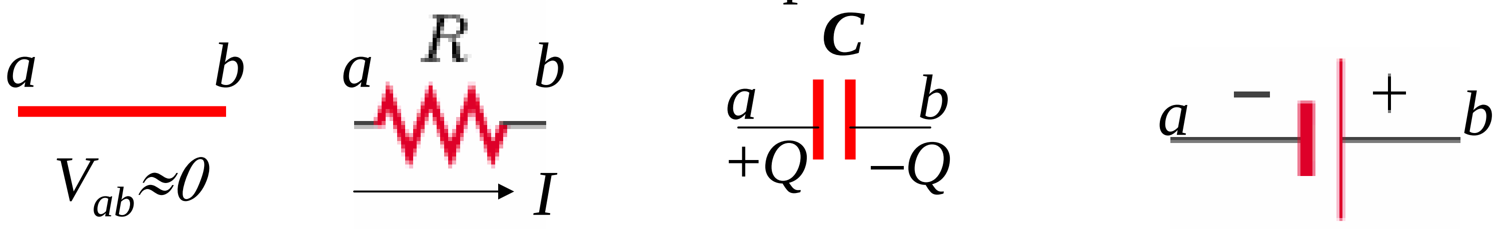
Announcements

1. Hour exam results – hopefully available Wed.
    - Will go over exam at Tuesday's tutorial (6 PM)
        - Time change????
    - Fresh start on homework
2. Today's topic – RC circuits
    - Kirchhoff's analysis
    - Differential equations
    - Time dependent currents and charges

Analysis of DC circuits:

Elements:

wire	resistor	capacitor	emf

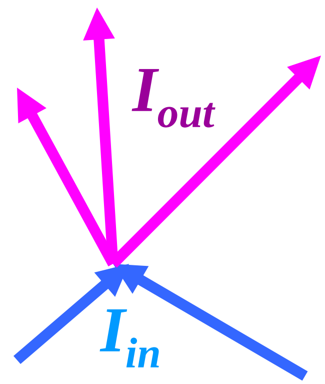


$V_{ab} \approx 0$ $\quad$ $V_{ab} = -IR$ $\quad$ $V_{ab} = -Q/C$ $\quad$ $V_{ab} = \mathcal{E}$

The principles:

