1. Consider a three-dimensional charge distribution of the form:

\[ \rho(\mathbf{r}) = \frac{q}{\pi^{3/2}a^3} e^{-(r/a)^2} \]

where \( q \) and \( a \) are constants. In the following, you may wish to show and use the result that for this particular charge density,

\[ \frac{1}{4\pi\varepsilon_0} \int d^3r' \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} = \frac{q}{4\pi\varepsilon_0} \frac{\text{erf}(r/a)}{r}. \]

(a) Find the electrostatic potential \( \Phi(\mathbf{r}) \), as a function of the distance \( \mathbf{r} \) from the center of the charge distribution \( \rho(\mathbf{r}) \).

(b) Now suppose that a grounded metal plate is placed at a distance \(-d\hat{z}\) from the center of the charge distribution. Find the electrostatic potential due to \( \rho(\mathbf{r}) \) and the boundary condition \( \Phi(x, y, z = -d) = 0 \) for a general point \( \mathbf{r} \) with \( z > 0 \).