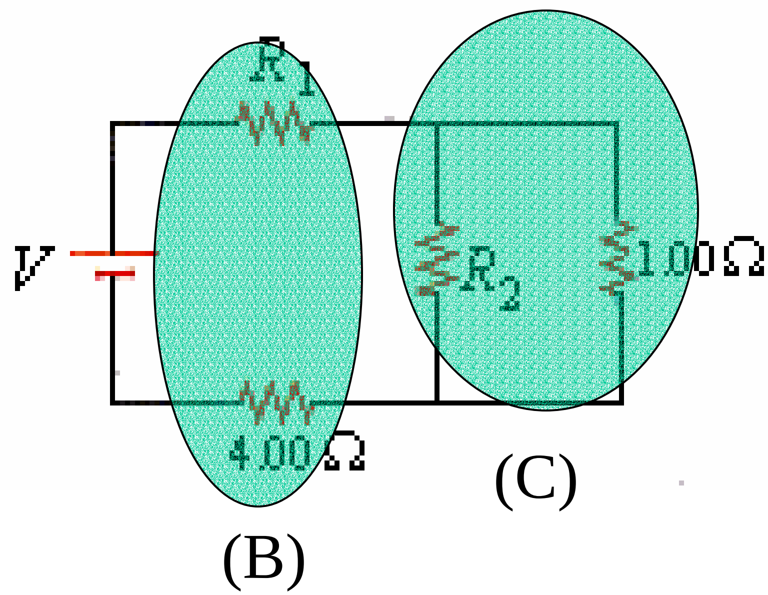
Figure P28.6.

$$R'_{eq} = 4\Omega + R_{eq} + 9\Omega$$

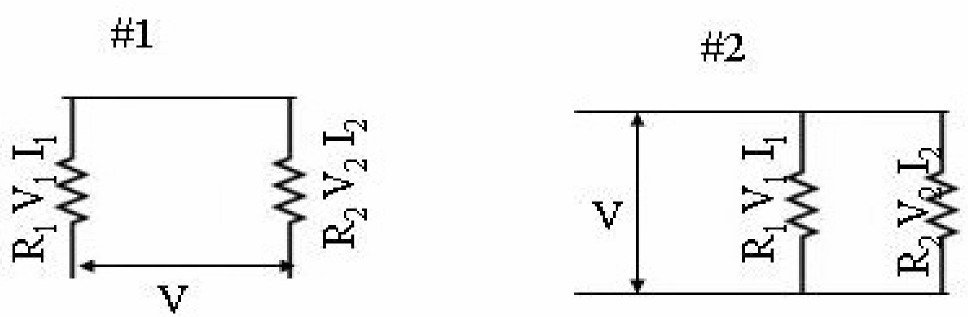
Peer instruction question:

Which of these are NOT in parallel?





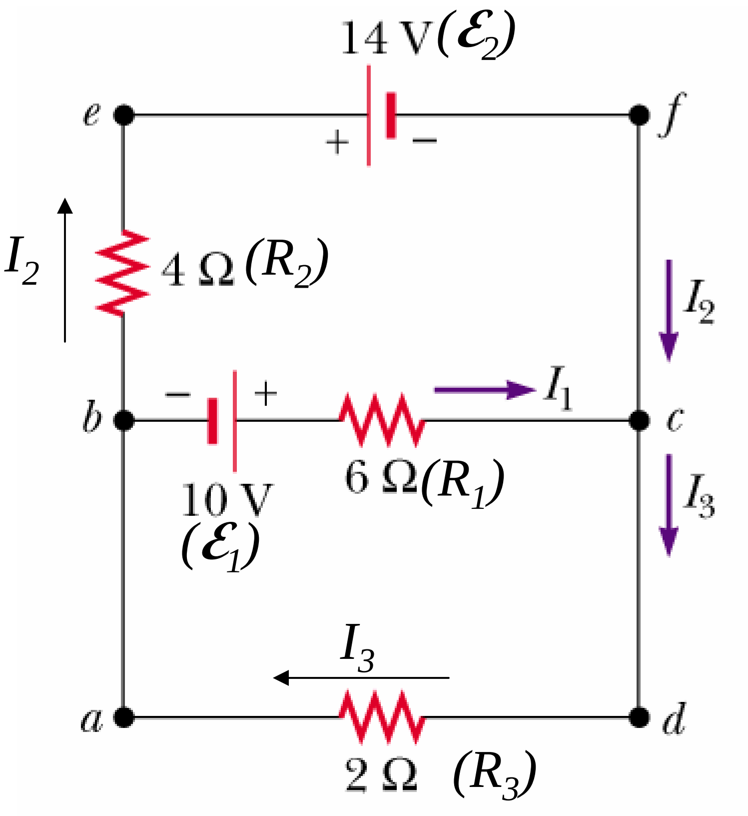
Online Quiz for Lecture 8
Resistance curcuits -- Jan. 31, 2005