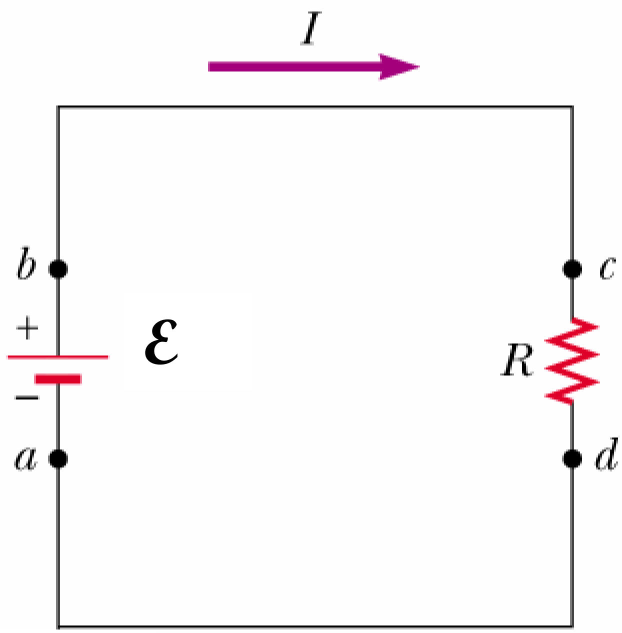
Kirchhoff's rules

At any wire junction: $\sum I_{in} = \sum I_{out}$



For any closed wire loop: $\sum \Delta V = 0$

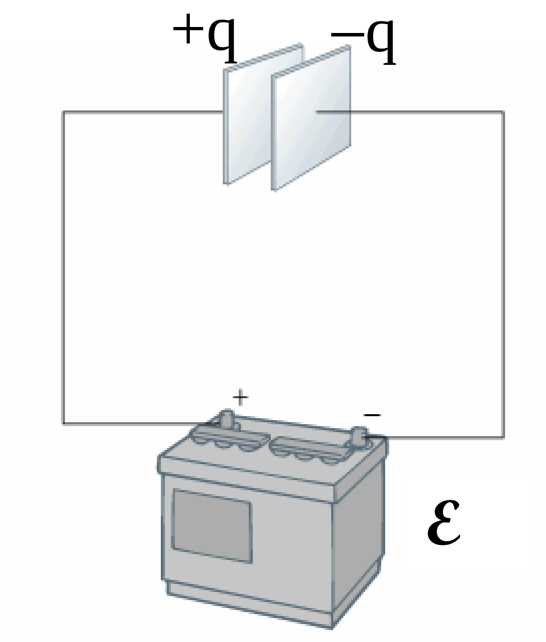
Circuit with an emf and resistor:



$$\mathcal{E} = I\,R$$

$$I = \frac{dq}{dt} = \frac{\mathcal{E}}{R}$$

Circuit with an emf and a capacitor:



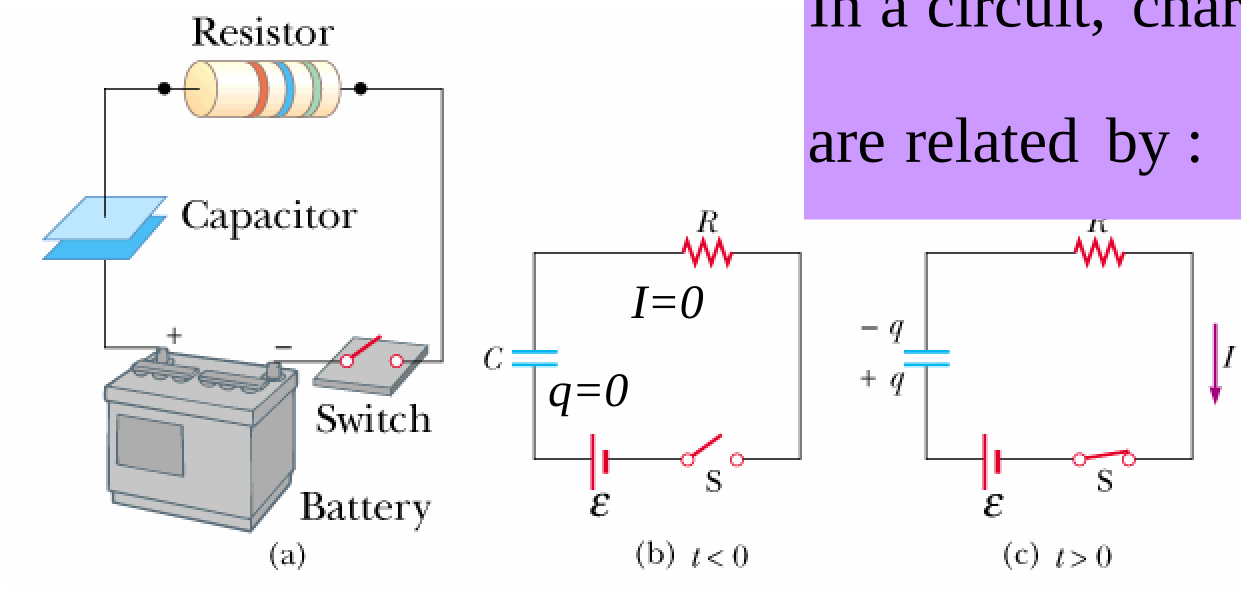
$$\mathcal{E} = \frac{q}{C}$$

$$q = \mathcal{E}C$$

Circuit containing capacitor and resistor --

Charging a capacitor:

