Plan for Lecture 10 (Review Chapters 26-28):

1. Capacitance
2. Resistance
3. Voltage
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Remember to send in your chapter reading questions...
Professor George Holzwarth (gholz@wfu.edu) will administer the exam on 2/28/2012 and will lecture on magnetic fields, electric motors, etc. on 3/1/2012.
Reminder:

Second exam – Tuesday, February 28, 2012

- ~5 problems – show your work and reasoning for possible partial credit.
- Should bring 1 8½” x 11” sheet of paper to the exam (to be turned in with your exam papers).
- Should bring calculator for numerical work. Must not use cell phones or computers during the exam.
- There will be assigned seating in Olin 101 for the exam.
Advice:

1. Keep basic concepts and equations at the top of your head.
2. Practice problem solving and math skills
3. Develop an equation sheet that you can consult.
Problem solving steps

1. Visualize problem – labeling variables
2. Determine which basic physical principle(s) apply
3. Write down the appropriate equations using the variables defined in step 1.
4. Check whether you have the correct amount of information to solve the problem (same number of knowns and unknowns).
5. Solve the equations.
6. Check whether your answer makes sense (units, order of magnitude, etc.).
How many times did you use your equation sheet during the last exam?

A. 0
B. 1
C. 2
D. >2
Example 28.5:
Assume the capacitor is initially uncharged.

What is the current $I$ flowing through the battery just when the switch is closed?
A. $0$
B. $\mathcal{E}/2R$
C. $\mathcal{E}/R$
D. $2\mathcal{E}/R$

What is the current $I$ flowing through the battery after the switch has been closed for a long time?
A. $0$
B. $\mathcal{E}/2R$
C. $\mathcal{E}/R$
D. $2\mathcal{E}/R$
When switch is open:

\[
\begin{align*}
\mathcal{E} - IR_1 - \frac{Q}{C} - IR_2 &= 0 \\
I(R_1 + R_2) &= \mathcal{E} - \frac{Q}{C} \\
\frac{dQ}{dt}(R_1 + R_2) &= \mathcal{E} - \frac{Q}{C} \\
\frac{dQ}{dt} &= -\frac{1}{(R_1 + R_2)C}(Q - C\mathcal{E}) \\
Q(t) &= C\mathcal{E}(1 - e^{-t/\tau}) \quad \text{where } \tau \equiv (R_1 + R_2)C \\
\text{For } t >> \tau : \quad Q(t \to \infty) &= C\mathcal{E} \\
I(t \to \infty) &= \frac{C\mathcal{E}}{\tau} e^{-t/\tau} \bigg|_{t \to \infty} = 0
\end{align*}
\]
When switch is closed (reset $t=0$):

\[ \mathcal{E} - I_1 R_1 = 0 \]

\[ -\frac{Q}{C} - I_2 R_2 = 0 \]

\[ I_2 = \frac{dQ}{dt} \Rightarrow Q(t) = Q_0 e^{-t/\tau} \]

\[ \tau \equiv R_2 C \quad \text{and} \quad Q_0 \equiv C\mathcal{E} \]

\[ I_2 = \frac{dQ}{dt} = -\frac{C\mathcal{E}}{\tau} e^{-t/\tau} = -\frac{\mathcal{E}}{R_2} e^{-t/\tau} \]

Current through switch:

\[ I_1 = I_2 + I_s = 0 \]

\[ I_s = I_1 - I_2 = \frac{\mathcal{E}}{R_1} + \frac{\mathcal{E}}{R_2} e^{-t/\tau} \]
From webassign:

Consider the following figure, where $\mathcal{E}_1 = 6.6$ V and $\mathcal{E}_2 = 2.9$ V.