Consider the two circuits shown above connected to a V=20V voltage source with resistors R_1 and R_2. If $R_1 > R_2$, which of the following statements are true in the two cases?

A. $V_1 > V_2$ → **#1**
B. $V_1 < V_2$
C. $I_1 > I_2$
D. $I_1 < I_2$ → **#2**

Example of circuit analysis:



$$I_3 = I_1 + I_2$$

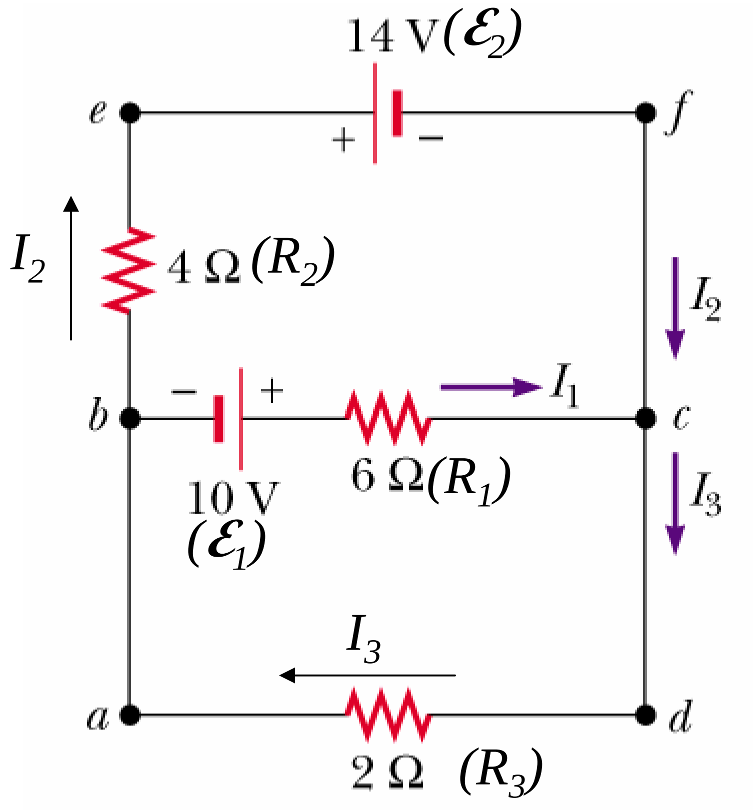
$$\mathcal{E}_1 - R_1 I_1 - R_3 I_3 = 0$$

$$-R_2 I_2 - \mathcal{E}_2 - R_3 I_3 = 0$$

➔**3 unknowns (*I_1* , *I_2* , *I_3*)**

3 equations

➔**unique solution**



$$I_3 = I_1 + I_2$$

$$\mathcal{E}_1 - R_1 I_1 - R_3 I_3 = 0$$

$$-R_2 I_2 - \mathcal{E}_2 - R_3 I_3 = 0$$

Linear Equation method :

$$I_1 + I_2 - I_3 = 0$$

$$R_1 I_1 + R_3 I_3 = \mathcal{E}_1$$

$$R_2 I_2 + R_3 I_3 = -\mathcal{E}_2$$

May be solved by

- **Subsitution method**
- **Elimination method**
- **Maple**
- **Graphics calculator**

Linear Equation method :