In a circuit, charge and current are related by : $I = \frac{dq}{dt}$



When the switch is closed :

$$\mathcal{E} - \frac{q}{C} - IR = 0 \qquad \Rightarrow \qquad \mathcal{E} - \frac{q}{C} - \frac{dq}{dt}R = 0$$

First order differential equation for charge $q(t)$ and current $I(t)= \frac{dq}{dt}$

$$\mathcal{E} - \frac{q}{C} - \frac{dq}{dt}R = 0$$

initial condition: $q(t=0)=0$

solution:

$$q(t) = C\mathcal{E}\left(1 - e^{-t/(RC)}\right)$$

$$I(t) \equiv \frac{dq}{dt} = \frac{\mathcal{E}}{R} e^{-t/(RC)}$$

characteristic time constant for RC circuit

$$\text{Solution:} \quad q(t) = C\mathcal{E}\left(1 - e^{-t/(RC)}\right)$$

$$\text{Check:} \quad \mathcal{E} - \frac{q}{C} - \frac{dq}{dt}R = \mathcal{E} - \frac{C\mathcal{E}\left(1 - e^{-t/(RC)}\right)}{C} - \frac{C\mathcal{E}e^{-t/(RC)}}{RC}R$$

$$= \mathcal{E} - \mathcal{E} + \mathcal{E}e^{-t/(RC)} - \mathcal{E}e^{-t/(RC)} = 0$$

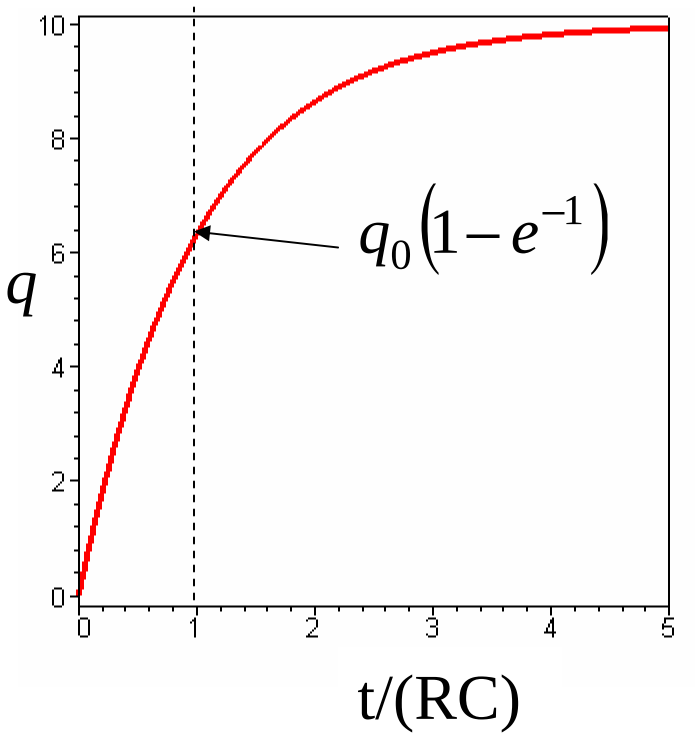
$$\text{Initialcondition:} \quad q(t=0) = C\mathcal{E}\left(1 - e^{-0/(RC)}\right)$$

$$= C\mathcal{E}(1-1) = 0$$

➔Mathematical analysis tells us that the solution is unique.

