PHY 114 A General Physics II
11 AM-12:15 PM Olin 101
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Main topics today (Chapt 29):

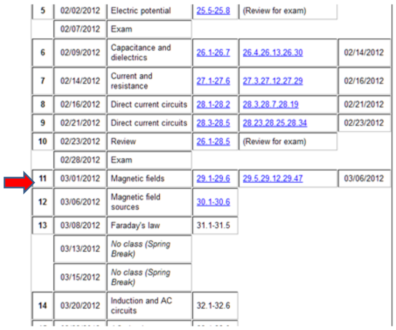
1. Magnetic field $\vec{B}$
- ***history*:**
- ***applications:***

2. Lorentz force law for charged particles moving in electric and magnetic fields

3. Magnetic force on a current-carrying wire

4. Torque on a current loop

5. Hall effect to measure B electrically



5	02/02/2012	Electric potential	25.5-25.8	(Review for exam)	
	02/07/2012	Exam			
6	02/09/2012	Capacitance and dielectrics	26.1-26.7	26.4,26.13,26.30	02/14/2012
7	02/14/2012	Current and resistance	27.1-27.6	27.3,27.12,27.29	02/16/2012
8	02/16/2012	Direct current circuits	28.1-28.2	28.3,28.7,28.19	02/21/2012
9	02/21/2012	Direct current circuits	28.3-28.5	28.23,28.25,28.34	02/23/2012
10	02/23/2012	Review	26.1-28.5	(Review for exam)	
	02/28/2012	Exam			
11	03/01/2012	Magnetic fields	29.1-29.6	29.5,29.12,29.47	03/06/2012
12	03/06/2012	Magnetic field sources	30.1-30.6		
13	03/08/2012	Faraday's law	31.1-31.5		
	03/13/2012	*No class (Spring Break)*			
	03/15/2012	*No class (Spring Break)*			
14	03/20/2012	Induction and AC circuits	32.1-32.6		

Remember to send in your chapter reading questions…

Brief history of Magnetism

1300 BC compass used to help travellers find directions in China.

800 BC Greeks observe that pieces of magnetite (Fe_3O_4) attract pieces of iron (Fe).

1269 AD Using a compass to point along field lines outside a spherical magnet, de Maricourt (France) observes 1 source and 1 sink (poles)

1269-1820 AD Everybody busy with wars and plagues for 550 years. No time for magnetism.

1820 AD Oersted (Denmark) observes that electric current in a wire deflects a nearby compass. Faraday(England), Henry(US), Maxwell (England) work on the physics of magnetism.

Late 1800's: motors, generators, power grids, electric lighting, electric refrigeration invented. GE, Siemens, Westinghouse, etc make these products widely available.

1889 Heaviside derives a key equation relating magnetic field B to force F on a moving charge or current.

1899 H. A. Lorentz derives the same equation, now known as the Lorentz Equation.

1900: EE programs developed at MIT, Stanford.

Applications of magnetic forces

"electric" motor, "electric" car, "electric" generator,
"electric" drill, solenoid actuator.

tape recorder, magnetic hard drive, CRT for oscilloscopes and TV
magnetic levitation for trains

science (Nmr, mass spectrometer)
medicine (MRI; magnetic navigation systems for catheter)

Recall what we did for $\vec{E}$

Electric field $\vec{E}$ has magnitude and direction
Source of E is charged particles (e^- , Na^+,....)

Potential energy $\Delta U = -q_0 \int_A^B \vec{E} \cdot d\vec{s}$

Voltage difference $\Delta V = \Delta U / q_0 = -\int_A^B \vec{E} \cdot d\vec{s}$

Force $\vec{F} = q\vec{E}$

electric charge$_1$ $\Leftrightarrow$ $\vec{E}$ $\Leftrightarrow$ electric charge$_2$

Math for F is easy

"Electric" cars are propelled by magnetic force

Motor in the Tesla electric car:
3-phase, 4-pole induction motor,
248 hp (185 kW).
Maximum torque: 270 N·m between
0 and 6,000 rpm

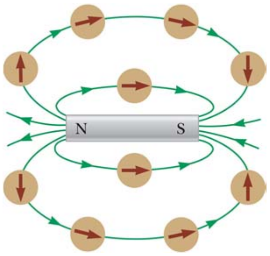
Magnetic Field $\vec{\mathbf{B}}$

- B has both magnitude and direction (vector)
- direction is given by the direction toward ` which the north pole of a compass needle points
- field lines can be traced out by a compass or by metal filings

