

**PHY 752 – Problem Set #10**

Read Chapters 6-7 in **Martin**: homework is due Wednesday, Feb. 9, 2011. This assignment is designed to explore the power of the variational method for a simple Hamiltonian problem.

1. Consider the Hamiltonian for the electron in a H atom:

$$\mathcal{H}(\mathbf{r}) = -\frac{\hbar^2}{2m}\nabla^2 - \frac{e^2}{r}. \quad (1)$$

For the following two functional forms, use the variational principle to find the best estimate of the ground state energy. That is, in each case, find the optimal value of  $\alpha$  which minimizes the energy

$$E(\alpha) \equiv \frac{\langle \Psi(r, \alpha) | \mathcal{H}(\mathbf{r}) | \Psi(r, \alpha) \rangle}{\langle \Psi(r, \alpha) | \Psi(r, \alpha) \rangle}. \quad (2)$$

(a)

$$|\Psi(r, \alpha)\rangle \equiv e^{-\alpha r}. \quad (3)$$

(b)

$$|\Psi(r, \alpha)\rangle \equiv e^{-\alpha r^2}. \quad (4)$$