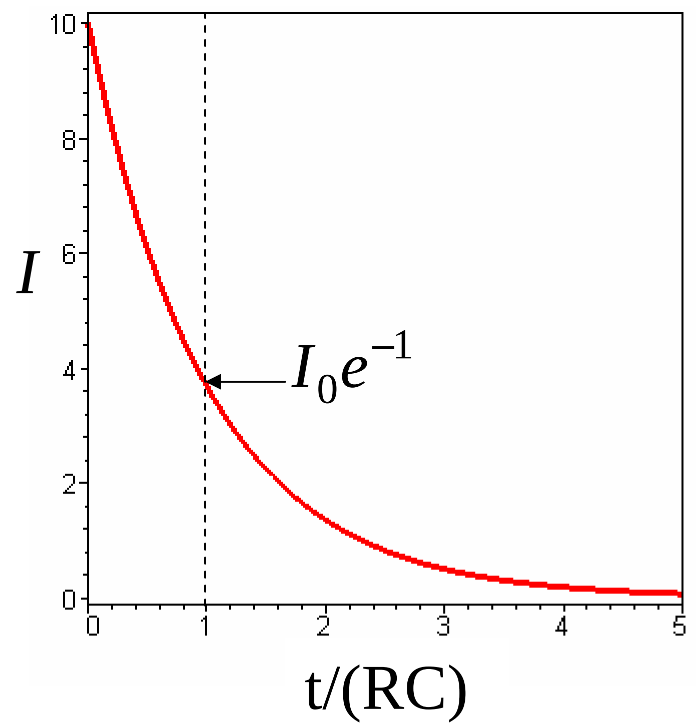
Charge

$$q(t) = C\mathcal{E}\left(1 - e^{-t/(RC)}\right)$$

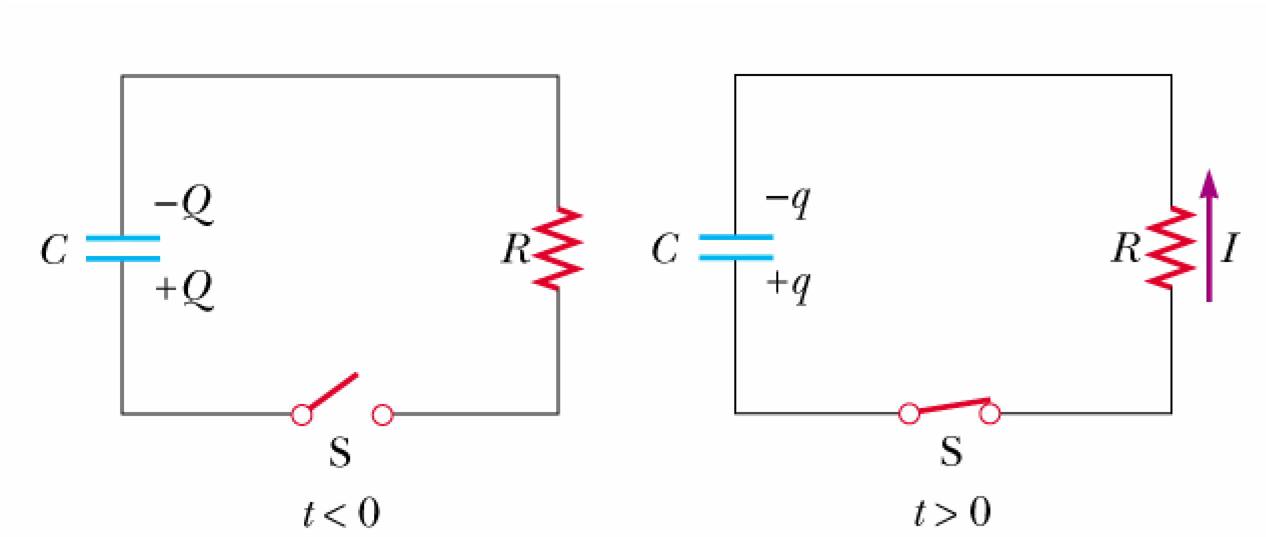


Current

$$I(t) \equiv \frac{dq}{dt} = \frac{\mathcal{E}}{R} e^{-t/(RC)}$$



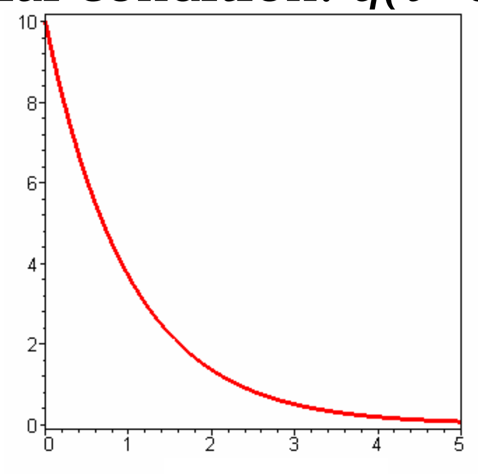
Discharging a capacitor



initial condition: *q(t=0)=Q*

$$-\frac{q}{C} - IR = 0 \qquad \Rightarrow \qquad -\frac{q}{C} - \frac{dq}{dt}R = 0$$

q



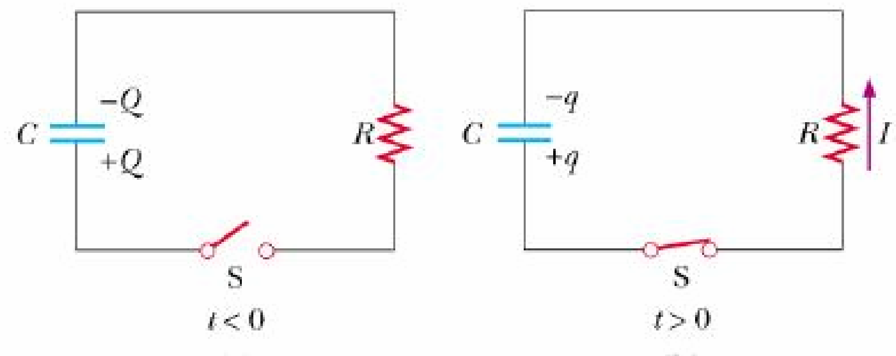
t/(RC)

solution :

$$q(t) = Q\left(e^{-t/(RC)}\right)$$

$$I(t) \equiv \frac{dq}{dt} = -\frac{Q}{RC}e^{-t/(RC)}$$