Section 29.1

Magnetic Field Lines around a bar magnet

- A compass shows the direction of the field lines.
- The field lines outside a bar magnet point from its North pole to the its South pole. Think of the N pole as the source of the field lines.
- compass points toward the south pole of the magnet.
- Compass doesn't measure field strength

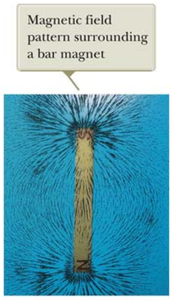


Section 29.1

demos with magneprobe, iron filings

Magnetic Field Lines, Bar Magnet

- Iron filings align along the magnetic field lines.
- The direction of the field is given by the direction in which the north pole of a compass points.

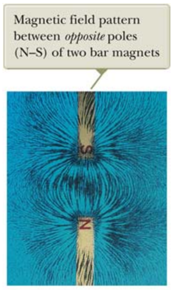

Magnetic field pattern surrounding a bar magnet

(a)

Section 29.1

Magnetic Field Lines, Opposite Poles

- Iron filings are used to show the pattern of the electric field lines.
- The direction of the field is the direction a north pole would point.
  - Compare to the electric field produced by an electric dipole

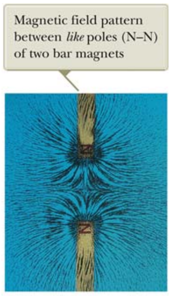

Magnetic field pattern between *opposite* poles (N–S) of two bar magnets

(b)

Section 29.1

Magnetic Field Lines, Like Poles

- Iron filings show the alignment of the magnetic field lines.
- The direction of B is the direction a north pole would point.
  - Compare to the electric field produced by like charges


Magnetic field pattern between *like* poles (N–N) of two bar magnets

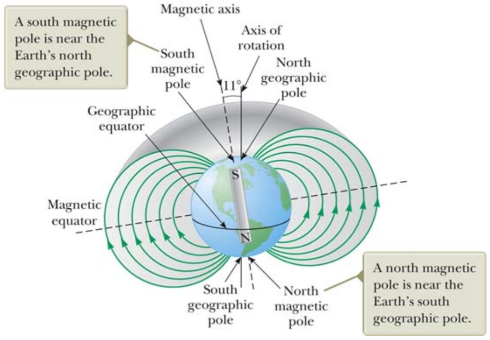
(c)

Section 29.1

Earth's Geographic & Magnetic Poles

- Earth's magnetic field behaves like there's a gigantic bar magnet inside Earth.
- The magnet's S pole is near Earth's geographical N pole. (Careful ! Confusing!)
- On surface of Earth, the magnetic field lines begin near the S geographic pole and end near Earth's N geographic pole.
- A compass needle points in the direction of B, so it points toward the N geographic pole.

Earth's Magnetic Field

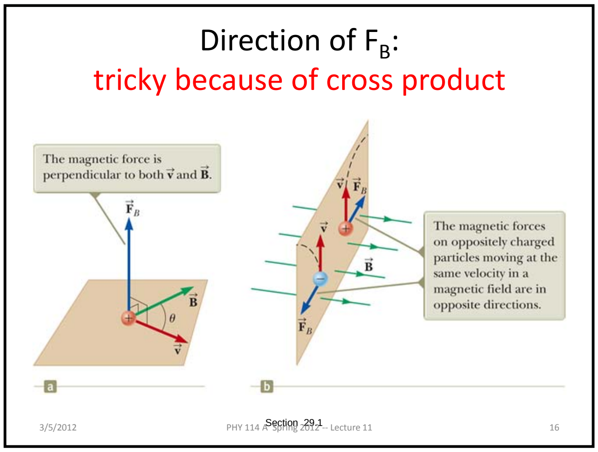


Force on a Charge Moving in a Magnetic Field $\vec{B}$

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

$\vec{\mathbf{F}}_B$ is the magnetic force
q is the charge
$\vec{\mathbf{v}}$ is the velocity of the charge
$\vec{\mathbf{B}}$ is the magnetic field

