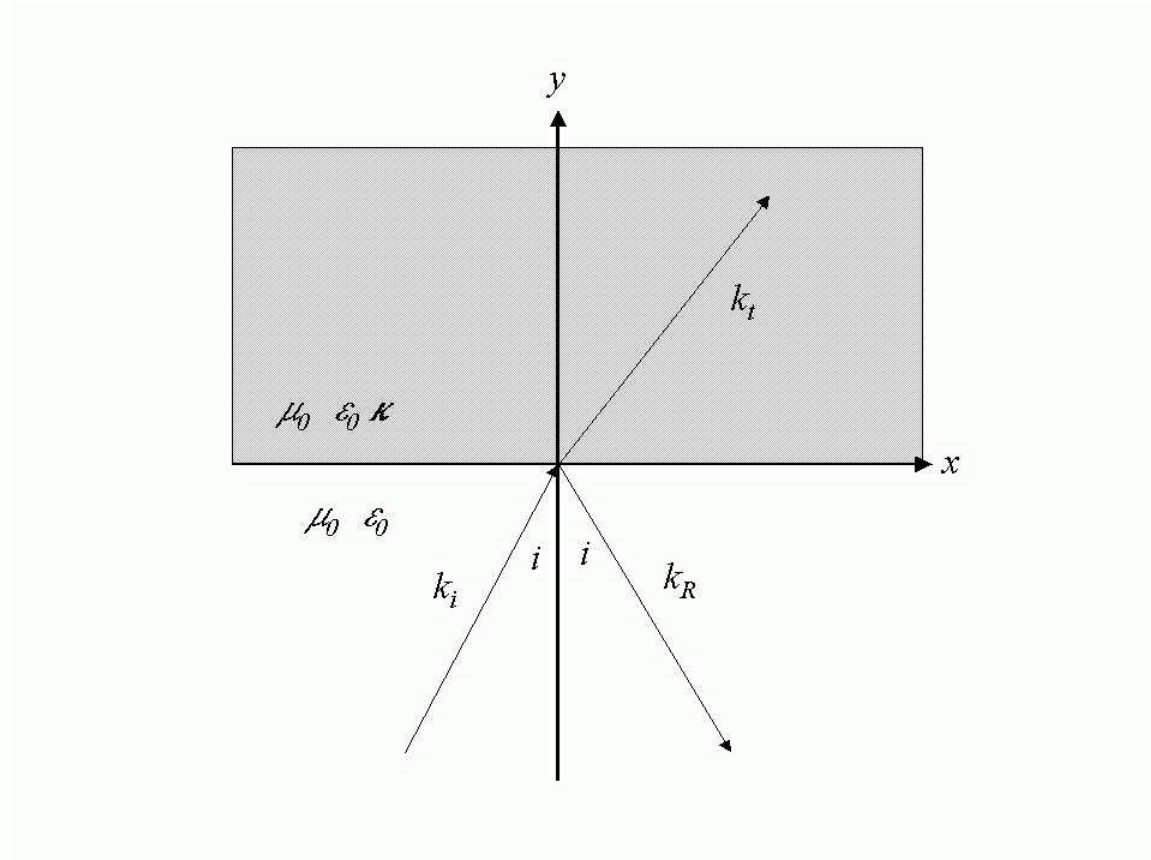


Notes for Lecture #22

Reflectivity for anisotropic media – Extension of Section 7.3 in Jackson's text



Consider the problem of determining the reflectance from an anisotropic medium as shown above. We will assume that the permeability is μ_0 and that the permittivity ϵ is a tensor which can be expressed in terms of a unit-less dielectric tensor $\kappa = \epsilon/\epsilon_0$. For simplicity, we will assume that the dielectric tensor for the medium is diagonal and is given by:

$$\kappa \equiv \begin{pmatrix} \kappa_{xx} & 0 & 0 \\ 0 & \kappa_{yy} & 0 \\ 0 & 0 & \kappa_{zz} \end{pmatrix}. \quad (1)$$

We will assume also that the wave vector in the medium is confined to the $x - y$ plane and will be denoted by

$$\mathbf{k}_t \equiv \frac{\omega}{c}(n_x \hat{\mathbf{x}} + n_y \hat{\mathbf{y}}), \quad (2)$$

where n_x and n_y are to be determined. We will assume that the complex representation of

electric field inside the medium is given by

$$\mathbf{E} = (E_x \hat{\mathbf{x}} + E_y \hat{\mathbf{y}} + E_z \hat{\mathbf{z}}) e^{i \frac{\omega}{c} (n_x x + n_y y) - i \omega t}. \quad (3)$$

