Plan for Lecture 12 (Chapters 30):

Sources of Magnetic fields

1. “Permanent” magnets

2. Biot-Savart Law; magnetic fields from a current-carrying wire

3. Ampere’ Law

4. Magnetic fields in a solenoid
<table>
<thead>
<tr>
<th>Date</th>
<th>Topic</th>
<th>Pages</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>02/07/2012</td>
<td>Exam</td>
<td></td>
<td></td>
</tr>
<tr>
<td>02/09/2012</td>
<td>Capacitance and dielectrics</td>
<td>26.1-26.7</td>
<td>02/14/2012</td>
</tr>
<tr>
<td>02/14/2012</td>
<td>Current and resistance</td>
<td>27.1-27.6</td>
<td>02/16/2012</td>
</tr>
<tr>
<td>02/16/2012</td>
<td>Direct current circuits</td>
<td>28.1-28.2</td>
<td>02/21/2012</td>
</tr>
<tr>
<td>02/21/2012</td>
<td>Direct current circuits</td>
<td>28.3-28.5</td>
<td>02/23/2012</td>
</tr>
<tr>
<td>02/23/2012</td>
<td>Review</td>
<td>26.1-28.5</td>
<td>(Review for exam)</td>
</tr>
<tr>
<td>02/28/2012</td>
<td>Exam</td>
<td></td>
<td></td>
</tr>
<tr>
<td>03/01/2012</td>
<td>Magnetic fields</td>
<td>29.1-29.6</td>
<td>03/06/2012</td>
</tr>
<tr>
<td>03/06/2012</td>
<td>Magnetic field sources</td>
<td>30.1-30.6</td>
<td>03/08/2012</td>
</tr>
<tr>
<td>03/08/2012</td>
<td>Faraday’s law</td>
<td>31.1-31.5</td>
<td>03/20/2012</td>
</tr>
<tr>
<td>03/13/2012</td>
<td>No class (Spring Break)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>03/15/2012</td>
<td>No class (Spring Break)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>03/20/2012</td>
<td>Induction and AC circuits</td>
<td>32.1-32.6</td>
<td></td>
</tr>
<tr>
<td>03/22/2012</td>
<td>AC circuits</td>
<td>33.1-33.9</td>
<td></td>
</tr>
<tr>
<td>03/27/2012</td>
<td>Electromagnetic waves</td>
<td>34.1-34.3</td>
<td></td>
</tr>
</tbody>
</table>
Review of magnetic forces:

\[ \mathbf{F}_B = q \mathbf{v} \times \mathbf{B} \]

\[ d\mathbf{F}_B = I \, ds \times \mathbf{B} \]

**Right hand rule**

http://www.fotosearch.com/CSP760/k7601927/
Vector cross product

\[ \hat{i} \times \hat{j} = \hat{k} \]
\[ \hat{j} \times \hat{k} = \hat{i} \]
\[ \hat{k} \times \hat{i} = \hat{j} \]
\[ \hat{j} \times \hat{i} = -\hat{k} \]
Sources of magnetic fields \( \mathbf{B} \)

Permanent magnet materials – Fe, \( \text{Fe}_2\text{O}_3 \), Co, Ni, alloys

Internal atomic level magnet dipole moments \( \vec{\mu} \)

\[ \Rightarrow \text{Energy incentive for neighboring magnetic dipoles to align at temperatures below Curie temperature.} \]

Visualization of intrinsic spin magnetic moment of electron.

In an unmagnetized substance, the atomic magnetic dipoles are randomly oriented.

When an external field $\vec{B}$ is applied, the domains with components of magnetic moment in the same direction as $\vec{B}$ grow larger, giving the sample a net magnetization.

As the field is made even stronger, the domains with magnetic moment vectors not aligned with the external field become very small.

“Permanent” magnets controlled by temperature and external magnetic fields.
Magnetic fields generated by moving charge:

**Electrical field generated by charge distribution**

\[
\rho(r) = \sum_{i, r_i \approx r} \frac{Q_i}{\Delta V_i}
\]

\[
E(r) = \frac{1}{4\pi \varepsilon_0} \int \rho(r') \frac{r - r'}{|r - r'|^3} d^3 r'
\]

**Magnetic field generated by current distribution**

\[
J(r) = \sum_{i, r_i \approx r} \frac{Q_i v_i}{\Delta V_i}
\]

\[
B(r) = \frac{\mu_0}{4\pi} \int J(r') \times \frac{r - r'}{|r - r'|^3} d^3 r'
\]
Biot-Savart Law:

Units: $B \to T$ (Tesla)