1 $I_1 + I_2 - I_3 = 0$

2 $R_1 I_1 + R_3 I_3 = \mathcal{E}_1$

3 $R_2 I_2 + R_3 I_3 = -\mathcal{E}_2$

➔

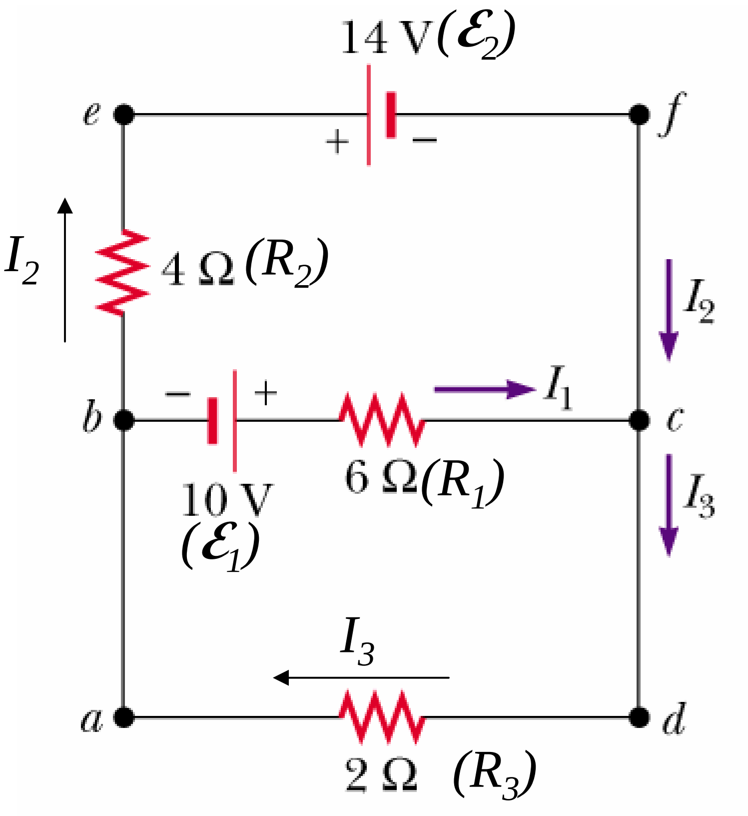
1 $I_3 = I_1 + I_2$

$(R_1 + R_3) I_1 + R_3 I_2 = \mathcal{E}_1$

$R_3 I_1 + (R_2 + R_3) I_2 = -\mathcal{E}_2$

2 $$I_2 = \frac{\mathcal{E}_1}{R_3} - I_1 \frac{R_1 + R_3}{R_3}$$

3 $$\left(R_3 - (R_2 + R_3) \frac{R_1 + R_3}{R_3} \right) I_1 = -\mathcal{E}_2 - (R_2 + R_3) \frac{\mathcal{E}_1}{R_3}$$



$$I_3 = I_1 + I_2$$

$$I_2 = \frac{\mathcal{E}_1}{R_3} - I_1 \frac{R_1 + R_3}{R_3}$$

$$\left(R_3 - (R_2 + R_3)\frac{R_1 + R_3}{R_3} \right) I_1 = -\mathcal{E}_2 - (R_2 + R_3)\frac{\mathcal{E}_1}{R_3}$$

R_1 = 6Ω, R_2=4Ω, R_3=2Ω

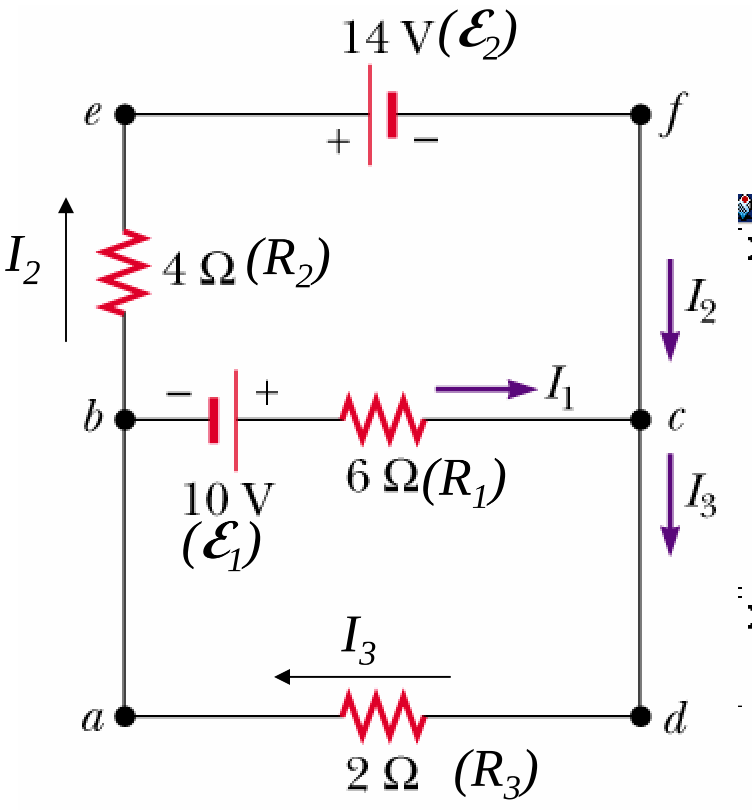
$\mathcal{E}_1$=10V, $\mathcal{E}_2$=14V