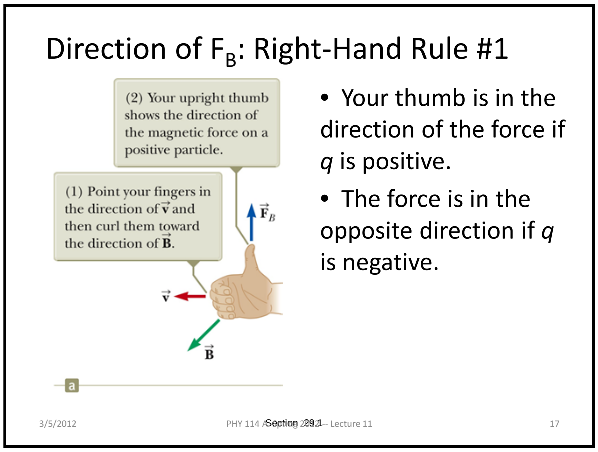
This equation defines B, just as $\vec{F}_E = q\vec{E}$ defines $\vec{E}$.



Direction of F_B: tricky because of cross product

The magnetic force is perpendicular to both $\vec{v}$ and $\vec{B}$.

The magnetic forces on oppositely charged particles moving at the same velocity in a magnetic field are in opposite directions.

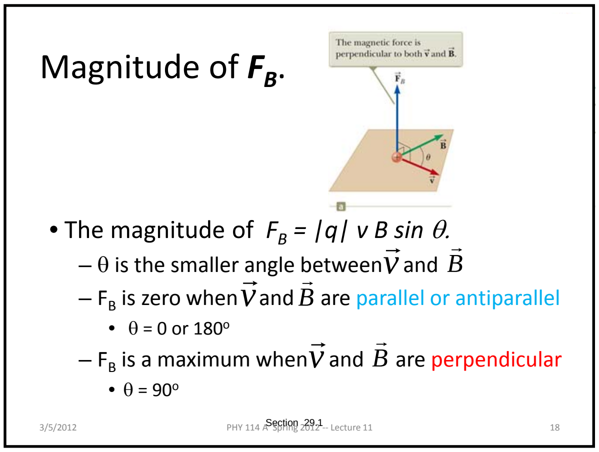


Direction of F_B: Right-Hand Rule #1

(2) Your upright thumb shows the direction of the magnetic force on a positive particle.

(1) Point your fingers in the direction of $\vec{v}$ and then curl them toward the direction of $\vec{B}$.

- Your thumb is in the direction of the force if *q* is positive.
- The force is in the opposite direction if *q* is negative.



Magnitude of F_B.

The magnetic force is perpendicular to both $\vec{v}$ and $\vec{B}$.

- The magnitude of $F_B = |q|\ v B \sin\theta$.
  - θ is the smaller angle between $\vec{V}$ and $\vec{B}$
  - F_B is zero when $\vec{V}$ and $\vec{B}$ are parallel or antiparallel
    - $\theta = 0$ or 180°
  - F_B is a maximum when $\vec{V}$ and $\vec{B}$ are perpendicular
    - $\theta = 90^\circ$

The SI unit of Magnetic Field is the Tesla

- *Recall* $F = qvB\sin(\theta)$

  $\text{so } B = \dfrac{F}{qv\sin(\theta)}$

- The SI unit of magnetic field is the tesla (T).

  $$T = \frac{N}{C\cdot(m/s)} = \frac{N}{A\cdot m} = \frac{Wb}{m^2}$$

  Wb is a weber
- The gauss (G) is also a commonly used unit:

  1 T = 10^4 G

Section 29.1

Typical values of T

Best superconducting lab magnet	30 T
Medical MRI magnet (supercon)	5
electric motor	2
Small bar magnet	0.01
At surface of earth	$0.5\text{x}10^{-4}$
Inside human brain	10^{-13}
————————————————	
Surface of a magnetar (1992)	10^{10}

Demo deflection of cathode ray

Differences Between Electric and Magnetic Fields

- Direction of force
  - The electric force acts along the direction of E.
  - The magnetic force acts perpendicular to B.
- Motion
  - The electric force acts on a charged particle regardless of whether the particle is moving.
  - The magnetic force acts on a charged particle only when the particle is in motion.
- Work
  - The electric force does work in displacing a charged particle.
  - The magnetic force associated with a steady magnetic field does no work when a particle is displaced.
    - This is because the force is perpendicular to the displacement of its point of application.

Section 29.1

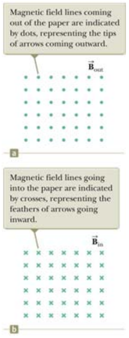
Work in constant B, cont.