(a) What is the expected reading of the ideal ammeter?

\[ A \]

(b) What is the expected reading of the ideal voltmeter?

\[ V \]

\[ V - \mathcal{E}_1 + \mathcal{E}_2 = 0 \quad \implies V = 3.7V \]

\[ \mathcal{E}_1 - 10I_2 - 6I_1 = 0 \]

\[ \mathcal{E}_2 - 5I_3 - 6I_4 = 0 \]

\[ I_1 = I_2 + I_5 \quad I_1 = I_4 \]

\[ I_3 = I_5 + I_4 \]
\[ V - \mathcal{E}_1 + \mathcal{E}_2 = 0 \quad \Rightarrow V = 3.7V \]

\[ \mathcal{E}_1 - 10I_2 - 6I_1 = 0 \]

\[ \mathcal{E}_2 - 5I_3 - 6I_4 = 0 \]

\[ I_1 = I_2 + I_5 \quad I_1 = I_4 \]

\[ I_3 = I_5 + I_4 \]

\[ 2I_1 = I_2 + I_3 \]

\[ \mathcal{E}_1 - 10I_2 - 6I_1 = 0 \]

\[ 2(\mathcal{E}_2 - 5I_3 - 6I_4 = 0) \]

\[ \Rightarrow \mathcal{E}_1 + 2\mathcal{E}_2 - 20I_1 - 18I_1 = 0 \]

\[ I_1 = \frac{\mathcal{E}_1 + 2\mathcal{E}_2}{38} = 0.33 A \]
A charged capacitor is connected to a resistor and switch as in the figure below. The circuit has a time constant of 0.800 s. Soon after the switch is closed, the charge on the capacitor is 80.0% of its initial charge.

(a) Find the time interval required for the capacitor to reach this charge.

\[ t \, \text{s} \]

(b) If \( R = 200 \, \text{k}\Omega \), what is the value of \( C \)?

\[ \mu\text{F} \]

\[ Q(t) = Q_0 e^{-t/\tau} \]

If \( \frac{Q(t)}{Q_0} = 0.8 = e^{-t/\tau} \)

\[ \Rightarrow \ln\left(e^{-t/\tau}\right) = -\frac{t}{\tau} = \ln 0.8 = -0.223 \]

\[ t = 0.8 \, \text{s} \cdot 0.223 = 0.18 \, \text{s} \]
Review of Chapters 26-28
General formulation of capacitance

Ignoring sign:

\[ Q = C \Delta V \]

Charge on + and – terminals of capacitor

Voltage drop across capacitor terminals

Depends on geometry and material composition of capacitor

Units:

\[ C = \frac{\text{Coulombs}}{\text{Volt}} \equiv \text{Farad} \]

For parallel plate capacitors in vacuum (or air):

\[ C = A \varepsilon_0 / d \]

For parallel plate capacitors in dielectric medium:

\[ C = A \kappa \varepsilon_0 / d \]

\[ \kappa \equiv \text{dielectric constant (typically } \kappa > 1 \) \]
More complicated circuits –
Capacitors connected in parallel

\[
\Delta V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \equiv \frac{Q_{tot}}{C_{eq}}
\]

\[
Q_{tot} = Q_1 + Q_2 = C_1 \Delta V + C_2 \Delta V
\]

\[
= (C_1 + C_2) \Delta V
\]

\[
\Rightarrow C_{eq} = C_1 + C_2
\]
More complicated circuits –
Capacitors connected in series

\[ \Delta V = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} \equiv \frac{Q_{tot}}{C_{eq}} \]

\[ Q_{tot} = Q_1 = Q_2 = C_{eq} \Delta V \]

\[ \Rightarrow C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} \]
Energy storage within a capacitor

Electrons move from the plate to the wire, leaving the plate positively charged.

Separation of charges represents potential energy.

Electrons move from the wire to the plate.

$dU = \frac{q}{C} dq$

$U = \int_{0}^{Q} \frac{q}{C} dq = \frac{1}{2} \frac{Q^2}{C}$

$U = \frac{1}{2} C(\Delta V)^2$
Up to now, we have been considering equilibrium configurations of charges -- electrostatics. Now we will consider steady-state motions of charges.

