1. Suppose that an electromagnetic wave of pure (real) frequency $\omega$ is traveling along the $z$-axis of a wave guide having a square cross section with side dimension $a$ composed of a medium having a real permittivity constant $\epsilon$ and a real permeability constant $\mu$. Suppose that the wave is known to have the form:

$$E(r, t) = \mathbb{R}\left\{H_0 e^{ikz-i\omega t} (i\mu \omega) \frac{\pi}{a} \sin\left(\frac{\pi x}{a}\right) \hat{y}\right\}$$

$$H(r, t) = \mathbb{R}\left\{H_0 e^{ikz-i\omega t} \left[ -ik \frac{\pi}{a} \sin\left(\frac{\pi x}{a}\right) \hat{x} + \cos\left(\frac{\pi x}{a}\right) \hat{z}\right]\right\}.$$

Here $H_0$ denotes a real amplitude, and the parameter $k$ is assumed to be real and equal to

$$k \equiv \sqrt{\omega^2 - \left(\frac{\pi}{a}\right)^2},$$

where $\omega > \frac{\pi}{a}$. Find the form of the time-averaged Poynting vector

$$\langle S \rangle_{\text{avg}} \equiv \frac{1}{2} \mathbb{R}\{E(r, t) \times H^*(r, t)\}$$

for this electromagnetic wave.