- The kinetic energy of a charged particle moving through a constant ***B*** cannot be altered by ***B*** alone.
- When a charged particle moves with a given ***v*** through a constant magnetic field, the field can alter the direction of the ***v***, but not the speed or the kinetic energy.

Section 29.1

Depiction of a uniform magnetic field.

- When vectors are perpendicular to the page, dots and crosses are used.
  - dots represent the points of arrows coming out of the page.
  - crosses represent the tail feathers on the arrows going into the page.
- The same notation applies to other vectors.



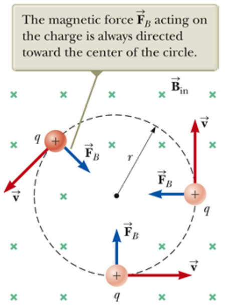
Magnetic field lines coming out of the paper are indicated by dots, representing the tips of arrows coming outward.

Magnetic field lines going into the paper are indicated by crosses, representing the feathers of arrows going inward.

Section 29.2

Moving charged particles in a uniform B field travel in circles or helices.

- Simplest case: $q\vec{v} \perp \vec{B}$
- $\vec{F}$ always points toward the center of a circular path.
- $\vec{F}$ causes a centripetal acceleration of the particle
- direction of ***v*** changes, but not its magnitude
- magnitude of F = qvB


The magnetic force $\vec{F}_B$ acting on the charge is always directed toward the center of the circle.

Section 29.2

Find the radius of the circle

- uniform circular motion requires a mechanical force: $F = \frac{mv^2}{r}$
- Equating the magnetic and centripetal forces:

$$F_B = qvB = \frac{mv^2}{r}$$

- Solve for r:

$$r = \frac{mv}{qB}$$

  - r is proportional to the linear momentum of the particle and inversely proportional to the magnetic field and the charge.

Section 29.2

Bending of an Electron Beam in a uniform B-field

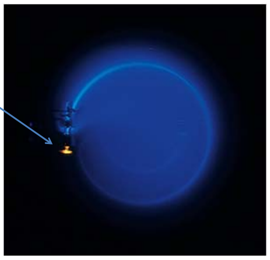
- A beam of electrons with velocity ***v*** is generated by an "electron gun"

$$K = q\Delta V = \tfrac{1}{2}mv^2$$

- electrons then enter a uniform magnetic field that is perpendicular to ***v***.

- The electrons trace out in a curved path but their speed is unchanged.

- Conservation of energy gives *v*



Class question: direction of B?

Section 29.2

More About Motion of the Particle in uniform B (makes a circle)

- The angular speed of the particle is

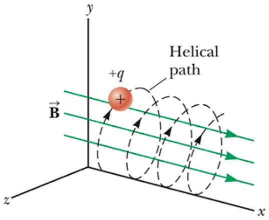
$$\omega = \frac{v}{r} = \frac{qB}{m}$$

  - The angular speed, ω, is also referred to as the **cyclotron frequency.**

- The period of the motion is

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

Motion of a charged particle if v is not perpendicular to B



- Assume ***B*** points along x
- If ***v*** of the charged particle is not $\perp$ ***B***, break up v into two parts.

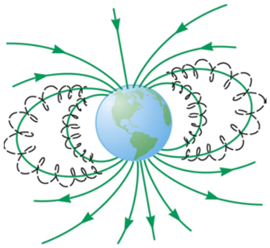
  1) a part parallel to B; $F_B = 0$.

  2) a part $\perp$ to B. This gives non-zero F_B. Particles moves in a circle in the yz plane.

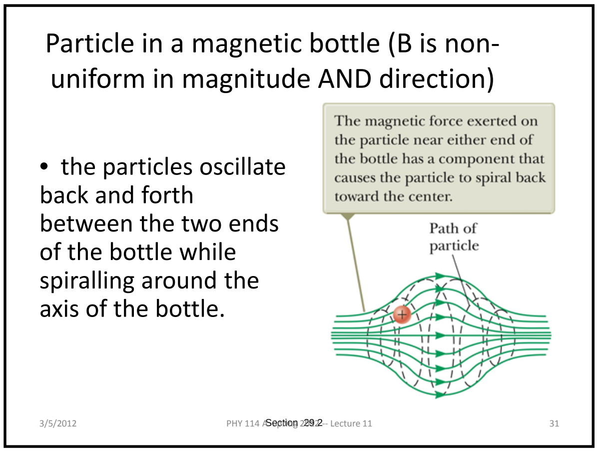
The overall motion is a helix.

