March 27, 2011

PHY 712 – Problem Set # 20

Continue reading Chapter 9 of **Jackson**. This problem is due Fri. Apr. 1, 2011.

1. Suppose that you have a source with the following charge and current density distributions:

$$\rho(\mathbf{r},t) = \mathrm{Re}\left(\frac{2Dz\mathrm{e}^{-r^2/R^2}}{R^5\pi^{3/2}}\mathrm{e}^{-i\omega t}\right).$$

$$\mathbf{J}(\mathbf{r},t) = \mathrm{Re}\left(\hat{\mathbf{z}}\frac{-i\omega D\mathrm{e}^{-r^2/R^2}}{R^3\pi^{3/2}}\mathrm{e}^{-i\omega t}\right).$$

   In this expression, the constant D denotes the dipole moment, R is a length parameter, and ω is the (constant) harmonic frequency.

   (a) Show that this source is consistent with the continuity equation.

   (b) Write an expression for the scalar and vector potentials $\Phi(\mathbf{r},t)$ and $\mathbf{A}(\mathbf{r},t)$, evaluating as many of the integrals as is feasible.

   (c) Write the forms of $\Phi(\mathbf{r},t)$ and $\mathbf{A}(\mathbf{r},t)$ for distances $r >> R$.

   (d) Find the electric and magnetic fields $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$ for distances $r >> R$.

   (e) Find the time averaged Poynting vector for this source for distances $r >> R$.