Online Quiz for Lecture 9
RC circuits -- Feb. 7, 2005



Consider the RC circuit shown in the diagram above. At t=0, the capacitor initially has a charge Q -- q(t=0)=Q. The capacitance is 1 x 10^{-6} F and the resistance is R=5000W.

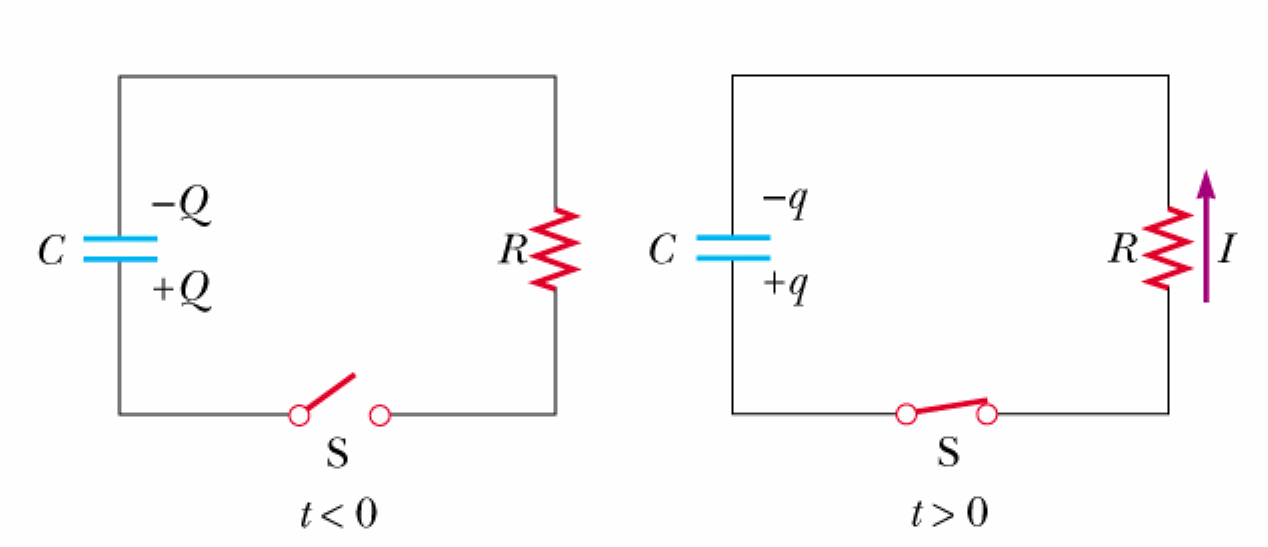
1. How long will it take for the charge to be reduced to 37% of its initial value?
   (A) 0.0001 s (B) 0.005 s (C) 0.023 s (D) 5 s (E) 23 s
2. How long will it take for the charge to be reduced to 1% of its initial value?
   (A) 0.0001 s (B) 0.005 s (C) 0.023 s (D) 5 s (E) 23 s .