- radius of helix is same as previous equation for r, but with *v* replaced by $v_\perp = \sqrt{v_y^2 + v_z^2}$

Van Allen Radiation Belts



- The Van Allen radiation belts consist of charged particles surrounding the Earth in doughnut-shaped regions.
- The particles are trapped by the Earth's nonuniform magnetic field.
- The particles spiral from pole to pole.
  - May result in auroras



Particle in a magnetic bottle (B is non-uniform in magnitude AND direction)

- the particles oscillate back and forth between the two ends of the bottle while spiralling around the axis of the bottle.

The magnetic force exerted on the particle near either end of the bottle has a component that causes the particle to spiral back toward the center.

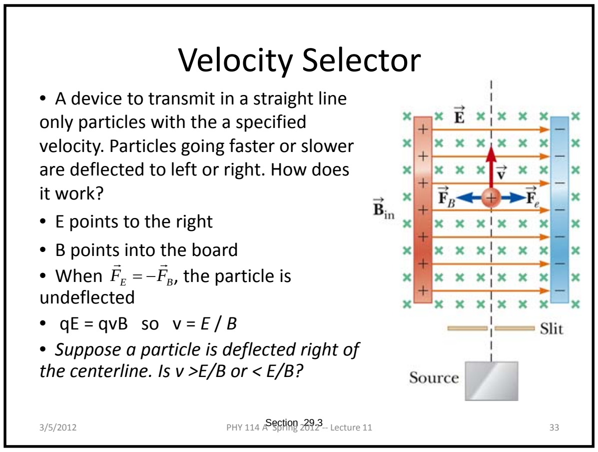
Charged Particles Moving in Electric and Magnetic Fields

- In many applications, charged particles will move in the presence of both electric and magnetic fields.
- The total force is called the Lorentz force

(O. Heaviside, 1889; H. A. Lorentz, 1899).

$$\vec{F} = \vec{F}_E + \vec{F}_B$$

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

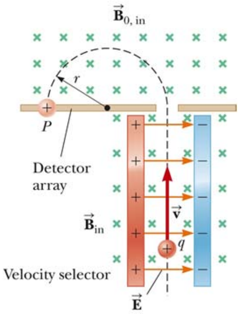


Velocity Selector

- A device to transmit in a straight line only particles with the a specified velocity. Particles going faster or slower are deflected to left or right. How does it work?
- E points to the right
- B points into the board
- When $\vec{F}_E = -\vec{F}_B$, the particle is undeflected
- qE = qvB so v = *E* / *B*
- *Suppose a particle is deflected right of the centerline. Is v >E/B or < E/B?*

Mass Spectrometer

- A mass spectrometer separates ions according to their mass-to-charge ratio.
- In one design, a beam of ions passes through a velocity selector and enters a second magnetic field.
- After entering the second magnetic field, the ions move in a semicircle of radius r before striking a detector at P.
- If the ions are positively charged, they deflect to the left.
- If the ions are negatively charged, they deflect to the right.



Mass Spectrometer, cont.

- The mass-to-charge ratio (m/q) can be determined by measuring the radius of curvature r. But

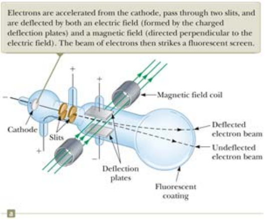
$$r = \frac{mv}{qB_0} \quad \text{so} \quad \frac{m}{q} = \frac{rB_o}{v} = \frac{rB_oB}{E}$$

- In practice, you can measure the masses of various isotopes of a given atom, with all the ions carrying the same charge.
  - The mass ratios can be determined even if the charge is unknown.

Thomson's *e*/*m* Experiment

(J.J. Thomson, "Cathode Rays", *Philosophical Magazine*, 44, 293-305 (1897))

- "Cathode Rays" (electrons) are generated with a heated cathode and nearby anode with a hole (electron gun).
- They are deflected by $\perp$ electric and magnetic fields.
- The beam of electrons strikes a fluorescent screen.
- e/m is determined from known values of v, E, and B


Electrons are accelerated from the cathode, pass through two slits, and are deflected by both an electric field (formed by the charged deflection plates) and a magnetic field (directed perpendicular to the electric field). The beam of electrons then strikes a fluorescent screen.

