Notes on examples of character tables

From the reference: L. P. Bouckaert, R. Smoluchowski, and E. Wigner, Phys. Rev. **50**, 58 (1936) the character table for the O_h group is given by

TABLE I. *Characters of small representations of* Γ, R, H.

Γ, R, H	E	$3C_4^2$	$6C_4$	$6C_2$	$8C_3$	J	$3JC_4^2$	$6JC_4$	$6JC_2$	$8JC_3$
Γ_1	1	1	1	1	1	1	1	1	1	1
Γ_2	1	1	−1	−1	1	1	1	−1	−1	1
Γ_{12}	2	2	0	0	−1	2	2	0	0	−1
Γ_{15}'	3	−1	1	−1	0	3	−1	1	−1	0
Γ_{25}'	3	−1	−1	1	0	3	−1	−1	1	0
Γ_1'	1	1	1	1	1	−1	−1	−1	−1	−1
Γ_2'	1	1	−1	−1	1	−1	−1	1	1	−1
Γ_{12}'	2	2	0	0	−1	−2	−2	0	0	1
Γ_{15}	3	−1	1	−1	0	−3	1	−1	1	0
Γ_{25}	3	−1	−1	1	0	−3	1	1	−1	0

Here, the class notations are given by

1. E – $\{xyz\}$
2. C_4^2 – $\{\bar{x}\bar{y}z, x\bar{y}\bar{z}, \bar{x}y\bar{z}\}$
3. C_4 – $\{\bar{y}xz, y\bar{x}z, x\bar{z}y, xz\bar{y}, zy\bar{x}, \bar{z}yx\}$
4. C_2 – $\{yx\bar{z}, z\bar{y}x, \bar{x}zy, \bar{y}\bar{x}z, \bar{z}\bar{y}\bar{x}, \bar{x}\bar{z}\bar{y}\}$
5. C_3 – $\{zxy, yzx, z\bar{x}\bar{y}, y\bar{z}\bar{x}, \bar{z}x\bar{y}, \bar{y}z\bar{x}, \bar{z}\bar{x}y, \bar{y}\bar{z}x\}$
6. J – $\{\bar{x}\bar{y}\bar{z}\}$
7. JC_4^2 –
8. JC_4 –
9. JC_2 –
10. JC_3 –

For comparison, the group $D_4(422)$ is given in your text book on page 327 where the class notations are given by

1. E – $\{xyz\}$
2. C_4^2 – $\{\bar{x}\bar{y}z\}$
3. C_4 – $\{\bar{y}xz, y\bar{x}z\}$
4. $C_2' \equiv O_h[C_4^2]$ – $\{\bar{x}y\bar{z}, x\bar{y}\bar{z}\}$
5. $C_2'' \equiv O_h[C_2]$ – $\{\bar{y}\bar{x}\bar{z}, yx\bar{z}\}$

We can now evaluate the character table to determine the "campatability" of $D_4(422)$ with O_h. The following character table from the back of your book has been "augmented" with the characters of the O_h group

	E	C_4^2	$2C_4$	$2C_2'$	$2C_2''$
A_1	1	1	1	1	1
A_2	1	1	1	-1	-1
B_1	1	1	-1	1	-1
B_2	1	1	-1	-1	1
E	2	-2	0	0	0
Γ_1	1	1	1	1	1
Γ_2	1	1	-1	1	-1
Γ_{12}	2	2	0	2	0
$\Gamma_{15'}$	3	-1	1	-1	-1
$\Gamma_{25'}$	3	-1	-1	-1	1
$\Gamma_{1'}$	1	1	1	1	1
$\Gamma_{2'}$	1	1	-1	1	-1
$\Gamma_{12'}$	2	2	0	2	0
Γ_{15}	3	-1	1	-1	-1
Γ_{25}	3	-1	-1	-1	1