Solving:

I_1=2 A

I_2=-3 A

I_3=-1 A



Asking Maple to do the algebra:

lecture8.mws - [Server 1]

```
> solve({I1+I2=I3,R1*I1+R3*I3=E1,R2*I2+R3*I3=-E2},{I1,I2,I3});
```

$$\left\{ I3 = -\frac{R1\,E2 - R2\,E1}{R2\,R1 + R2\,R3 + R1\,R3},\ I2 = -\frac{R3\,E1 + R1\,E2 + E2\,R3}{R2\,R1 + R2\,R3 + R1\,R3},\ I1 = \frac{R3\,E1 + E2\,R3 + R2\,E1}{R2\,R1 + R2\,R3 + R1\,R3} \right\}$$

```
> solve({I1+I2=I3,6*I1+2*I3=10,4*I2+2*I3=-14},{I1,I2,I3});
```

$$\{ I2 = -3,\ I1 = 2,\ I3 = -1 \}$$

Summary of Kirchhoff's analysis:

➤**LOOP RULE:** The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.

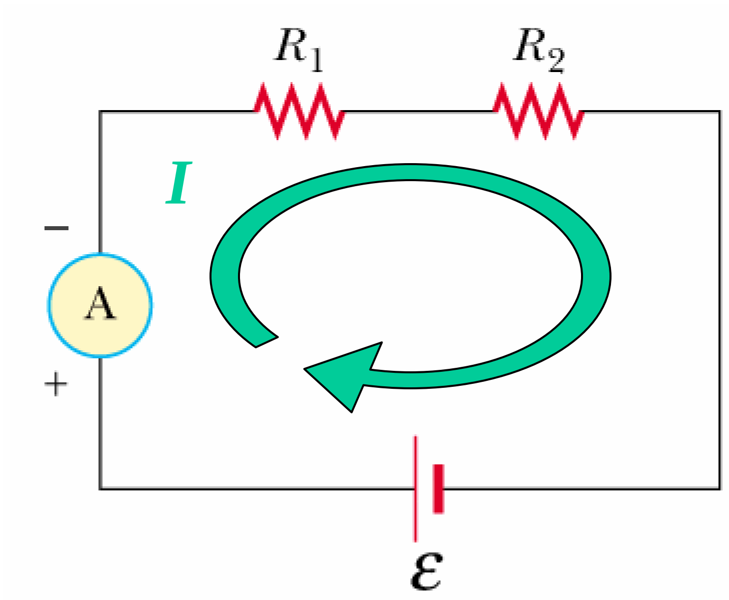
➤**JUNCTION RULE:** The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction.