Cyclotron (E. O. Lawrence and M. S. Livingston, 1934)

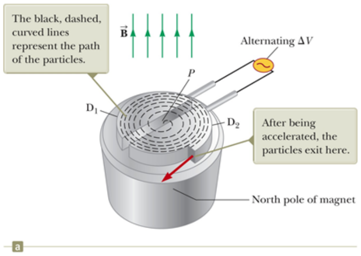
- A **cyclotron** is a device that can accelerate charged particles to very high speeds.
- The energetic particles produced are used to bombard atomic nuclei and thereby produce reactions.
- Key idea: give charged particles an electrical kick of energy

$$\Delta K = q\Delta V$$

Use a transverse B to make them go in a circle.

The radius of the circle increases with velocity of particle, but the period of circular motion is independent of v

Cyclotron, cont'd



- D_1 and D_2 are called *dees* because of their shape.
- A high frequency alternating voltage is applied to the dees.
- A uniform magnetic field is perpendicular to them.
- A positive ion is released near the center and moves in a semicircular path.

Cyclotron, final

- The frequency of the electrical voltage is adjusted to be the same as the frequency of circular travel:

$$\nu_{circ\_travel} = \nu_{oscillator}$$

$$\frac{qB}{2\pi m} = \nu_{oscillator}$$

The cyclotron's operation is based on the fact that this frequency is independent of the speed and radius of the particle. Clever observation (Nobel prize).

- However, the energy of the particle depends on R: $K = \frac{1}{2}mv^2 = \frac{q^2B^2R^2}{2m}$

- When the energy of the ions in a cyclotron exceeds about 20 MeV, relativistic effects come into play.
- Most accelerators currently used in research are *synchrotrons.*

Magnetic Force on a Current Carrying Conductor (think “electric” motor)

- When a current-carrying wire is placed in a magnetic field, it experiences a force
- The direction of the force is given by the right-hand rule.

Force on a Wire, equation

- The magnetic force exerted on one moving charge in the wire is

$$\vec{\mathbf{F}} = q\vec{\mathbf{v}}_d \times \vec{\mathbf{B}}$$

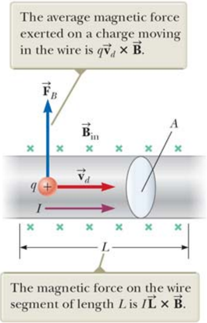
- Current **I** is given by (eq. 27.4)

$$I = \frac{\Delta Q}{\Delta t} = nqv_d A$$

- The total force is the product of the force on one charge and the number of charges in a length L of wire:

$$\vec{\mathbf{F}} = \left(q\vec{\mathbf{v}}_d \times \vec{\mathbf{B}}\right) nAL$$

Substituting **I** for nqv$_d$A gives



The average magnetic force exerted on a charge moving in the wire is $q\vec{\mathbf{v}}_d \times \vec{\mathbf{B}}$.

The magnetic force on the wire segment of length L is $I\vec{\mathbf{L}} \times \vec{\mathbf{B}}$.

Force on a straight wire, Equation cont.

$$\vec{\mathbf{F}}_B = I\vec{\mathbf{L}} \times \vec{\mathbf{B}}$$

$\vec{\mathbf{L}}$ is a vector that points in the direction of the current.

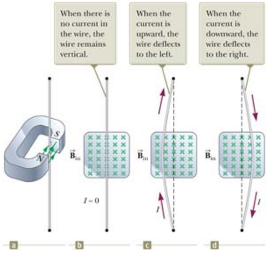
- Its magnitude is the length *L* of the segment.

$\vec{\mathbf{B}}$ is the magnetic field.

Demo: Force on a Wire

- The magnetic field is into the page
- The current is up the page
- The force is to the left
- The wire deflects to the left

•DO DEMO


When there is no current in the wire, the wire remains vertical.

When the current is upward, the wire deflects to the left.

When the current is downward, the wire deflects to the right.

