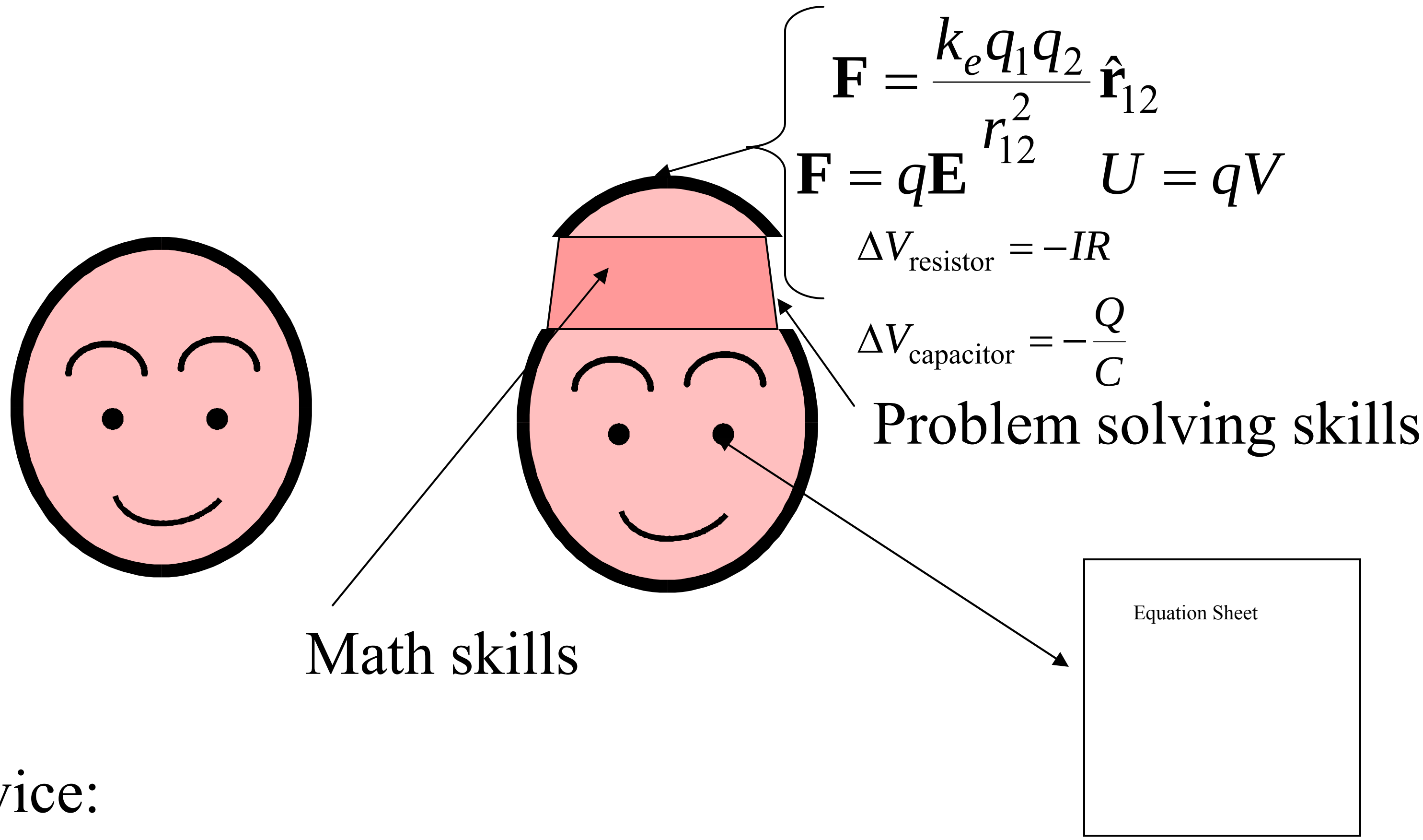
Announcements

1. **First hour exam – Friday, February 4, 2005 – covering Chapters 22-28.**
   - 5 problems – show your work and reasoning for possible partial credit.
   - May bring 1 8½" x 11" sheet of paper to the exam (to be turned in with your exam papers).
   - The exam will be proctored by Professor G. Holzwarth.
2. Practice problems available on website.
3. Today's lecture –
   Advice for studying Systematic review



Advice:

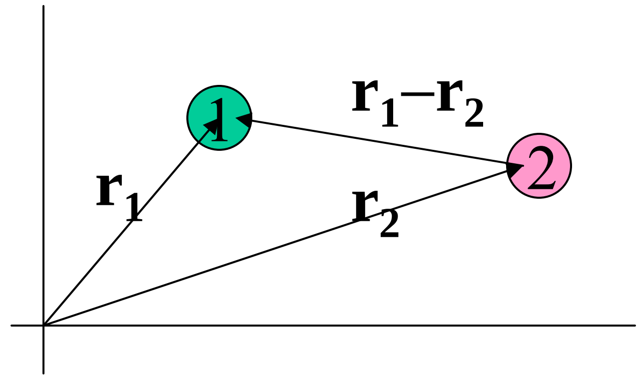
1. Keep basic concepts and equations at the top of your head.
2. Practice problem solving and math skills
3. Develop an equation sheet that you can consult.

Problem solving steps

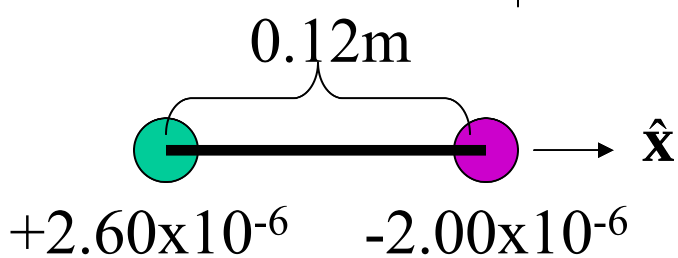
1. Visualize problem – labeling variables
2. Determine which basic physical principle(s) apply
3. Write down the appropriate equations using the variables defined in step 1.
4. Check whether you have the correct amount of information to solve the problem (same number of knowns and unknowns).
5. Solve the equations.
6. Check whether your answer makes sense (units, order of magnitude, etc.).