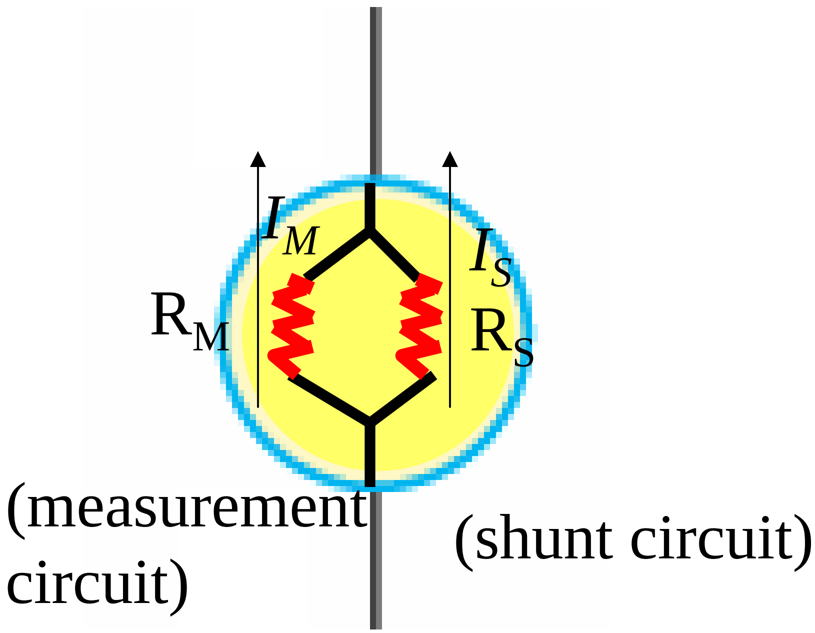
Sign conventions:

➤**RESISTANCE RULE:** For a move through a resistance in the direction of the current, the change in potential is $-iR$; in the opposite direction it is $+iR$.
EMF RULE: For a move through an ideal emf device in the direction of the emf arrow, the change in potential is +; in the opposite direction it is -.

Practical circuits:

Ammeter





I_M **should be small (sensitive device)**

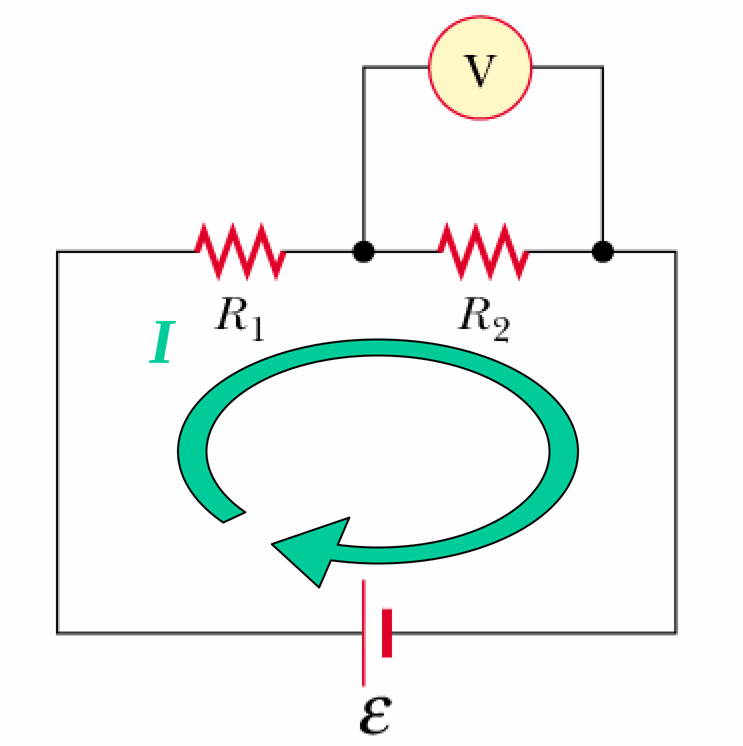
$R_S \ll R_1 , R_2$

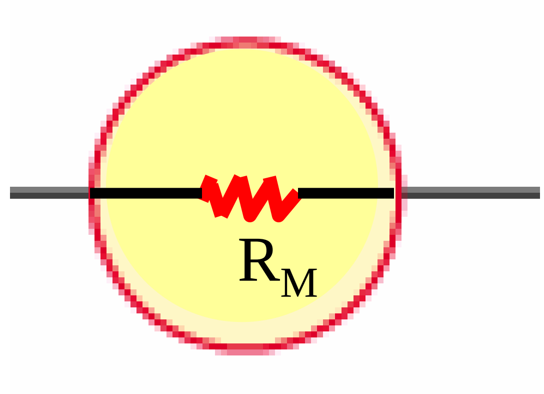
$R_M \gg R_S$

$I = I_M + I_S$

Practical circuits:

Voltmeter





$R_M >> R_1$, R_2

I_M should be small (sensitive device)

$I = I_M + I_2$

PROBLEM 31

In Fig. 28-33, $\mathcal{E}_1 = 3.00$ V, $\mathcal{E}_2 = 1.00$ V, $R_1 = 5.00$ W, $R_2 = 2.00$ W, $R_3 = 4.00$ W, and both batteries are ideal. What is the rate at which energy is dissipated in (a) R_1, (b) R_2, and (c) R_3? What is the power of (d) battery 1 and (e) battery 2?