Force on a Wire of <u>Arbitrary</u> Shape

•Consider a small segment of the wire,

$$d\vec{\mathbf{s}}$$

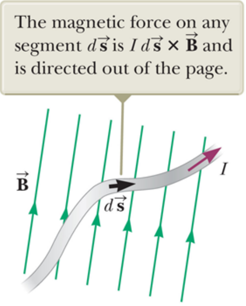
•The force exerted on this segment is

$$d\vec{\mathbf{F}}_B = I\, d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$$

- The total force is

$$\vec{\mathbf{F}}_B = I \int_{\mathbf{a}}^{b} d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$$

- Next: build a motor. Motor needs torque, not force.

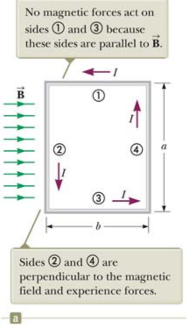

The magnetic force on any segment $d\vec{\mathbf{s}}$ is $I\, d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$ and is directed out of the page.

Torque on a rectangular current loop

- The loop carries a current I in a uniform magnetic field.
- sides 1 & 3: the wires are parallel to the field so

$$\vec{\mathbf{L}} \times \vec{\mathbf{B}} = 0$$

- F = 0 on sides 1 & 3
- torque = 0
- Now look at sides 2 and 4.

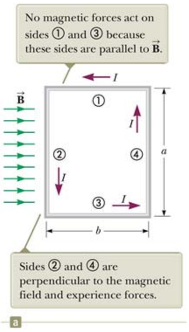

No magnetic forces act on sides ① and ③ because these sides are parallel to $\vec{\mathbf{B}}$.

Sides ② and ④ are perpendicular to the magnetic field and experience forces.

Torque on sides 2 and 4 of the current loop,

- The magnitude of the magnetic force on side 2 and side 4 is :

  $F_2 = F_4 = I\,a\,B$
- The direction of F_2 is out of the page.
- The direction of F_4 is into the page.
- Thus, net force = 0
- BUT net torque is NOT zero.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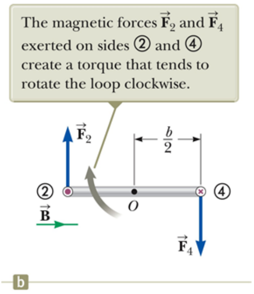
Sides ② and ④ are perpendicular to the magnetic field and experience forces.

Torque on a Current Loop, 3

- The forces are equal in magnitude but in opposite directions
- They produce a torque around an axis through O .


The magnetic forces $\vec{\mathbf{F}}_2$ and $\vec{\mathbf{F}}_4$ exerted on sides ② and ④ create a torque that tends to rotate the loop clockwise.

Torque on a Current Loop, equation

- Recall the equation for torque (Eq 11.1):

$$\vec{\tau} = \vec{r} \times \vec{F}$$

- The magnitude of the torque is:

$$\tau_{\max} = F_2 \frac{b}{2} + F_4 \frac{b}{2} = (IaB)\frac{b}{2} + (IaB)\frac{b}{2}$$

$$= IabB = IAB$$

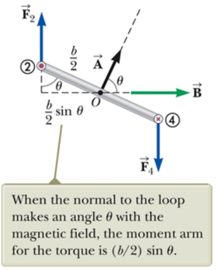
(ab=area A of loop)

This orientation of the loop gives the maximum torque. Plane of the loop is parallel to the magnetic field. If ***A*** is a vector normal to the loop, ***A*** is perpendicular to ***B***.

Torque on a Loop, General orientation

- If B makes an angle of θ< 90º with a line perpendicular to the plane of the loop, the lever arm for the torques is reduced to (b/2)sin θ so

$$\tau = IAB\sin\theta$$



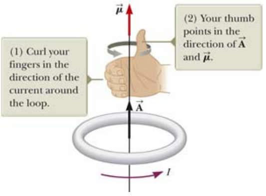
When the normal to the loop makes an angle θ with the magnetic field, the moment arm for the torque is $(b/2)\sin\theta$.

Torque on a Current Loop, Summary

- The torque has a maximum value when the field is perpendicular to the normal to the plane of the loop. $\vec{\tau} = I\vec{\mathbf{A}} \times \vec{\mathbf{B}}$
- The torque is zero when the field is parallel to the normal to the plane of the loop.
- $\vec{\mathbf{A}}$ *is perpendicular to the plane of the loop and has a magnitude equal to the area of the loop.*

Direction of A, the normal to a current loop

- The right-hand rule is used to determine the direction of $\vec{\mathbf{A}}$
- Curl your fingers in the direction of the current in the loop.
- Your thumb points in the direction of $\vec{\mathbf{A}}$.