Coulomb's law describes the electrical force between two charged particles:

$$\mathbf{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{|\mathbf{r}_1 - \mathbf{r}_2|^2} \hat{\mathbf{r}}_{12} = -\mathbf{F}_{21}$$

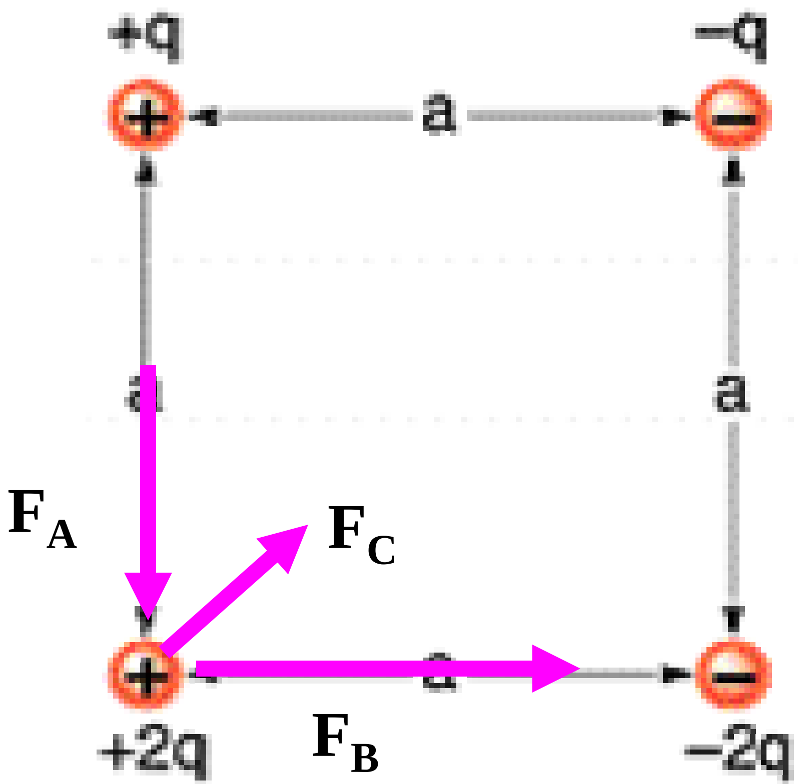


Example:



$$\mathbf{r}_1 - \mathbf{r}_2 = 0.12m(-\hat{\mathbf{x}})$$

$$\mathbf{F}_{12} = 8.99 \times 10^9 \frac{Nm^2}{C^2} \frac{(2.6 \times 10^{-6} C)(-2.0 \times 10^{-6} C)}{(0.12m)^2} (-\hat{\mathbf{x}}) = 3.25 N\hat{\mathbf{x}}$$

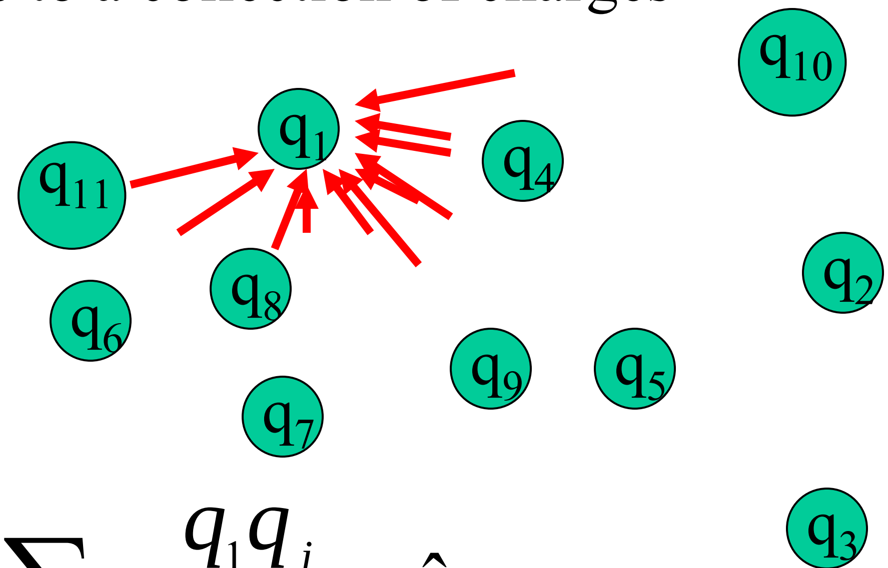


Net force is **vector** sum.

$$\mathbf{F}_A = \frac{1}{4\pi\varepsilon_0}\frac{2q^2}{a^2}\left(-\hat{\mathbf{y}}\right) \quad \mathbf{F}_B = \frac{1}{4\pi\varepsilon_0}\frac{4q^2}{a^2}\left(\hat{\mathbf{x}}\right) \quad \mathbf{F}_C = \frac{1}{4\pi\varepsilon_0}\frac{2q^2}{2a^2}\left(\cos 45^o\,\hat{\mathbf{x}} + \sin 45^o\,\hat{\mathbf{y}}\right)$$

$$\mathbf{F} = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}\left\{\left(4+\cos 45^o\right)\hat{\mathbf{x}} + \left(-2+\sin 45^o\right)\hat{\mathbf{y}}\right\}$$