$$e^{-t/\tau} = 0.01$$

$$-t/\tau = \ln(0.01) = -4.605$$

$$\Rightarrow t = \tau \cdot (4.605) \approx 0.023$$

RC circuit -- Discharging a capacitor through a resistor



initial condition:

$$q(t=0)=Q$$

after the switch is closed:

$$-\frac{q}{C} - IR = 0$$

$$\Rightarrow \quad -\frac{q}{C} - \frac{dq}{dt}R = 0$$

Analysis:

$$-\frac{q}{C}-\frac{dq}{dt}R=0$$

$$\frac{dq}{dt}=-\frac{1}{RC}q$$

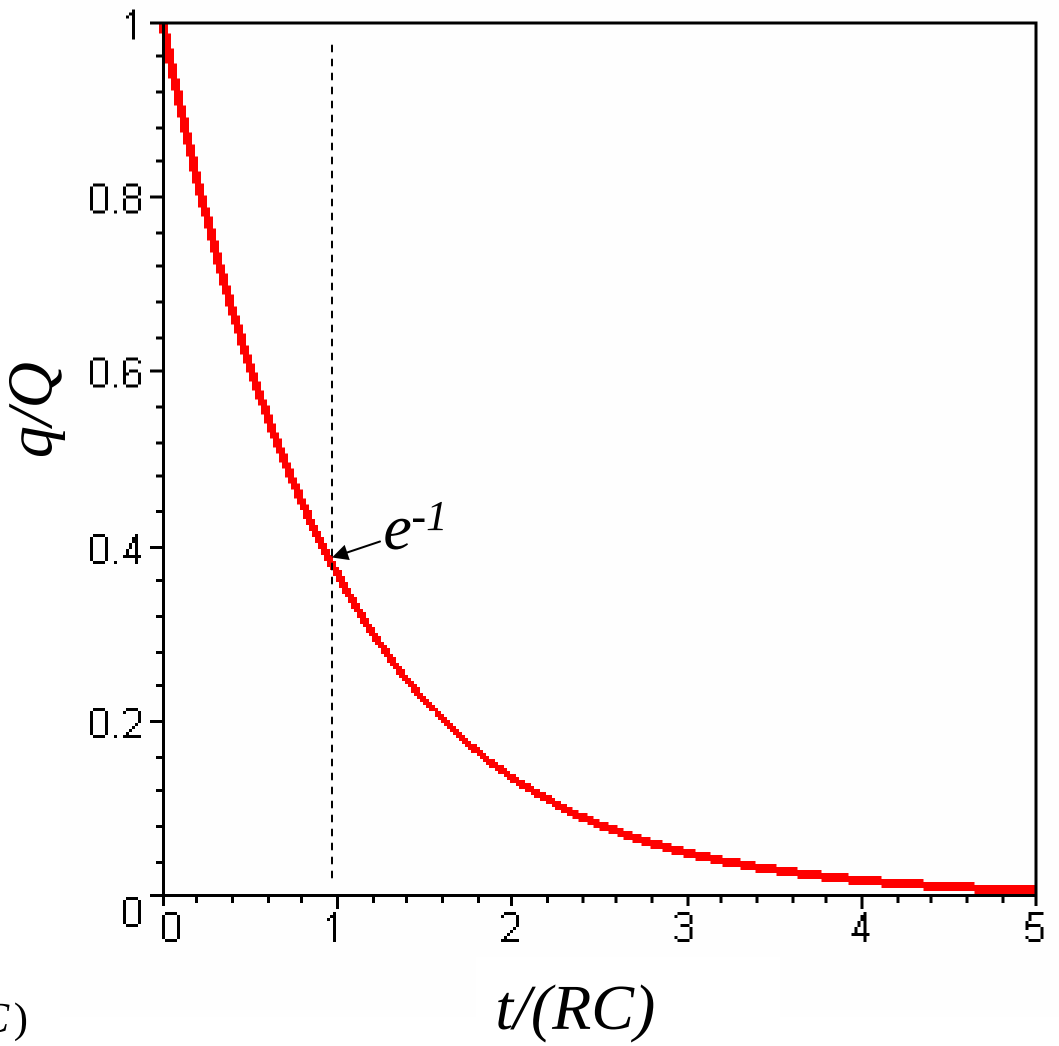
$$\frac{dq}{q}=-\frac{1}{RC}dt$$

$$\ln\left(\frac{q(t)}{Q}\right)=-\frac{t}{RC}$$

Result:

$$q(t)=Q\left(e^{-t/(RC)}\right)$$

$$I(t)\equiv\frac{dq}{dt}=-\frac{Q}{RC}e^{-t/(RC)}$$



General method for solving first-order linear differential equation

Assume we want to find *q(t)* in terms of constants *A,B,C*.

$$\alpha \frac{dq}{dt} + \beta q + \gamma = 0$$

$$\text{Try solution form: } \quad q(t) = Xe^{Yt} + Z$$

