April 12, 2006

PHY 712 – Problem Set # 26

1. Consider the following time-varying charge density

$$\rho(r,\theta,\phi,t) = \frac{\sqrt{2}}{4\pi a^4} \mathrm{e}^{-3r/(2a)} r\cos\theta\cos(\omega t), \tag{1}$$

and corresponding current density

$$\mathbf{J}(r,\theta,\phi,t) = -\frac{\sqrt{2}\hbar}{8\pi a^4 m} \mathrm{e}^{-3r/(2a)} \sin(\omega t)\left(\frac{\hat{\mathbf{r}} r\cos\theta}{2a} + \hat{\mathbf{z}}\right), \tag{2}$$

where a denotes the bohr radius and m denotes the electron mass. The frequency is given by $\omega = 3\hbar/(8ma^2)$.

(a) Check that $\rho(r,\theta,\phi,t)$ and $\mathbf{J}(r,\theta,\phi,t)$ satisfy the continuity equation.

(b) Write each of the sources as the real part of a purely harmonic time dependence such as

$$\rho(\mathbf{r},t) = \Re\left\{\mathrm{e}^{-i\omega t}\tilde{\rho}(\mathbf{r})\right\}. \tag{3}$$

$$\mathbf{J}(\mathbf{r},t) = \Re\left\{\mathrm{e}^{-i\omega t}\tilde{\mathbf{J}}(\mathbf{r})\right\}. \tag{4}$$

(c) Using the appropriate Green's function, express the scalar potential $\Phi(\mathbf{r},t)$ and the vector potential $\mathbf{A}(\mathbf{r},t)$ in terms of spatial integrals over the sources.

(d) Evaluate these integrals far from the sources ($r >> a$). (You may wish to consult sections 9.6 as well as the famous page 109 of your text.)

(e) Discuss your result in comparison with the simple dipole radiator presented in section 9.2 of your text.