Forces due to a collection of charges

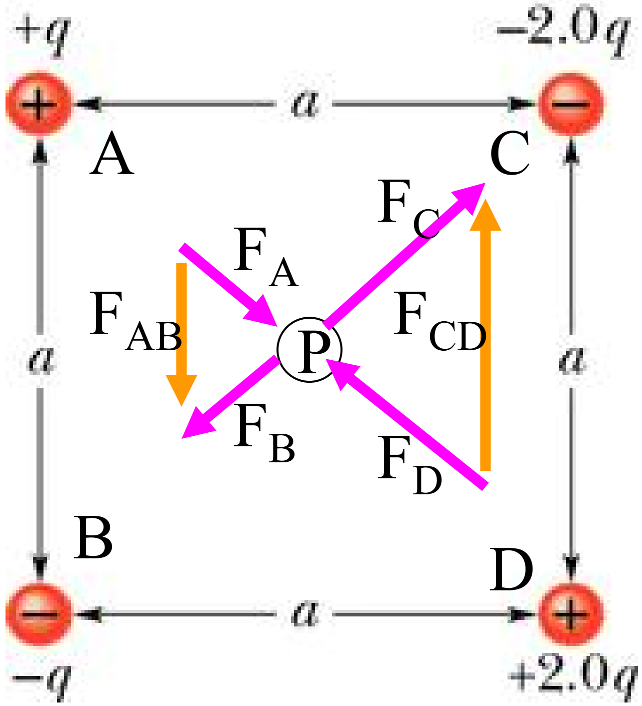


$$\mathbf{F}_1 = \frac{1}{4\pi\varepsilon_0}\sum_j \frac{q_1 q_j}{|\mathbf{r}_1 - \mathbf{r}_j|^2}\hat{\mathbf{r}}_{1j}$$

Electric field at $\mathbf{r}_1$:

$$\mathbf{E}(\mathbf{r}_1) = \frac{\mathbf{F}_1}{q_1} = \frac{1}{4\pi\varepsilon_0}\sum_j \frac{q_j}{|\mathbf{r}_1 - \mathbf{r}_j|^2}\hat{\mathbf{r}}_{1j}$$

Example: Find the electrostatic field at the point P:



In this example, you can see graphically that the horizontal components cancel and the net field is vertical.

$$\mathbf{E}(P) = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(a^2/2\right)}\left\{\sin 45^o(-1-1+2+2)\hat{\mathbf{y}}\right\}$$

Electrostatic potential energy

$$U(\mathbf{r}) = -\int_{\mathbf{r}_{ref}}^{\mathbf{r}} \mathbf{F} \cdot d\mathbf{r}$$

Electrostatic potential

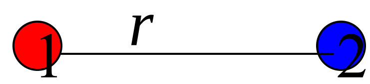
$$V(\mathbf{r}) = -\int_{\mathbf{r}_{ref}}^{\mathbf{r}} E \cdot d\mathbf{r}$$

Volt=J/C

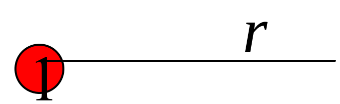
N/C

For a point charge, a convenient choice for potential reference is $r_{ref}=\infty$.

For point charge:



$$U(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$



$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}$$

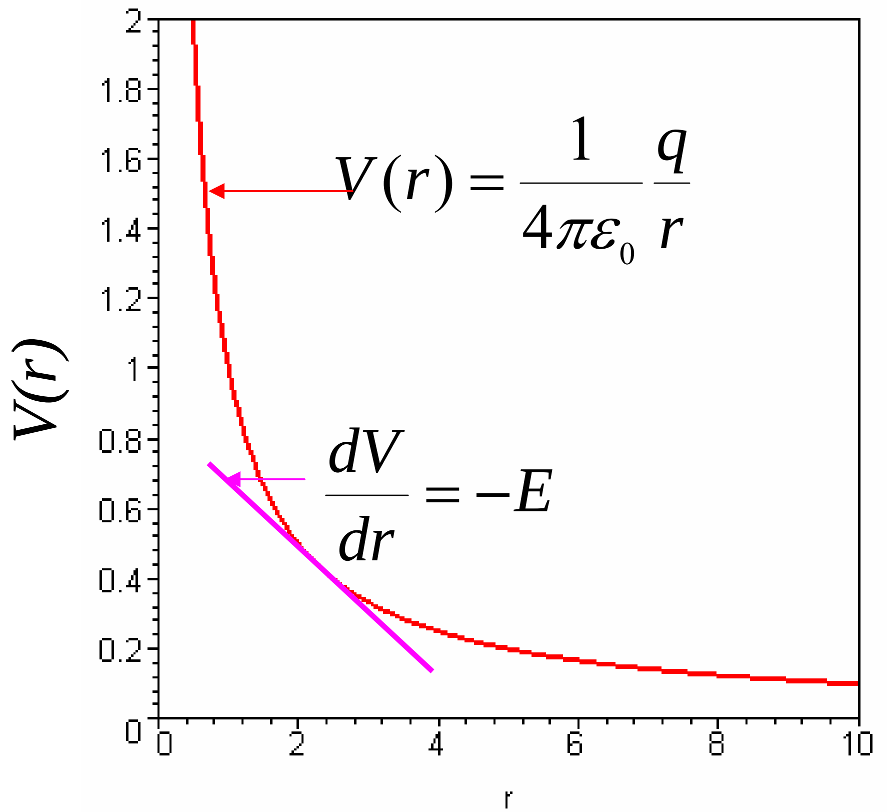
(Note: $U=V/q_2$)

Some details:

$$U(r) = -\frac{1}{4\pi\varepsilon_0}\int_{\infty}^{r}\frac{q_1 q_2}{r'^2}dr' = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}\bigg|_{\infty}^{r} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}\bigg|$$