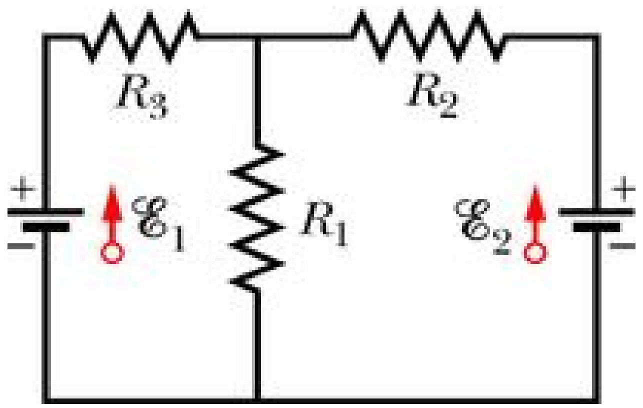
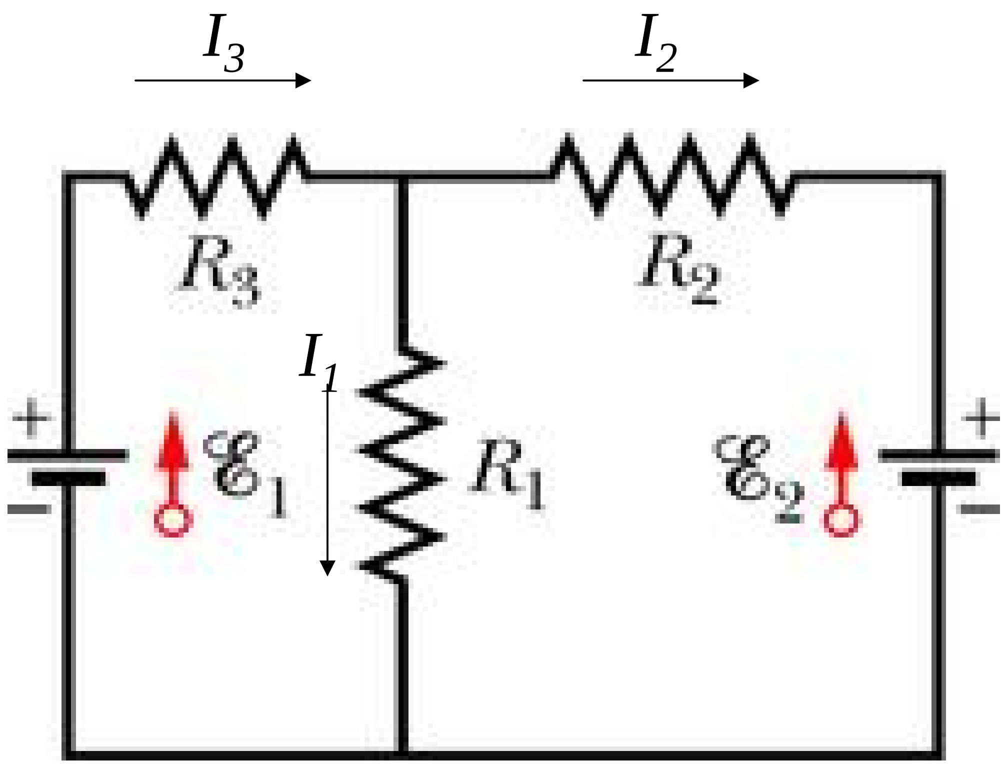


Fig. 28-33 Problem 31.



I_3
I_2
R_3
R_2
I_1
R_1
+
−
$\mathscr{E}_1$
$\mathscr{E}_2$
+
−