In terms of this electric field and the magnetic field $\mathbf{H} = \mathbf{B}/\mu_0$, where \mathbf{H} is assumed to have the same complex spatial and temporal form as (3), the four Maxwell's equations are given by:

$$\begin{aligned} \nabla \cdot \mathbf{H} &= 0 & \nabla \cdot \boldsymbol{\kappa} \cdot \mathbf{E} &= 0 \\ \nabla \times \mathbf{E} - i \omega \mu_0 \mathbf{H} &= 0 & \nabla \times \mathbf{H} + i \omega \epsilon_0 \boldsymbol{\kappa} \cdot \mathbf{E} &= 0 \end{aligned} \quad (4)$$

Using these equations, we obtain the following equations for electric field amplitudes within the medium:

$$\begin{pmatrix} \kappa_{xx} - n_y^2 & n_x n_y & 0 \\ n_x n_y & \kappa_{yy} - n_x^2 & 0 \\ 0 & 0 & \kappa_{zz} - (n_x^2 + n_y^2) \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0. \quad (5)$$

Once the electric field amplitudes are determined, the magnetic field can be determined according to:

$$\mathbf{H} = \frac{1}{\mu_0 c} \{ E_z (n_y \hat{\mathbf{x}} - n_x \hat{\mathbf{y}}) + (E_y n_x - E_x n_y) \hat{\mathbf{z}} \} e^{i \frac{\omega}{c} (n_x x + n_y y) - i \omega t}. \quad (6)$$

The incident and reflected electro-magnetic fields are given in your textbook. In the notation of the figure the wavevector for the incident wave is given by:

$$\mathbf{k}_i = \frac{\omega}{c} (\sin i \hat{\mathbf{x}} + \cos i \hat{\mathbf{y}}), \quad (7)$$

and the wavevector for reflected wave is given by:

$$\mathbf{k}_R = \frac{\omega}{c} (\sin i \hat{\mathbf{x}} - \cos i \hat{\mathbf{y}}). \quad (8)$$

In this notation, Snell's law requires that $n_x = \sin i$. The continuity conditions at the $y = 0$ plane involve continuity requirements on the following fields:

$$\mathbf{H}(x, 0, z, t), \quad E_x(x, 0, z, t), \quad E_z(x, 0, z, t), \quad \text{and} \quad D_y(x, 0, z, t), \quad (9)$$

at all times t .

Below we consider two different polarizations for the electric field.

Solution for s-polarization

In this case, $E_x = E_y = 0$, and $n_x^2 = \kappa_{zz} - n_y^2$. The fields in the medium are given by:

$$\mathbf{E} = E_z \hat{\mathbf{z}} e^{i \frac{\omega}{c} (n_x x + n_y y) - i \omega t} \quad \mathbf{H} = \frac{1}{\mu_0 c} \{ E_z (n_y \hat{\mathbf{x}} - n_x \hat{\mathbf{y}}) \} e^{i \frac{\omega}{c} (n_x x + n_y y) - i \omega t}. \quad (10)$$

The amplitude E_z can be determined from the matching conditions:

$$\begin{aligned} E_0 + E_0'' &= E_z \\ (E_0 - E_0'') \cos i &= E_z n_y \\ (E_0 + E_0'') \sin i &= E_z n_x. \end{aligned} \quad (11)$$

In this case, the last equation is redundant. The other two equations can be solved for the reflected amplitude:

$$\frac{E_0''}{E_0} = \frac{\cos i - n_y}{\cos i + n_y}. \quad (12)$$

This is very similar to the result given in Eq. 7.39 of **Jackson** for the isotropic media.

Solution for p-polarization

In this case, $E_z = 0$ and

$$n_y^2 = \frac{\kappa_{xx}}{\kappa_{yy}} (\kappa_{yy} - n_x^2). \quad (13)$$

In terms of the unknown amplitude E_x , the electric field in the medium is given by:

$$\mathbf{E} = E_x \left(\hat{\mathbf{x}} - \frac{\kappa_{xx} n_x}{\kappa_{yy} n_y} \hat{\mathbf{y}} \right) e^{i \frac{\omega}{c} (n_x x + n_y y) - i \omega t}. \quad (14)$$

The corresponding magnetic field is given by:

$$\mathbf{H} = -\frac{E_x}{\mu_0 c} \frac{\kappa_{xx}}{n_y} \hat{\mathbf{z}} e^{i \frac{\omega}{c} (n_x x + n_y y) - i \omega t}. \quad (15)$$

The amplitude E_x can be determined from the matching conditions:

$$\begin{aligned} (E_0 - E_0'') \cos i &= E_x \\ (E_0 + E_0'') &= \frac{\kappa_{xx}}{n_y} E_x \\ (E_0 + E_0'') \sin i &= \frac{\kappa_{xx} n_x}{n_y} E_x. \end{aligned} \quad (16)$$

Again, the last equation is redundant, and the solution for the reflected amplitude is given by:

$$\frac{E_0''}{E_0} = \frac{\kappa_{xx} \cos i - n_y}{\kappa_{xx} \cos i + n_y}. \quad (17)$$

This result reduces to Eq. 7.41 in **Jackson** for the isotropic case.