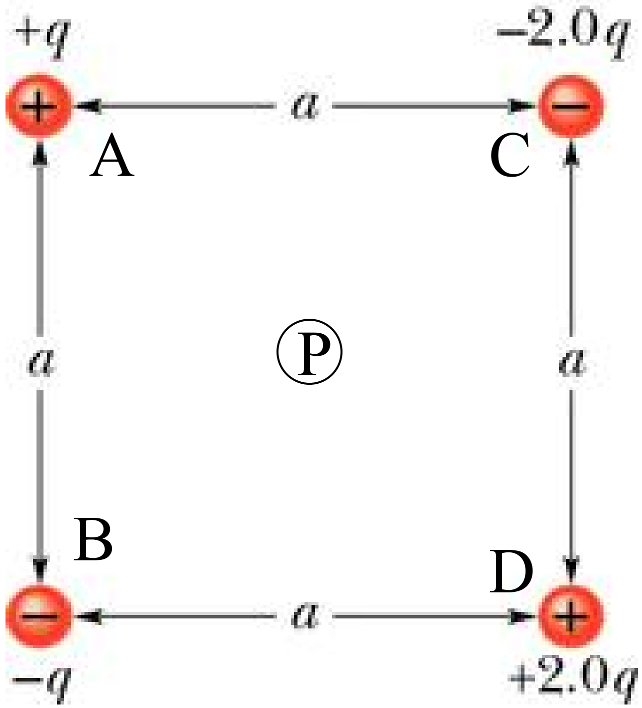
$$V(r) = -\frac{1}{4\pi\varepsilon_0}\int_{\infty}^{r}\frac{q}{r'^2}dr' = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}\bigg|_{\infty}^{r} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}\bigg|$$

Electrostatic potential of a point charge continued --



$$\mathbf{E}(r) = -\nabla V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$$

Example: Find the electrostatic potential at the point P:



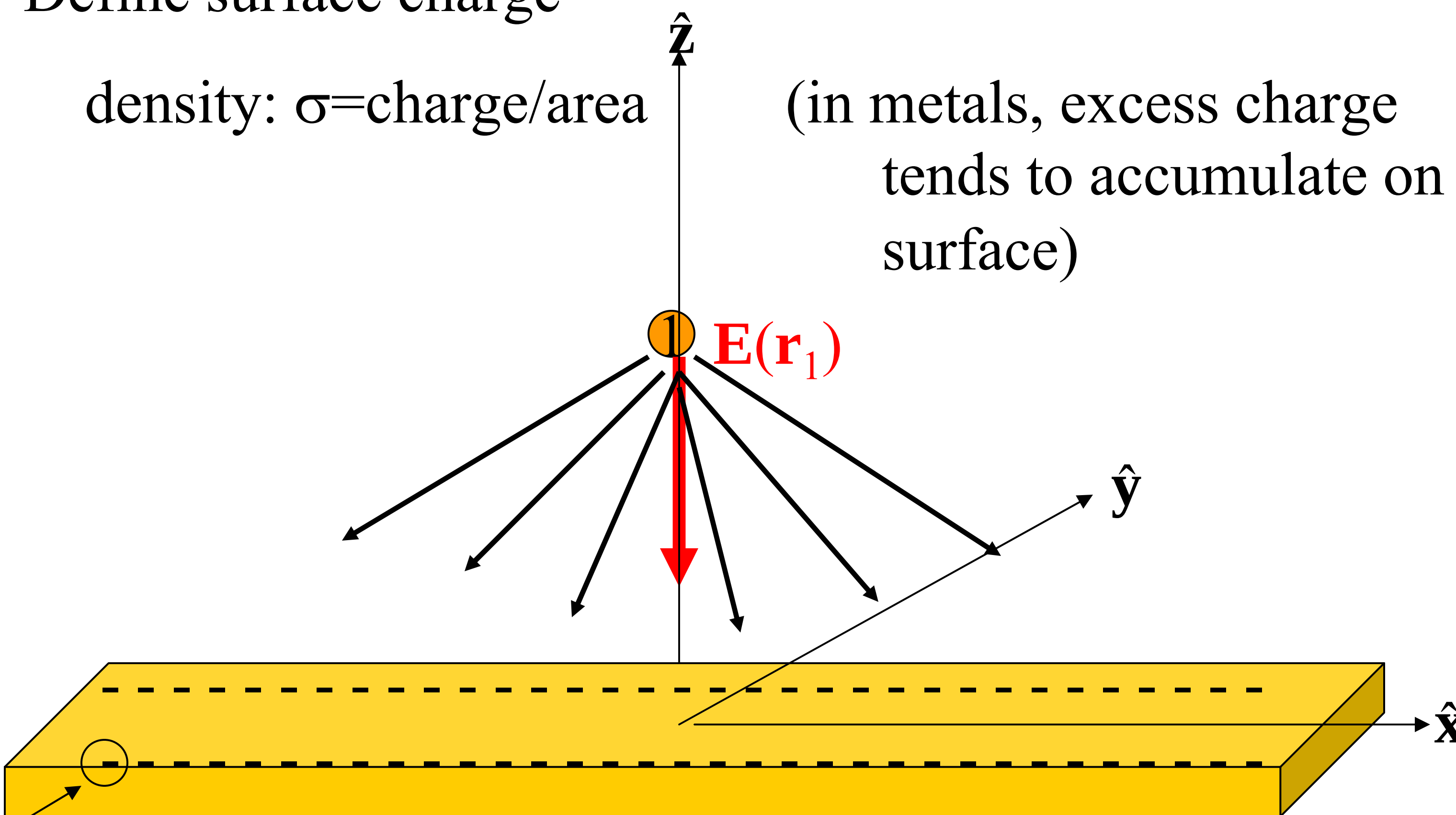
The potential is a scalar quantity :

$$V(P) = \frac{1}{4\pi\varepsilon_0} \frac{q}{\left(a/\sqrt{2}\right)} \{1 - 1 - 2 + 2\} = 0$$

