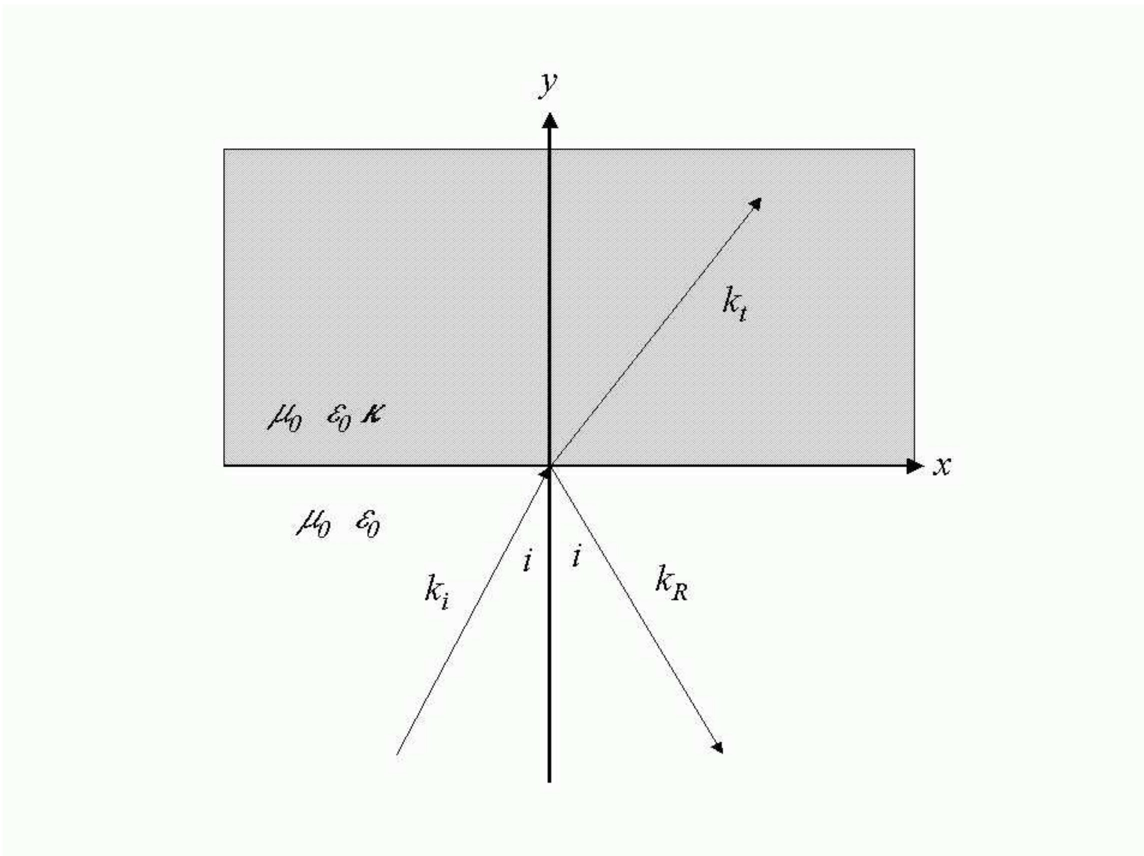
February 26, 2011

Notes for Lecture #19

Reflectivity for anisotropic media – Extension of Section 7.3 in Jackson's text



Consider the problem of determining the reflectance from an anisotropic medium as shown above. We will assume that the permeability is μ_0 and that the permittivity ϵ is a tensor which can be expressed in terms of a unit-less dielectric tensor $\kappa = \epsilon/\epsilon_0$. For simplicity, we will assume that the dielectric tensor for the medium is diagonal and is given by:

$$\kappa \equiv \begin{pmatrix} \kappa_{xx} & 0 & 0 \\ 0 & \kappa_{yy} & 0 \\ 0 & 0 & \kappa_{zz} \end{pmatrix}. \tag{1}$$

We will assume also that the wave vector in the medium is confined to the $x - y$ plane and will be denoted by

$$\mathbf{k}_t \equiv \frac{\omega}{c}(n_x\hat{\mathbf{x}} + n_y\hat{\mathbf{y}}), \tag{2}$$

where n_x and n_y are to be determined. We will assume that the complex representation of electric field inside the medium is given by

$$\mathbf{E} = (E_x\hat{\mathbf{x}} + E_y\hat{\mathbf{y}} + E_z\hat{\mathbf{z}})\mathrm{e}^{i\frac{\omega}{c}(n_x x + n_y y) - i\omega t}. \tag{3}$$

In terms of this electric field and the magnetic field $\mathbf{H} = \mathbf{B}/\mu_0$, where $\mathbf{H}$ is assumed to have the same complex spatial and temporal form as (3), the four Maxwell's equations are given by:

$$\begin{array}{cc} \nabla \cdot \mathbf{H} = 0 & \nabla \cdot \kappa \cdot \mathbf{E} = 0 \\ \nabla \times \mathbf{E} - i\omega\mu_0\mathbf{H} = 0 & \nabla \times \mathbf{H} + i\omega\epsilon_0\kappa \cdot \mathbf{E} = 0 \end{array} \tag{4}$$