$J = \frac{I}{\text{area}} \to A/m^2$

Constant: $\mu_0$ “permeability” of free space

$4\pi \times 10^{-7} \text{Tm/A}$

$$B(r) = \frac{\mu_0}{4\pi} \int J(r') \times \frac{r - r'}{|r - r'|^3} d^3r'$$

For thin wire of constant current $I$, where $s$ denotes direction along wire:

$$B(r) = \frac{\mu_0 I}{4\pi} \int ds \times \frac{r - s}{|r - s|^3}$$
Magnetic field produced by straight wire:

\[ |d\vec{s}| = dx \]

\[ \mathbf{B}(r_P) = \hat{z} \frac{\mu_0 I}{4\pi} \int_{x_{\min}}^{x_{\max}} \frac{adx}{(a^2 + x^2)^{3/2}} \]

\[ = \hat{z} \frac{\mu_0 I}{4\pi a} \int_{\theta_1}^{\theta_2} \cos \theta \, d\theta \]

\[ \mathbf{B}(r_P) = \hat{z} \frac{\mu_0 I}{4\pi a} \left( \sin \theta_1 - \sin \theta_2 \right) \]
\[ \mathbf{B}(\mathbf{r}_P) = \hat{z} \frac{\mu_0 I}{4\pi a} \left( \sin \theta_1 - \sin \theta_2 \right) \]

For \( x_{\text{min}} \to -\infty, \ x_{\text{max}} \to \infty \):

\[ \theta_1 \to -90^\circ, \ \theta_2 \to 90^\circ \]

\[ \mathbf{B}(\mathbf{r}_P) = \hat{z} \frac{\mu_0 I}{2\pi a} \]
Consider 2 (infinitely long) wires. Vote for the least incorrect answer:
A. The two wires cancel each other completely
B. The two wires generate exactly twice the magnet field compared to a single wire
C. The two wires attract each other.
D. The two wires repel each other.
Consider 2 (infinitely long) wires. Vote for the least incorrect answer:
A. The two wires cancel each other completely
B. The two wires generate exactly twice the magnet field compared to a single wire
C. The two wires attract each other.
D. The two wires repel each other.
\[ \mathbf{F} = \ell \mathbf{I} \times \mathbf{B} \]

\( \Rightarrow \) wires repel each other
Magnetic field produced by wire loop:

\[ \mathbf{B}(\mathbf{r}_P) = \hat{x} \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{a^2 \, d\theta}{(a^2 + x^2)^{3/2}} \]

\[ = \hat{x} \frac{\mu_0 I}{4\pi} \frac{2\pi a^2}{(a^2 + x^2)^{3/2}} = \hat{x} \frac{\mu_0 I a^2}{2(a^2 + x^2)^{3/2}} \]
Easier way for situations with high symmetry
– Ampere’s Law

\[ \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{in} \]

In this case
\[ \oint \mathbf{B} \cdot d\mathbf{s} = 2\pi a B = \mu_0 I \]
\[ \Rightarrow B = \frac{\mu_0 I}{2\pi a} \]
Example of Ampere’s law:

The figure below shows a wire coming out of the screen. Which of the paths for \[ \int \mathbf{B} \cdot ds \] has the smallest magnitude?

A. a
B. b
C. c
D. d
Magnetic field in a solenoid

Ideal solenoid
Magnetic field inside ideal solenoid:

Ampère’s law applied to the rectangular dashed path can be used to calculate the magnitude of the interior field.

\[ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \]

\[ \int \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{in} \]

\[ \int \mathbf{B} \cdot d\mathbf{s} = B \ell = \mu_0 NI \]

\[ \Rightarrow B = \frac{\mu_0 NI}{\ell} \]

Ampère’s law applied to the circular path whose plane is perpendicular to the page can be used to show that there is a weak field outside the solenoid.
\[ B = \frac{\mu_0 NI}{\ell} \]

\[ B \approx 21 \text{ T} \]
(Experimental MRI at FSU)

http://www.magnet.fsu.edu/education/tutorials/magnetacademy/mri/images/mri-scanner.jpg
MRI signal – detects H nuclei using the magnetic moment of H – $\mu_H$.

$$E = -\mu_H \cdot B$$
Summary:

Ampere's law:

Integral form:  \[ \oint \mathbf{B} \cdot ds = \mu_0 I_{in} \]

Differential form:  \[ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \]

Gauss's law:

Integral form:  \[ \int \mathbf{E}(\mathbf{r}) \cdot d\mathbf{A} = \frac{Q_{in}}{\varepsilon_0} \]

Differential form:  \[ \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \]

Note:

\[ \int \mathbf{B}(\mathbf{r}) \cdot d\mathbf{A} = 0 \]

\[ \nabla \cdot \mathbf{B} = 0 \]