Calculation of the net electric field for a distribution of surface charge:

Define surface charge density: σ=charge/area

(in metals, excess charge tends to accumulate on the surface)

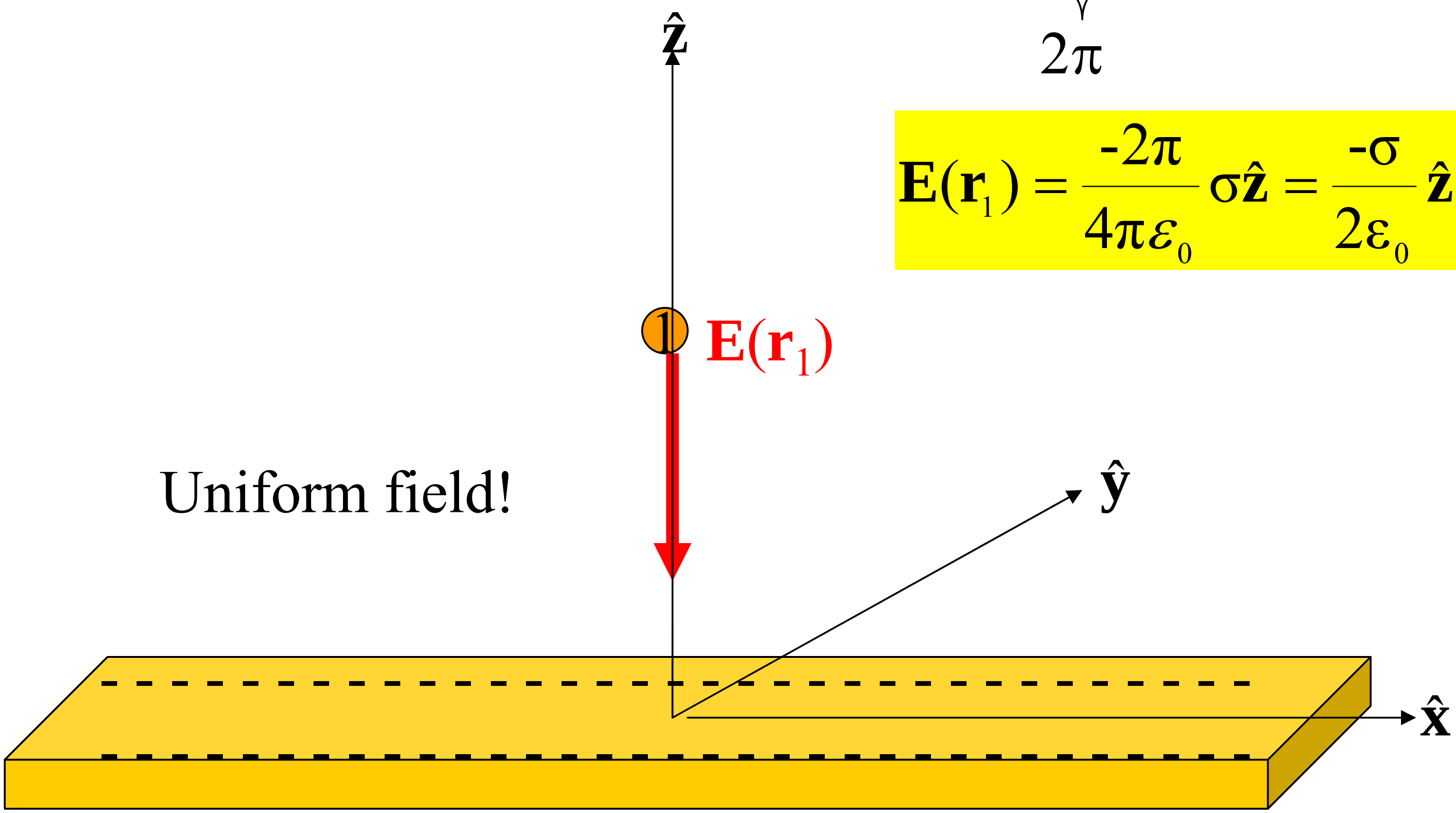


$q_i = \sigma dx_i dy_i$

Some details:

$$\mathbf{E}(\mathbf{r}_1) = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{|\mathbf{r}_1 - \mathbf{r}_i|^2}\hat{\mathbf{r}}_{1i} = \frac{\sigma}{4\pi\varepsilon_0}(-\hat{\mathbf{z}})\underbrace{\int_{-\infty}^{\infty}\frac{z\,dx_i\,dy_i}{\left(z^2+x_i^2+y_i^2\right)^{3/2}}}_{2\pi}$$

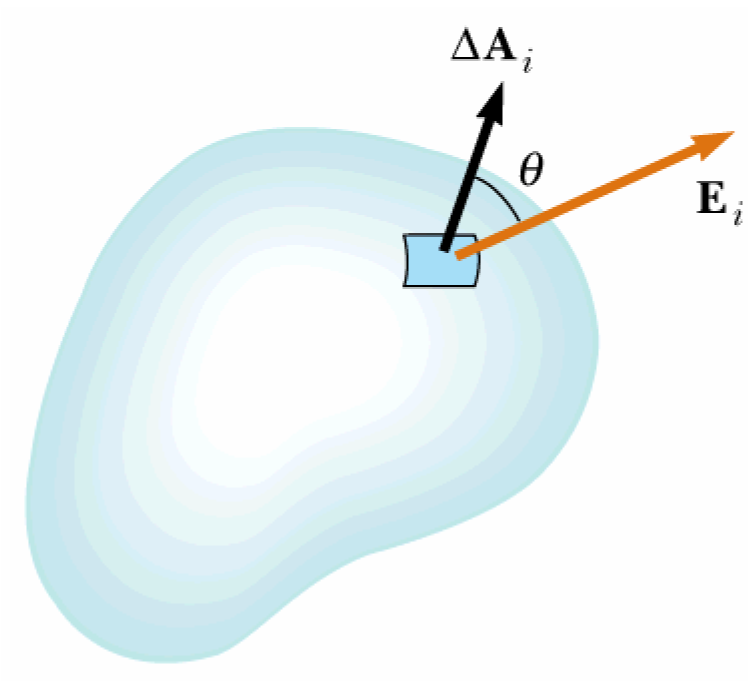
$$\boxed{\mathbf{E}(\mathbf{r}_1) = \frac{-2\pi}{4\pi\varepsilon_0}\sigma\hat{\mathbf{z}} = \frac{-\sigma}{2\varepsilon_0}\hat{\mathbf{z}}}$$



Uniform field!