$$\text{Substitute into equation: } \alpha X(Y)e^{Yt} + \beta\left(Xe^{Yt} + Z\right) + \gamma = 0$$

This must be true for all times *t*.

$$\beta Z + \gamma = 0 \qquad \Rightarrow Z = -\gamma/\beta$$

$$\alpha XY + \beta X = 0 \qquad \Rightarrow Y = -\beta/\alpha$$

X determined from initial conditions

$$\text{If} \quad q(t=0) = Q \qquad \Rightarrow X = Q + \gamma/\beta$$

$$q(t) = Qe^{-\beta t/\alpha} + \frac{\gamma}{\beta}\left(e^{-\beta t/\alpha} - 1\right)$$

Extra credit:

Find several other examples of physical systems that obey a first order differential equation.

4. [HRW6 28.P.055.] In the circuit of Fig. 28-57, $\mathscr{E}$ = 1.2 kV, C = 6.5 μF, $R_1 = R_2 = R_3$ = 0.73 MΩ. With C completely uncharged, switch S is suddenly closed (at t = 0).

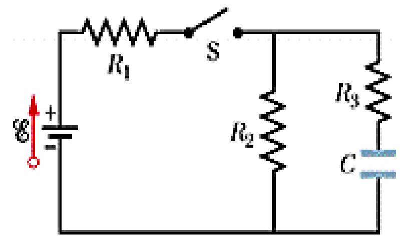


Figure 28-57.

Determine the current through each resistor for
t = 0

A (I_1)

A (I_2)

A (I_3)

and $t = \infty$

A (I_1)

A (I_2)

A (I_3)