January 23, 2006

PHY 712 – Problem Set #5

1. Consider a three-dimensional charge distribution of the form:

$$\rho(\mathbf{r}) = \frac{q}{\pi^{3/2}a^3}\mathrm{e}^{-(r/a)^2}$$

where q and a are constants. In the following, you may wish to show and use the result that for this particular charge density,

$$\frac{1}{4\pi\epsilon_0}\int d^3r' \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} = \frac{q}{4\pi\epsilon_0}\frac{\mathrm{erf}(r/a)}{r}.$$

(a) Find the electrostatic potential $\Phi(\mathbf{r})$, as a function of the distance $\mathbf{r}$ from the center of the charge distribution $\rho(\mathbf{r})$.

(b) Now suppose that a grounded metal plate is placed at a distance $-d\hat{\mathbf{z}}$ from the center of the charge distribution. Find the electrostatic potential due to $\rho(\mathbf{r})$ and the boundary condition $\Phi(x, y, z = -d) = 0$ for a general point $\mathbf{r}$ with $z > 0$.