An alternative method for calculating electric fields –
Gauss's Law

Define electric "flux":



$$\Phi_E \equiv \int \mathbf{E} \cdot d\mathbf{A}$$
$$= \int E\,dA\cos\theta$$

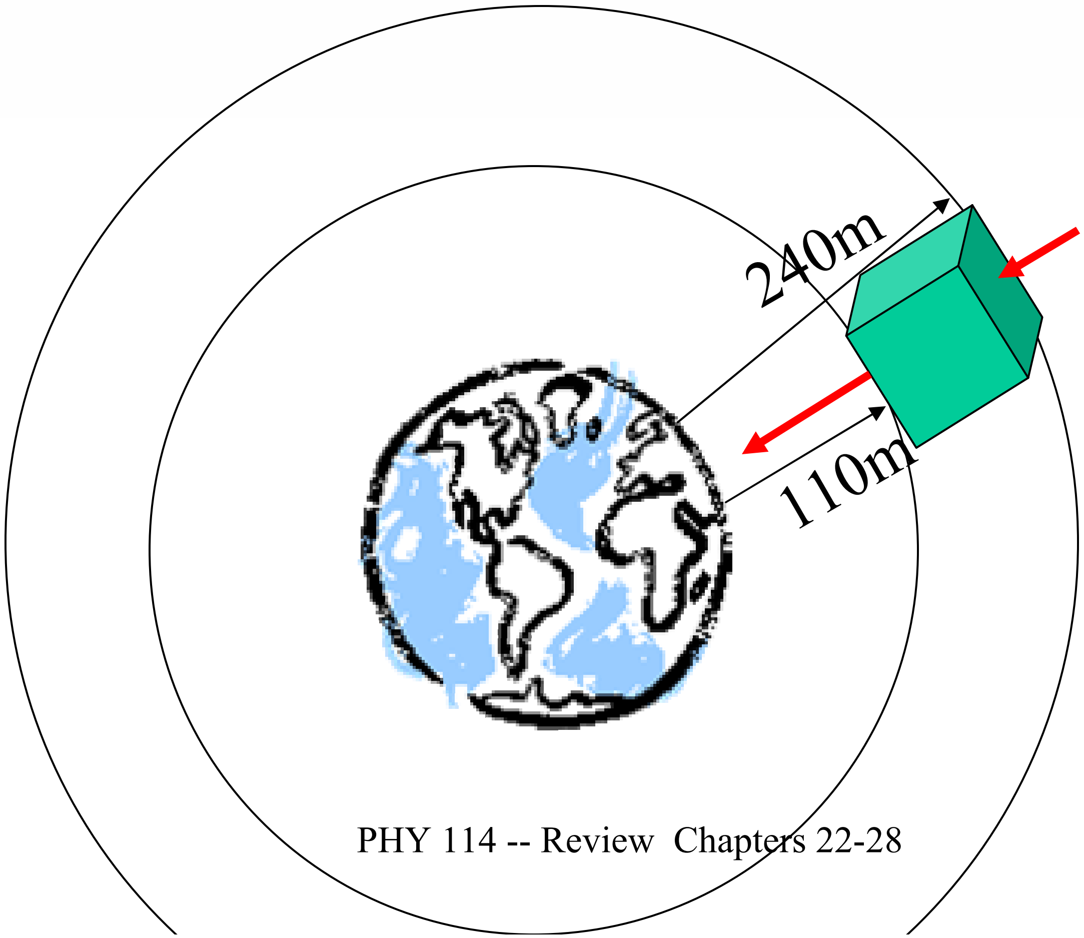
Gauss's law says:

$$\oint \mathbf{E} \cdot d\mathbf{A} = 4\pi k_e q_{in} = \frac{q_{in}}{\varepsilon_0}$$

