1. Diamond is a version of carbon. The position of the carbon atoms takes the form

\[ \mathbf{r} = d (n_1 + x) \mathbf{\hat{x}} + d (n_2 + y) \mathbf{\hat{y}} + d (n_3 + z) \mathbf{\hat{z}} \]

where \( d = 356.683 \) pm, \((n_1, n_2, n_3)\) are arbitrary integers, and \((x, y, z)\) takes on the following eight values:

\[(x, y, z) \in \{(0, 0, 0), (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}), (0, \frac{1}{4}, \frac{1}{4}), (\frac{1}{4}, 0, \frac{1}{4}), (\frac{1}{4}, \frac{1}{4}, 0), (\frac{1}{2}, \frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, \frac{1}{4}, \frac{1}{4}), (\frac{1}{4}, \frac{1}{2}, \frac{1}{4})\}\]

Thus there are eight carbon atoms per cell of size \(d^3\).

(a) For what values of \((x, y, z)\) will \( \mathbf{T} = dx \mathbf{\hat{x}} + dy \mathbf{\hat{y}} + dz \mathbf{\hat{z}} \) be a translation vector; i.e., if there is a carbon atom at \( \mathbf{r} \), there will always be a carbon atom at \( \mathbf{r} + \mathbf{T} \)? To make your answer finite, only include values with \(0 \leq x, y, z < 1\).

(b) Find primitive vectors \( \mathbf{a}, \mathbf{b}, \mathbf{c} \) such that all translation vectors take the form \( \mathbf{T} = m_1 \mathbf{a} + m_2 \mathbf{b} + m_3 \mathbf{c} \), where \((m_1, m_2, m_3)\) are integers. Demonstrate it explicitly for those vectors you found in part (a) (which will probably be trivial), and also for the three vectors \( d \mathbf{\hat{x}}, d \mathbf{\hat{y}} \) and \( d \mathbf{\hat{z}} \).

(c) What are the lengths of these vectors \(a, b, c\) and the angles between them, \(\alpha, \beta, \gamma\)?