(1) Curl your fingers in the direction of the current around the loop.

(2) Your thumb points in the direction of $\vec{\mathbf{A}}$ and $\vec{\mu}$.

Magnetic Dipole Moment

- The product $I\,\vec{\mathbf{A}}$ is defined as the **magnetic dipole moment**, $\vec{\mu}$, of the loop.
  - Often called the magnetic moment
- SI units: A · m^2
- Torque in terms of magnetic moment:

$$\vec{\tau} = \vec{\mu} \times \vec{\mathbf{B}}$$

  - Analogous to $\vec{\tau} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$ for electric dipole
  - Valid for any orientation of the field and the loop
  - Valid for a loop of any shape

Potential Energy of magnetic dipole

- The potential energy of a magnetic dipole in a magnetic field depends on the orientation of the dipole in the magnetic field given by

$$U = -\mu \cdot \mathbf{B}$$
$$U = -\mu B \cos\theta$$

U_{min} = $-\mu$B and occurs when the dipole moment is in the same direction as the field (compass needle).

U_{max} = $+\mu$B and occurs when the dipole moment is in the direction opposite the field.

What is the force on the magnetic dipole?

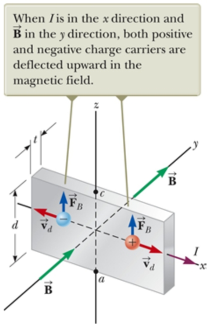
Hall Effect: a way to measure B electrically

(Edwin Hall, 1879)

- When a current carrying conductor is placed in a magnetic field, a potential difference is generated in a direction perpendicular to both the current and the magnetic field.
- charge carriers are deflected to one side of the conductor as a result of the magnetic forces they experience.

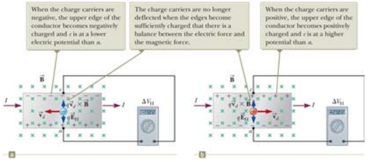
Hall Voltage

- This shows an arrangement for observing the Hall effect.
- The Hall voltage is measured between points *a* and *c*.


When I is in the x direction and $\vec{B}$ in the y direction, both positive and negative charge carriers are deflected upward in the magnetic field.

Section 29.6

Hall Voltage, cont.


When the charge carriers are negative, the upper edge of the conductor becomes negatively charged and *c* is at a lower electric potential than *a*.

The charge carriers are no longer deflected when the edges become sufficiently charged that there is a balance between the electric force and the magnetic force.

When the charge carriers are positive, the upper edge of the conductor becomes positively charged and *c* is at a higher potential than *a*.

- When the charge carriers are electrons, they experience an upward magnetic force which deflects them upward, so an excess of positive charge is left at the lower edge.
- This accumulation of charge establishes an electric field in the conductor.
- It increases until the electric force balances the magnetic force.
- If the charge carriers are positive, an excess of negative charges accumulates on the lower edge.

Section 29.6

Hall Voltage: cont'd

- $\Delta V_H = E_H d = v_d B d$

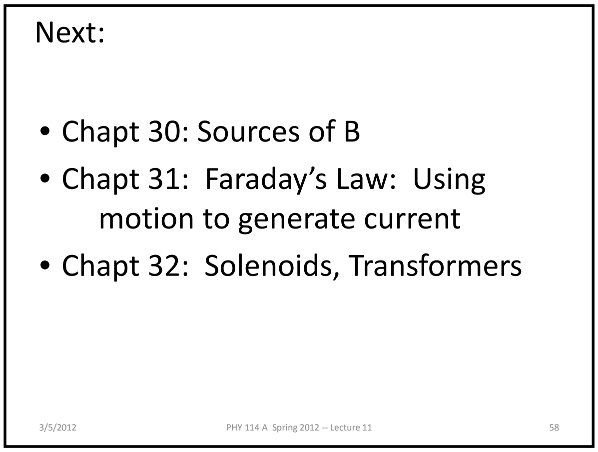
  d is the width of the conductor

  v_d is the drift velocity

  If v_d and d are known, we can measure B

  If B and d are known, we can determine the number and type of charge carriers in a test sample.

Section 29.6



Next:

- Chapt 30: Sources of B
- Chapt 31: Faraday’s Law: Using motion to generate current
- Chapt 32: Solenoids, Transformers