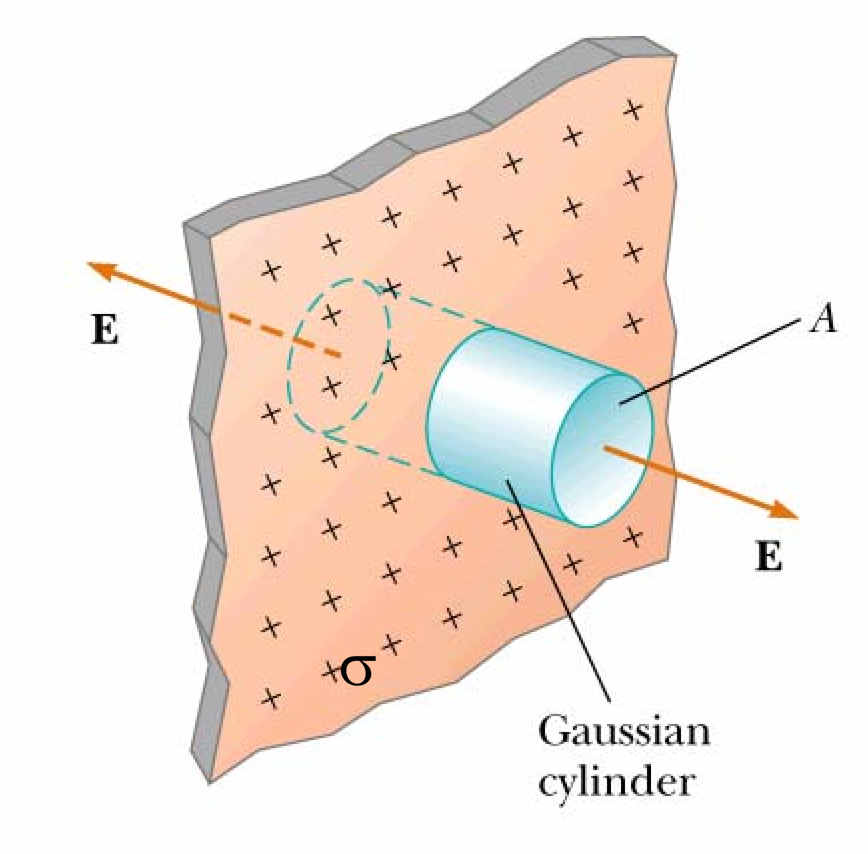
Integral of surrounding surface

2. [HRW6 24.P.009.] It is found experimentally that the electric field in a certain region of Earth's atmosphere is directed vertically down. At an altitude of 240 m the field has magnitude 60.0 N/C; at an altitude of 110 m, 100 N/C. Find the net amount of charge contained in a cube 130 m on edge, with horizontal faces at altitudes of 110 and 240 m. Neglect the curvature of Earth.

[ ] C



Electrostatic field from charged sheet



2 ends of cylinder

$$2EA = \frac{1}{\varepsilon_0}(\text{charge inside})$$

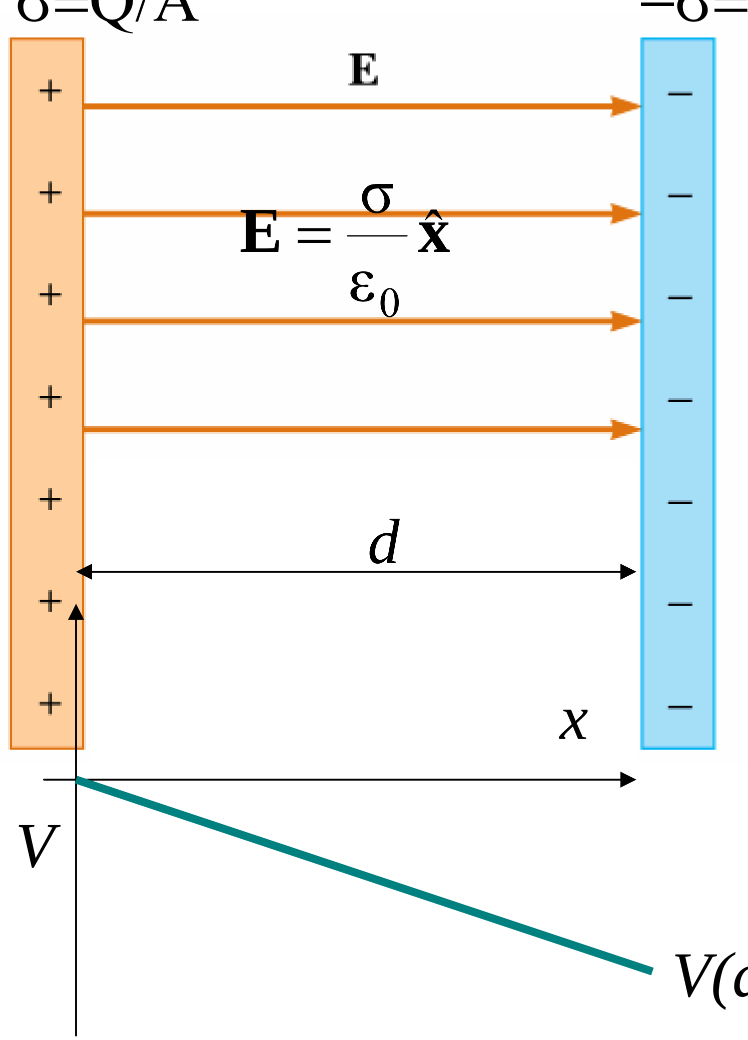
$$\Rightarrow E = \frac{\sigma A}{2A\varepsilon_0} = \frac{\sigma}{2\varepsilon_0}$$

$$\Rightarrow E = \frac{\sigma}{2\varepsilon_0}$$

permittivity constant $= 8.854 \times 10^{-12}\ C^2/(Nm^2)$

Electrostatic potential between two parallel plates

$\sigma = Q/A$ $\quad -\sigma = -Q/A$



$V(d) = -Ed = -\sigma\, d/\varepsilon_0$

Example:

If $V = 1$ Volt
and $d = 0.01$ m

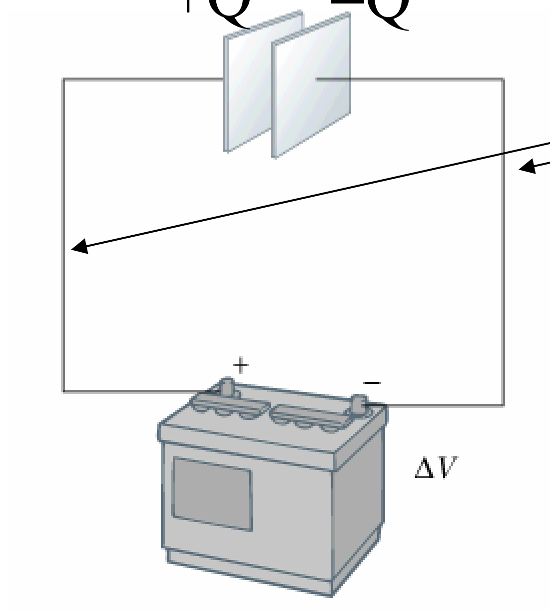
$\sigma = 8.854\text{x}10^{-10}\text{C/m}^2$

$$\Delta V = -\frac{Q}{C} \quad \text{where} \quad C \equiv \frac{\varepsilon_0 A}{d}$$