Using these equations, we obtain the following equations for electric field amplitudes within the medium:

$$\begin{pmatrix} \kappa_{xx} - n_y^2 & n_x n_y & 0 \\ n_x n_y & \kappa_{yy} - n_x^2 & 0 \\ 0 & 0 & \kappa_{zz} - (n_x^2 + n_y^2) \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0. \tag{5}$$

Once the electric field amplitudes are determined, the magnetic field can be determined according to:

$$\mathbf{H} = \frac{1}{\mu_0 c}\left\{E_z(n_y\hat{\mathbf{x}} - n_x\hat{\mathbf{y}}) + (E_y n_x - E_x n_y)\hat{\mathbf{z}}\right\}\mathrm{e}^{i\frac{\omega}{c}(n_x x + n_y y) - i\omega t}. \tag{6}$$

The incident and reflected electro-magnetic fields are given in your textbook. In the notation of the figure the wavevector for the incident wave is given by:

$$\mathbf{k}_i = \frac{\omega}{c}(\sin i\hat{\mathbf{x}} + \cos i\hat{\mathbf{y}}), \tag{7}$$

and the wavevector for reflected wave is given by:

$$\mathbf{k}_R = \frac{\omega}{c}(\sin i\hat{\mathbf{x}} - \cos i\hat{\mathbf{y}}). \tag{8}$$

In this notation, Snell's law requires that $n_x = \sin i$. The continuity conditions at the $y = 0$ plane involve continuity requirements on the following fields:

$$\mathbf{H}(x, 0, z, t), \quad E_x(x, 0, z, t), \quad E_z(x, 0, z, t), \quad \text{and} \quad D_y(x, 0, z, t), \tag{9}$$

at all times t.

Below we consider two different polarizations for the electric field.

Solution for s-polarization

In this case, $E_x = E_y = 0$, and $n_y^2 = \kappa_{zz} - n_x^2$. The fields in the medium are given by:

$$\mathbf{E} = E_z\hat{\mathbf{z}}\mathrm{e}^{i\frac{\omega}{c}(n_x x + n_y y) - i\omega t} \qquad \mathbf{H} = \frac{1}{\mu_0 c}\left\{E_z(n_y\hat{\mathbf{x}} - n_x\hat{\mathbf{y}})\right\}\mathrm{e}^{i\frac{\omega}{c}(n_x x + n_y y) - i\omega t}. \tag{10}$$

The amplitude E_z can be determined from the matching conditions:

$$\begin{aligned} E_0 + E_0'' &= E_z \\ (E_0 - E_0'')\cos i &= E_z n_y \\ (E_0 + E_0'')\sin i &= E_z n_x . \end{aligned} \tag{11}$$

In this case, the last equation is redundant. The other two equations can be solved for the reflected amplitude:

$$\frac{E_0''}{E_0} = \frac{\cos i - n_y}{\cos i + n_y}. \tag{12}$$

This is very similar to the result given in Eq. 7.39 of **Jackson** for the isotropic media.

Solution for p-polarization

In this case, $E_z = 0$ and

$$n_y^2 = \frac{\kappa_{xx}}{\kappa_{yy}}(\kappa_{yy} - n_x^2). \tag{13}$$

In terms of the unknown amplitude E_x, the electric field in the medium is given by:

$$\mathbf{E} = E_x \left(\hat{\mathbf{x}} - \frac{\kappa_{xx} n_x}{\kappa_{yy} n_y} \hat{\mathbf{y}} \right) \mathrm{e}^{i\frac{\omega}{c}(n_x x + n_y y) - i\omega t}. \tag{14}$$

The corresponding magnetic field is given by:

$$\mathbf{H} = -\frac{E_x}{\mu_0 c} \frac{\kappa_{xx}}{n_y} \hat{\mathbf{z}} \mathrm{e}^{i\frac{\omega}{c}(n_x x + n_y y) - i\omega t}. \tag{15}$$

The amplitude E_x can be determined from the matching conditions:

$$\begin{aligned} (E_0 - E_0'')\cos i &= E_x \\ (E_0 + E_0'') &= \tfrac{\kappa_{xx}}{n_y} E_x \\ (E_0 + E_0'')\sin i &= \tfrac{\kappa_{xx} n_x}{n_y} E_x . \end{aligned} \tag{16}$$

Again, the last equation is redundant, and the solution for the reflected amplitude is given by:

$$\frac{E_0''}{E_0} = \frac{\kappa_{xx}\cos i - n_y}{\kappa_{xx}\cos i + n_y}. \tag{17}$$

This result reduces to Eq. 7.41 in **Jackson** for the isotropic case.