Electrical current

\[ I = \frac{dQ}{dt} \]  
units: \( \frac{\text{Coulomb}}{\text{second}} \equiv \text{Ampere (A)} \)

By convention, \( +I \) denotes the direction of positive charge flow (or the opposite direction of negative charge flow).

\[ I = nq\nu_d A \]

\( \Delta x \)

\( \nu_d \Delta t \)

# charges/volume

drift velocity
Model to describe drift velocity

Newton's law: \[ m_e \frac{dv}{dt} = -eE - \frac{m_e v}{\tau} \]

term representing collisions on a time scale of \( \tau \)

Under steady-state conditions: \[ \left\langle m_e \frac{dv}{dt} \right\rangle = 0 = -eE - \left\langle \frac{m_e v}{\tau} \right\rangle \]

\[ \Rightarrow \left\langle v(t \to \infty) \right\rangle \equiv v_d = \frac{-eE \tau}{m_e} \]

Corresponding current: \[ I = n(-e)v_d A \]

\[ I = \frac{ne^2 \tau}{m_e} AE \]
A potential difference $\Delta V = V_b - V_a$ maintained across the conductor sets up an electric field $\vec{E}$, and this field produces a current $I$ that is proportional to the potential difference.

\[
\Delta V = E \ell
\]

\[
I = \frac{ne^2\tau}{m_e} AE = \frac{ne^2\tau A}{m_e \ell} \Delta V
\]

\[
\sigma \equiv \frac{1}{\rho}
\]

\[
I = \frac{1}{R} \Delta V
\]

\[
R = \rho \frac{\ell}{A}
\]
Switch open, no current flowing, battery storing energy

Switch closed, current flowing, battery discharging
Resistors in series:

\[ \Delta V - IR_1 - IR_2 = 0 \]

\[ I = \frac{\Delta V}{R_1 + R_2} \]

\[ R_{\text{series}} = R_1 + R_2 \]
Resistors in parallel

ΔV - I₁R₁ = 0
ΔV - I₂R₂ = 0

I = I₁ + I₂ = \frac{ΔV}{R₁} + \frac{ΔV}{R₂}

ΔV = I \left( \frac{1}{R₁} + \frac{1}{R₂} \right) ≡ IR_{Parallel}

R_{Parallel} = \frac{1}{\frac{1}{R₁} + \frac{1}{R₂}}
Step-by-step reduction of resistor circuit to its equivalent
Circuit analysis – Kirchhoff’s rules

Conservation of charge (current) at a junction:

\[ \sum_{\text{junction}} I = 0 \]

\[ I_1 - I_2 - I_3 = 0 \]
Circuit analysis – Kirchhoff’s rules
Conservation of potential around a closed circuit:

\[ \sum \Delta V = 0 \]

\[ \Delta V_1 + \Delta V_2 + \Delta V_3 = 0 \]

Sign conventions: \( \Delta V = V_b - V_a \)
Example:

\[ \mathcal{E}_1 = 6.0 \text{ V} \]

\[ R_2 = 10 \ \Omega \quad R_1 = 8.0 \ \Omega \]

\[ \mathcal{E}_2 = 12 \text{ V} \]

\[ I \]

\[ \mathcal{E}_1 - IR_1 - \mathcal{E}_2 - IR_2 = 0 \]

Solve for \( I \):

\[ I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} = \frac{6 - 12}{8 + 10} = -0.33A \]

\( I < 0 \) means:

A. The circuit is wrong (cannot exist or will blow up).

B. The current flows opposite the arrow.
Example:

\[-14 - 2I_3 - 4I_2 = 0\]
\[10 - 6I_1 - 2I_3 = 0\]
\[I_1 + I_2 = I_3\]

\[I_1 = 2\,\text{A}\]
\[I_2 = -3\,\text{A}\]
\[I_3 = -1\,\text{A}\]