Capacitors in a circuit:

$Q = \Delta V\, C$

+Q –Q



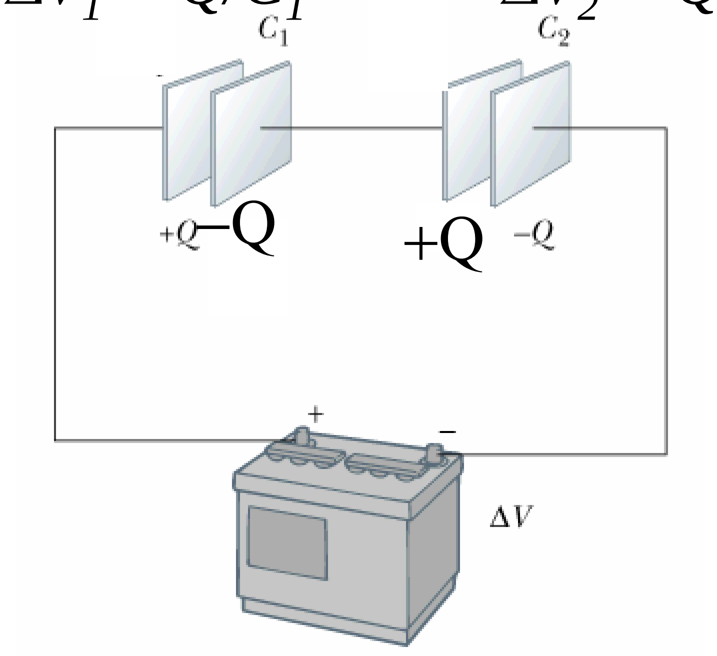
conductor

battery – source of charge (thanks to chemistry)

Two capacitors in a circuit –

Consider the following configuration:

$\Delta V_1 = Q/C_1$ $\Delta V_2 = Q/C_2$



$$\Delta V = \Delta V_1 + \Delta V_2$$

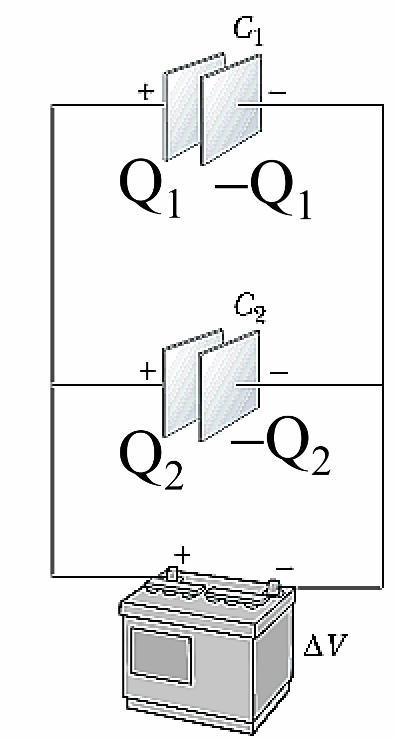
$$= \frac{Q}{C_1} + \frac{Q}{C_2} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

Capacitors connected in *series* are equivalent to C_{eq}:

$$\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i}$$

Two capacitors in a circuit –

Consider the following configuration:



$$\Delta V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$Q = Q_1 + Q_2 = C_1\Delta V + C_2\Delta V$$
$$= (C_1 + C_2)\Delta V$$

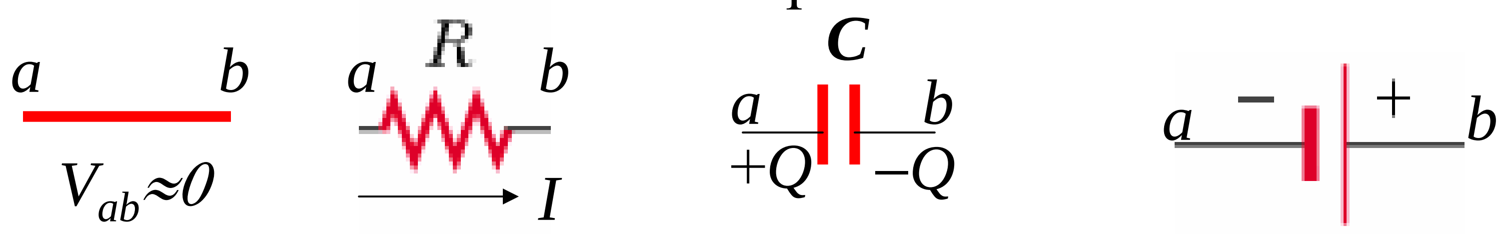
Capacitors connected in *parallel* are equivalent to C_{eq}:

$$C_{eq} = \sum_i C_i$$

Analysis of DC circuits:

Elements:

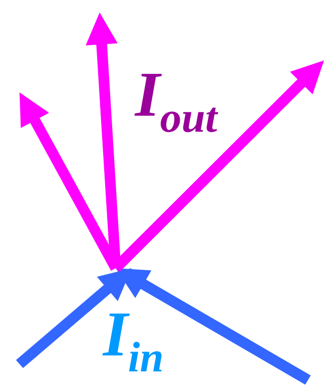
wire | resistor | capacitor | emf



$V_{ab} \approx 0$ | $V_{ab} = -IR$ | $V_{ab} = -Q/C$ | $V_{ab} = \mathcal{E}$

The principles:

Kirchhoff's rules



At any wire junction: $\sum I_{in} = \sum I_{out}$

For any closed wire loop: $\sum \Delta V = 0$