PHY 712 – Problem Set #4

Continue reading Chaper 1 & 2 in Jackson; homework is due Wednesday, Jan. 28, 2009.

1. Consider a one-dimensional charge distribution of the form:

$$\rho(x) = \begin{cases}
0 & \text{for } x \leq 0 \\
\rho_0 \sin\left(\frac{2\pi x}{a}\right) & \text{for } 0 \leq x \leq a \\
0 & \text{for } x \geq a,
\end{cases}$$

where ρ_0 and *a* are constants.

- (a) Solve the Poisson equation for the electrostatic potential $\Phi(x)$ with the boundary conditions $\Phi(0) = 0$ and $\frac{d\Phi}{dx}(0) = 0$.
 - i. Use the Green's function discussed in Lecture Notes #4:

$$G(x, x') = 4\pi x_{<}.$$

ii. Use the Green's function discussed in Lecture Notes #5:

$$G(x, x') = \frac{8\pi}{a} \sum_{n} \frac{\sin(n\pi x/a)\sin(n\pi x'/a)}{\left(\frac{n\pi}{a}\right)^2}.$$

(b) In both cases, check whether the Green's function-derived solutions satisfy the boundary conditions. If they do not, you will need to add contributions from solutions to the homogeneous equations as discussed in Lecture Notes #5. Obviously, you should obtain the same answer for both methods.